Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.3-a+b-x^n-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [385]. This is test number [26].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (385)	% 0.00 (0)
Mathematica	% 99.48 (383)	% 0.52 (2)
Maple	% 51.17 (197)	% 48.83 (188)
Maxima	% 43.38 (167)	% 56.62 (218)
Fricas	% 55.58 (214)	% 44.42 (171)
Sympy	% 37.40 (144)	% 62.60 (241)
Giac	% 34.81 (134)	% 65.19 (251)
Mupad	% 44.16 (170)	% 55.84 (215)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

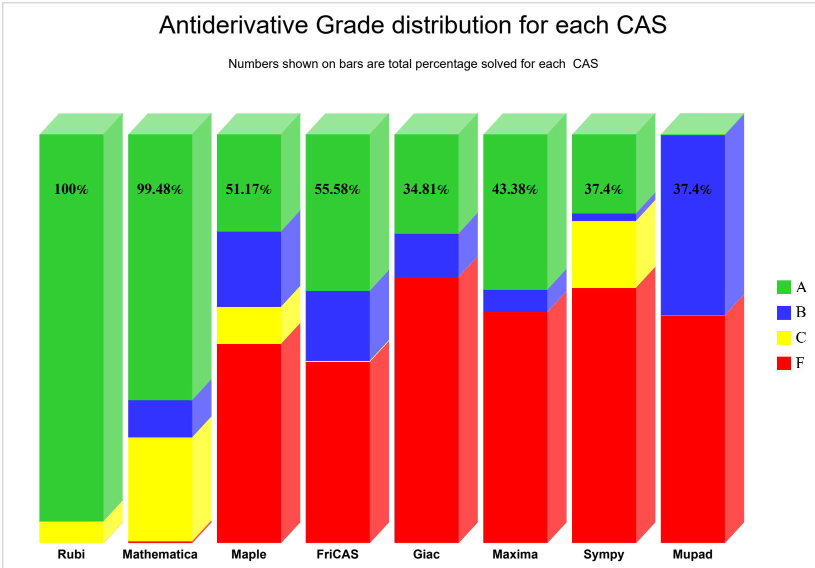
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

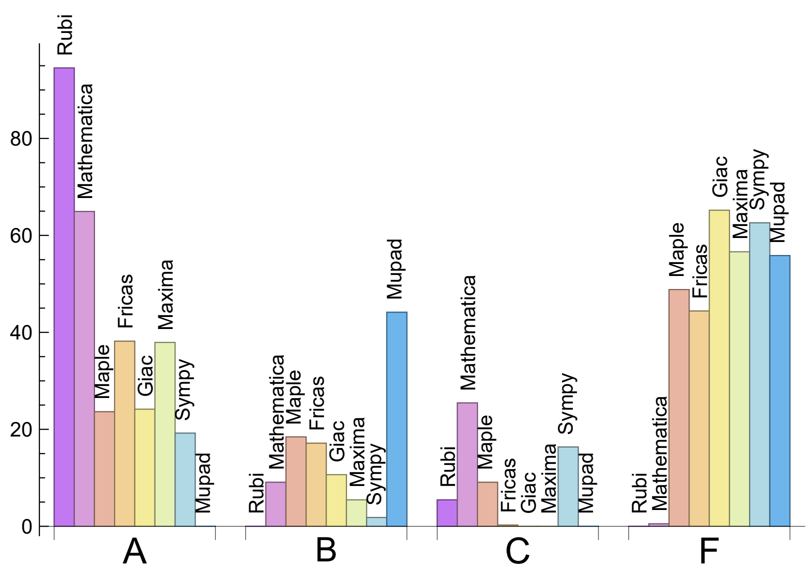
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.55	0.00	5.45	0.00
Mathematica	64.94	9.09	25.45	0.52
Maple	23.64	18.44	9.09	48.83
Maxima	37.92	5.45	0.00	56.62
Fricas	38.18	17.14	0.26	44.42
Sympy	19.22	1.82	16.36	62.60
Giac	24.16	10.65	0.00	65.19
Mupad	0.00	44.16	0.00	55.84

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	50.00 %	50.00 %	0.00 %
Maple	188	99.47 %	0.53 %	0.00 %
Maxima	218	100.00 %	0.00 %	0.00 %
Fricas	171	56.14 %	42.11 %	1.75 %
Sympy	241	47.72 %	46.47 %	5.81 %
Giac	251	89.64 %	0.00 %	10.36 %
Mupad	215	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

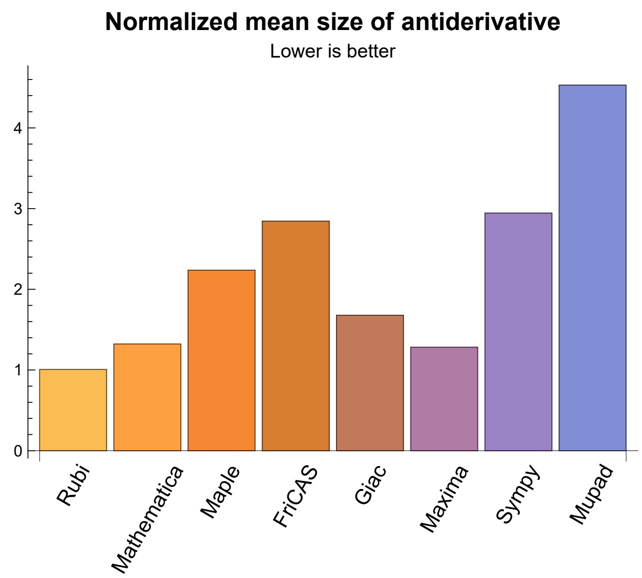
1.3 Performance

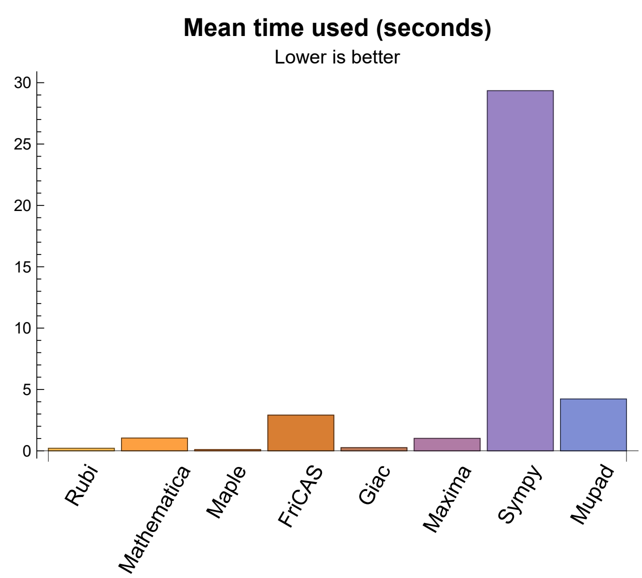
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	169.10	1.01	118.00	1.00
Mathematica	1.04	185.35	1.32	129.00	0.96
Maple	0.10	409.23	2.24	198.00	1.34
Maxima	1.02	185.69	1.28	144.00	1.14
Fricas	2.91	554.92	2.84	232.00	2.11
Sympy	29.35	314.96	2.94	153.50	1.20
Giac	0.26	261.01	1.68	200.00	1.36
Mupad	4.23	1166.19	4.53	148.50	1.22

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {34, 35, 36, 37, 38, 39, 86, 87, 88, 91, 92, 98, 99, 102, 103, 109, 110, 111, 115, 116}

Mathematica {34, 35, 36, 37, 38, 39, 75, 79, 81, 83, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 141, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 218, 221, 222, 269, 274, 307, 312, 317, 318, 319, 320, 328, 334, 335, 352, 384}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

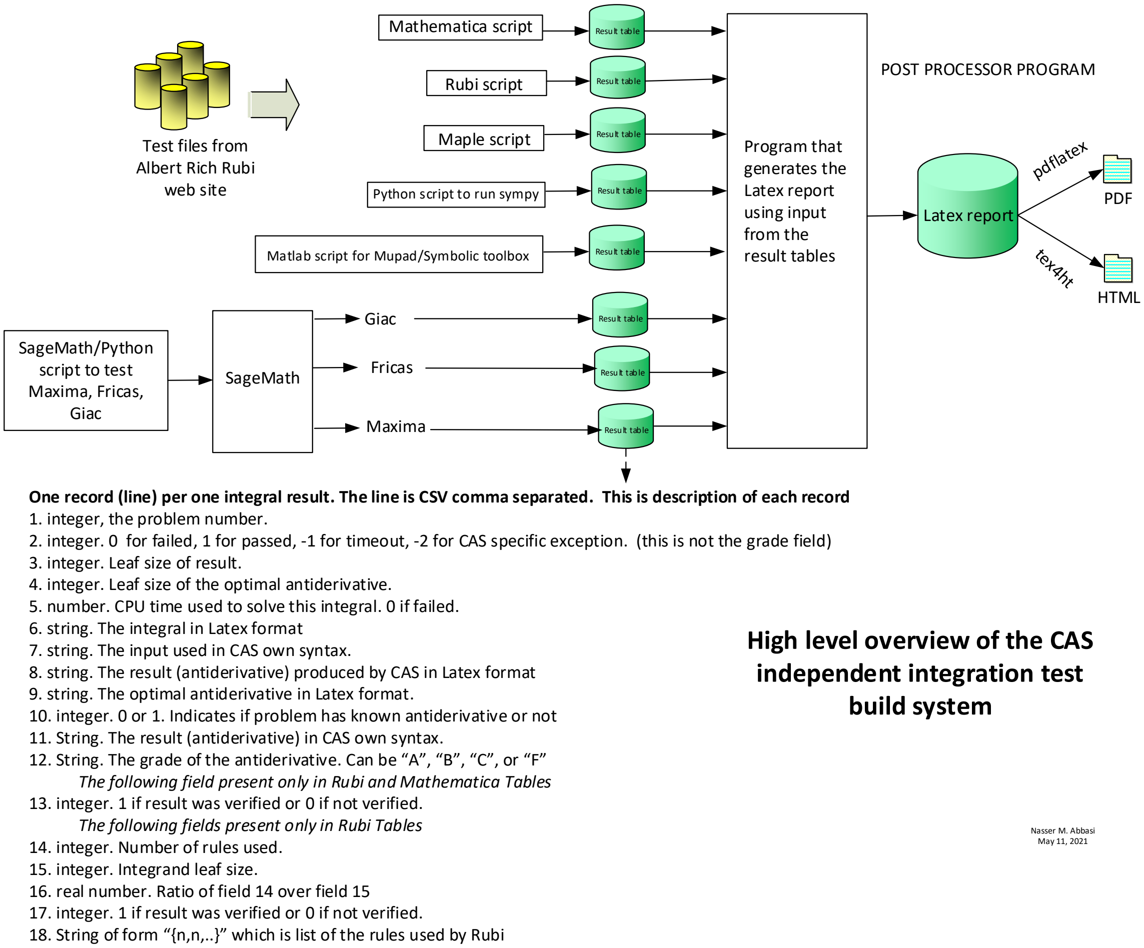
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 93, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { }

C grade: { 34, 35, 36, 37, 38, 39, 86, 87, 88, 91, 92, 98, 99, 102, 103, 109, 110, 111, 114, 115, 116 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 92, 103, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 195, 198, 216, 217, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 259, 266, 267, 268, 270, 272, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 313, 314, 315, 316, 320, 321, 322, 324, 325, 326, 327, 329, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 378, 380, 381, 383, 385 }

B grade: { 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 133, 136, 137, 142, 143, 144, 218, 221, 222, 269, 312, 317, 318, 319, 328, 334, 335, 352, 354, 384 }

C grade: { 27, 29, 34, 35, 36, 37, 38, 39, 56, 57, 59, 86, 87, 88, 90, 91, 98, 99, 100, 101, 102, 109, 110, 111, 112, 113, 114, 115, 141, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 223, 252, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 271, 273, 279, 298, 299, 300, 307, 323, 330, 331, 332, 333, 375, 377, 379 }

F grade: { 336, 382 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 17, 18, 19, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 163, 164, 165, 170, 171, 172, 191, 192, 224, 248, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 349, 351, 353, 355, 356, 357, 359, 361, 366, 369, 371, 374, 376, 378, 379 }

B grade: { 11, 12, 14, 15, 16, 20, 21, 22, 23, 157, 158, 160, 161, 162, 166, 167, 168, 169, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 324, 340, 341, 342, 363, 365, 367, 373, 375, 377 }

C grade: { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 216, 343, 344, 345, 346, 347, 348, 350, 352, 354, 358, 360, 362, 364, 368, 370, 372 }

F grade: { 13, 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 44, 45, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 85, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 240, 241, 245, 248, 252, 253, 255, 259, 260, 261, 262, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379 }

B grade: { 26, 27, 28, 32, 33, 40, 41, 42, 43, 46, 47, 56, 57, 58, 70, 71, 72, 73, 246, 247, 254 }

C grade: { }

F grade: { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 40, 41, 44, 45, 46, 47, 56, 57, 58, 60, 61, 62, 63, 70, 71, 72, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 153, 154, 155, 156, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 266, 267, 268, 270, 275, 276, 277, 278, 279, 280, 287, 322, 323, 324, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 381, 385 }

B grade: { 7, 12, 13, 20, 21, 22, 23, 26, 27, 28, 29, 36, 42, 43, 59, 73, 74, 86, 87, 88, 98, 99, 110, 111, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 191, 192, 193, 194, 205, 206, 216, 230, 237, 251, 256, 257, 258, 263, 264, 265, 284, 285, 286, 292, 293, 294, 321, 372 }

C grade: { 380 }

F grade: { 34, 35, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 333, 334, 335, 336, 382, 383, 384 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 224, 225, 226, 227, 231, 232, 233, 238, 239, 240, 241, 245, 246, 247, 248, 255, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 293, 294, 324 }

B grade: { 30, 60, 234, 254, 261, 262, 270 }

C grade: { 27, 28, 29, 40, 41, 48, 49, 50, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 135, 140, 141, 219, 220, 288, 289, 290, 295, 298, 299, 300, 301, 308, 313, 314, 315, 316, 349, 351, 352, 353, 354, 355, 356, 359, 361, 362, 363, 364, 365, 366, 369, 371, 372, 373, 374, 378 }

F grade: { 19, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 61, 62, 63, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 142, 143, 144, 145, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 291, 292, 296, 297, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 350, 357, 358, 360, 367, 368, 370, 375, 376, 377, 379, 380, 381, 382, 383, 384, 385 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 165, 168, 169, 170, 171, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 265, 270, 275, 276, 277, 278, 279, 280, 340, 341, 346, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 368, 370, 378 }

B grade: { 160, 161, 166, 167, 172, 227, 230, 237, 244, 248, 257, 264, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 342, 343, 344, 345, 347, 355, 356, 357, 365, 366, 367, 369, 371, 372, 373, 374, 375, 376, 377 }

C grade: { }

F grade: { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 379, 380, 381, 382, 383, 384, 385 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 316, 324, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 374, 376, 378, 379 }

C grade: { }

F grade: { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 368, 370, 372, 373, 375, 377, 380, 381, 382, 383, 384, 385 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	97	104	97	87
normalized size	1	1.00	1.00	1.03	1.02	1.03	1.11	1.03	0.93
time (sec)	N/A	0.072	0.049	0.038	0.475	0.363	0.125	0.173	0.051

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	74	80	74	66
normalized size	1	1.00	1.00	1.04	1.00	1.06	1.14	1.06	0.94
time (sec)	N/A	0.043	0.015	0.044	0.562	0.364	0.079	0.148	0.034

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	51	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96
time (sec)	N/A	0.028	0.008	0.044	0.481	0.359	0.074	0.189	0.048

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	25
normalized size	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.013	0.005	0.040	0.536	0.353	0.066	0.149	0.037

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	195	128	369	71	133	123
normalized size	1	1.00	0.89	1.35	0.89	2.56	0.49	0.92	0.85
time (sec)	N/A	0.094	0.123	0.048	1.277	0.445	0.417	0.202	1.383

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	158	537	97	160	143
normalized size	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.082	0.096	0.052	1.157	0.447	0.578	0.191	1.398

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	175	249	192	743	133	180	173
normalized size	1	1.00	0.89	1.26	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.106	0.131	0.059	1.434	0.460	0.777	0.191	1.396

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	132	139	132	116
normalized size	1	1.00	1.00	1.02	1.02	1.08	1.14	1.08	0.95
time (sec)	N/A	0.071	0.019	0.050	0.547	0.379	0.091	0.159	1.199

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	91	90	91	75
normalized size	1	1.00	1.00	1.06	1.00	1.11	1.10	1.11	0.91
time (sec)	N/A	0.046	0.012	0.037	0.707	0.370	0.084	0.208	0.042

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	51	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96
time (sec)	N/A	0.029	0.007	0.039	0.574	0.356	0.072	0.159	0.045

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	189	505	156	211	152
normalized size	1	1.00	0.97	1.93	1.09	2.92	0.90	1.22	0.88
time (sec)	N/A	0.128	0.142	0.046	1.230	0.482	0.679	0.194	1.386

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	210	367	226	771	189	233	191
normalized size	1	1.00	1.03	1.81	1.11	3.80	0.93	1.15	0.94
time (sec)	N/A	0.244	0.221	0.062	1.282	0.463	1.135	0.190	1.412

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	234	0	267	1067	233	264	249
normalized size	1	1.00	0.91	0.00	1.03	4.14	0.90	1.02	0.97
time (sec)	N/A	0.233	0.276	180.000	1.321	0.449	1.623	0.197	1.428

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	253	661	364	873	371	391	250
normalized size	1	1.00	1.00	2.62	1.44	3.46	1.47	1.55	0.99
time (sec)	N/A	0.191	0.127	0.047	1.189	0.466	1.310	0.201	1.430

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	203	486	273	700	257	296	192
normalized size	1	1.00	0.98	2.34	1.31	3.37	1.24	1.42	0.92
time (sec)	N/A	0.148	0.106	0.046	1.319	0.444	1.003	0.185	1.402

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	190	507	156	211	152
normalized size	1	1.00	0.97	1.93	1.10	2.93	0.90	1.22	0.88
time (sec)	N/A	0.124	0.149	0.043	1.321	0.452	0.690	0.324	1.375

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	128	390	71	133	123
normalized size	1	1.00	0.89	1.34	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.078	0.075	0.046	1.075	0.434	0.436	0.191	1.379

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	293	254	447	278	1364
normalized size	1	1.00	0.78	0.77	1.02	0.88	1.55	0.97	4.74
time (sec)	N/A	0.147	0.107	0.050	1.235	0.497	79.722	0.266	7.705

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	336	406	489	432	0	443	2589
normalized size	1	1.00	0.97	1.17	1.41	1.25	0.00	1.28	7.48
time (sec)	N/A	0.270	0.235	0.054	1.298	7.437	0.000	0.203	16.807

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	313	905	509	1619	546	529	416
normalized size	1	1.00	0.98	2.83	1.59	5.06	1.71	1.65	1.30
time (sec)	N/A	0.298	0.303	0.060	1.346	0.470	12.429	0.190	0.390

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	260	708	397	1316	405	412	302
normalized size	1	1.00	0.97	2.65	1.49	4.93	1.52	1.54	1.13
time (sec)	N/A	0.226	0.238	0.057	1.156	0.487	8.537	0.190	1.493

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	227	529	306	1027	291	319	240
normalized size	1	1.00	0.97	2.26	1.31	4.39	1.24	1.36	1.03
time (sec)	N/A	0.220	0.156	0.053	1.219	0.458	4.330	0.200	0.296

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	205	367	220	768	189	227	191
normalized size	1	1.00	1.01	1.81	1.08	3.78	0.93	1.12	0.94
time (sec)	N/A	0.231	0.198	0.053	1.395	0.467	2.556	0.213	1.467

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	158	537	97	160	143
normalized size	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.084	0.091	0.054	1.439	0.465	1.418	0.171	1.428

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	337	406	489	440	0	443	2492
normalized size	1	1.00	0.97	1.17	1.41	1.27	0.00	1.28	7.20
time (sec)	N/A	0.255	0.195	0.056	1.290	7.375	0.000	0.201	15.930

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	381	606	784	897	0	664	3637
normalized size	1	1.00	0.91	1.45	1.87	2.14	0.00	1.58	8.68
time (sec)	N/A	0.493	0.630	0.062	1.257	85.305	0.000	0.222	24.310

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	62	0	322	399	80	0	-1
normalized size	1	1.00	0.55	0.00	2.88	3.56	0.71	0.00	-0.01
time (sec)	N/A	0.032	0.106	0.429	1.219	0.436	4.984	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	134	0	244	363	76	0	-1
normalized size	1	1.00	1.47	0.00	2.68	3.99	0.84	0.00	-0.01
time (sec)	N/A	0.020	0.084	0.366	1.216	0.444	5.525	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	62	0	130	372	70	0	-1
normalized size	1	1.00	0.73	0.00	1.53	4.38	0.82	0.00	-0.01
time (sec)	N/A	0.013	0.037	0.387	1.254	0.443	16.137	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	28	25	50	44	190	0	27
normalized size	1	1.00	0.60	0.53	1.06	0.94	4.04	0.00	0.57
time (sec)	N/A	0.009	0.015	0.046	0.523	0.420	91.681	0.000	1.348

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	37	85	69	0	0	44
normalized size	1	1.00	0.73	0.67	1.55	1.25	0.00	0.00	0.80
time (sec)	N/A	0.013	0.021	0.043	0.624	0.417	0.000	0.000	1.425

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	51	48	119	91	0	0	58
normalized size	1	1.00	0.69	0.65	1.61	1.23	0.00	0.00	0.78
time (sec)	N/A	0.019	0.022	0.048	0.584	0.431	0.000	0.000	1.391

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	62	59	153	113	0	0	73
normalized size	1	1.00	0.67	0.63	1.65	1.22	0.00	0.00	0.78
time (sec)	N/A	0.026	0.026	0.046	0.538	0.431	0.000	0.000	1.371

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	483	56	232	0	0	0	0	0	-1
normalized size	1	0.12	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.384	0.607	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	464	55	217	0	0	0	0	0	-1
normalized size	1	0.12	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	0.116	0.609	0.000	34.203	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	58	151	0	0	644	0	0	-1
normalized size	1	0.15	0.38	0.00	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.026	0.134	0.698	0.000	29.485	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	58	153	0	0	0	0	0	-1
normalized size	1	0.13	0.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.133	0.584	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	58	213	0	0	0	0	0	-1
normalized size	1	0.12	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.029	0.139	0.681	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	58	240	0	0	0	0	0	-1
normalized size	1	0.12	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.176	0.600	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	151	0	552	421	126	0	-1
normalized size	1	1.00	1.09	0.00	3.97	3.03	0.91	0.00	-0.01
time (sec)	N/A	0.057	0.137	0.375	1.527	0.881	9.174	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	141	0	436	399	121	0	-1
normalized size	1	1.00	1.18	0.00	3.63	3.32	1.01	0.00	-0.01
time (sec)	N/A	0.042	0.078	0.384	1.396	0.815	7.599	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	137	0	296	412	0	0	-1
normalized size	1	1.00	1.21	0.00	2.62	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.082	0.532	1.373	0.843	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	131	0	180	521	0	0	-1
normalized size	1	1.00	1.19	0.00	1.64	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.063	0.533	1.354	0.912	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	40	37	105	67	0	0	44
normalized size	1	1.00	0.53	0.49	1.38	0.88	0.00	0.00	0.58
time (sec)	N/A	0.021	0.015	0.042	0.498	0.861	0.000	0.000	1.432

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	51	48	155	91	0	0	56
normalized size	1	1.00	0.49	0.46	1.48	0.87	0.00	0.00	0.53
time (sec)	N/A	0.035	0.032	0.050	0.585	0.908	0.000	0.000	1.389

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	59	206	113	0	0	71
normalized size	1	1.00	0.63	0.60	2.10	1.15	0.00	0.00	0.72
time (sec)	N/A	0.035	0.035	0.046	0.574	0.914	0.000	0.000	1.436

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	73	70	257	135	0	0	86
normalized size	1	1.00	0.62	0.60	2.20	1.15	0.00	0.00	0.74
time (sec)	N/A	0.044	0.040	0.049	0.510	0.990	0.000	0.000	1.427

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	97	0	0	0	168	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	1.79	0.00	-0.01
time (sec)	N/A	0.034	0.049	0.407	0.000	0.789	8.409	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	126	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	1.34	0.00	-0.01
time (sec)	N/A	0.034	0.046	0.382	0.000	0.646	5.592	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	121	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	1.29	0.00	-0.01
time (sec)	N/A	0.034	0.047	0.368	0.000	0.631	5.758	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.045	0.556	0.000	0.637	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.069	0.510	0.000	0.698	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	85	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.118	0.535	0.000	0.773	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.126	0.566	0.000	0.646	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	106	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.141	0.553	0.000	0.693	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	75	0	406	482	170	0	-1
normalized size	1	1.00	0.43	0.00	2.33	2.77	0.98	0.00	-0.01
time (sec)	N/A	0.059	0.068	0.374	1.678	0.714	10.424	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	72	0	322	424	82	0	-1
normalized size	1	1.00	0.51	0.00	2.28	3.01	0.58	0.00	-0.01
time (sec)	N/A	0.045	0.095	0.375	1.437	0.872	5.347	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	141	0	244	362	78	0	-1
normalized size	1	1.00	1.27	0.00	2.20	3.26	0.70	0.00	-0.01
time (sec)	N/A	0.030	0.154	0.395	1.198	0.653	4.439	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	0	134	488	71	0	-1
normalized size	1	1.00	0.62	0.00	1.35	4.93	0.72	0.00	-0.01
time (sec)	N/A	0.024	0.052	0.380	1.224	0.686	12.828	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	51	54	190	0	33
normalized size	1	1.00	0.79	0.72	1.09	1.15	4.04	0.00	0.70
time (sec)	N/A	0.010	0.036	0.049	0.470	0.828	82.052	0.000	1.370

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	57	86	87	0	0	87
normalized size	1	1.00	0.65	0.63	0.95	0.96	0.00	0.00	0.96
time (sec)	N/A	0.028	0.039	0.045	0.608	0.724	0.000	0.000	1.424

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	80	81	120	121	0	0	105
normalized size	1	1.00	0.66	0.67	0.99	1.00	0.00	0.00	0.87
time (sec)	N/A	0.035	0.034	0.042	0.499	0.579	0.000	0.000	1.460

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	100	105	154	155	0	0	132
normalized size	1	1.00	0.66	0.70	1.02	1.03	0.00	0.00	0.87
time (sec)	N/A	0.047	0.062	0.046	0.657	0.671	0.000	0.000	1.453

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	3.12	0.00	-0.01
time (sec)	N/A	0.023	0.052	0.420	0.000	0.665	10.988	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	2.05	0.00	-0.01
time (sec)	N/A	0.022	0.143	0.385	0.000	0.706	7.402	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.021	0.138	0.403	0.000	0.938	4.966	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.022	0.069	0.374	0.000	0.691	5.072	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	66	0	0	0	78	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.025	0.047	0.390	0.000	0.702	16.363	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	78	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.83	0.00	-0.01
time (sec)	N/A	0.028	0.044	0.400	0.000	0.661	117.701	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	238	0	672	717	270	0	-1
normalized size	1	1.00	0.91	0.00	2.56	2.74	1.03	0.00	-0.00
time (sec)	N/A	0.165	5.190	0.385	1.294	0.854	13.131	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	203	0	552	634	131	0	-1
normalized size	1	1.00	0.93	0.00	2.52	2.89	0.60	0.00	-0.00
time (sec)	N/A	0.166	5.171	0.385	1.479	0.632	7.275	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	172	0	436	554	126	0	-1
normalized size	1	1.00	0.98	0.00	2.49	3.17	0.72	0.00	-0.01
time (sec)	N/A	0.096	5.150	0.381	1.275	0.732	6.437	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	168	0	301	652	0	0	-1
normalized size	1	1.00	1.06	0.00	1.89	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.102	5.163	0.559	1.137	0.583	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	180	0	190	719	0	0	-1
normalized size	1	1.00	1.18	0.00	1.25	4.73	0.00	0.00	-0.01
time (sec)	N/A	0.068	5.247	0.550	1.193	0.736	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	126	76	109	103	0	0	148
normalized size	1	1.00	1.62	0.97	1.40	1.32	0.00	0.00	1.90
time (sec)	N/A	0.021	0.097	0.049	0.508	0.617	0.000	0.000	1.427

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	106	115	159	152	0	0	176
normalized size	1	1.00	0.61	0.66	0.91	0.87	0.00	0.00	1.01
time (sec)	N/A	0.073	5.106	0.046	0.712	0.726	0.000	0.000	1.450

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	138	156	210	200	0	0	217
normalized size	1	1.00	0.65	0.74	1.00	0.95	0.00	0.00	1.03
time (sec)	N/A	0.127	5.176	0.047	0.503	0.645	0.000	0.000	1.430

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	169	197	261	246	0	0	257
normalized size	1	1.00	0.67	0.78	1.03	0.97	0.00	0.00	1.02
time (sec)	N/A	0.207	5.153	0.049	0.643	0.830	0.000	0.000	1.481

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	177	0	0	0	418	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	3.10	0.00	-0.01
time (sec)	N/A	0.067	3.587	0.416	0.000	0.667	12.604	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	171	0	0	0	270	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	2.03	0.00	-0.01
time (sec)	N/A	0.077	5.189	0.373	0.000	0.669	7.059	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	179	0	0	0	131	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.062	3.997	0.388	0.000	0.851	3.938	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	104	0	0	0	126	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.075	5.135	0.388	0.000	0.691	4.271	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	171	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	3.763	0.605	0.000	0.679	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	128	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	5.162	0.604	0.000	0.614	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	120	134	182	166	0	0	271
normalized size	1	1.00	1.10	1.23	1.67	1.52	0.00	0.00	2.49
time (sec)	N/A	0.036	0.043	0.053	0.626	0.667	0.000	0.000	1.556

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	62	655	0	0	643	0	0	-1
normalized size	1	0.19	1.98	0.00	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.028	1.232	0.701	0.000	15.642	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	60	443	0	0	535	0	0	-1
normalized size	1	0.22	1.62	0.00	0.00	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.711	0.559	0.000	1.746	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	59	161	0	0	469	0	0	-1
normalized size	1	0.25	0.69	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.224	0.570	0.000	1.015	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	207	168	0	0	0	0	0	-1
normalized size	1	1.40	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.151	0.526	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	238	256	0	0	0	0	0	-1
normalized size	1	1.33	1.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.801	0.574	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	621	621	0	0	0	0	0	-1
normalized size	1	2.75	2.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.580	2.673	0.558	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	1172	277	0	0	0	0	0	-1
normalized size	1	4.19	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.642	5.746	0.605	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	-1
normalized size	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.611	0.612	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	-1
normalized size	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.166	0.587	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.039	0.613	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	-1
normalized size	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.279	0.643	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0	-1
normalized size	1	1.00	6.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.867	0.629	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	62	698	0	0	819	0	0	-1
normalized size	1	0.18	1.99	0.00	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.029	1.083	0.398	0.000	12.135	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	60	450	0	0	631	0	0	-1
normalized size	1	0.20	1.50	0.00	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.643	0.582	0.000	1.335	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	241	78	0	0	0	0	0	-1
normalized size	1	1.32	0.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.041	0.538	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	276	99	0	0	0	0	0	-1
normalized size	1	1.27	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.145	0.555	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	625	625	0	0	0	0	0	-1
normalized size	1	2.39	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.934	2.032	0.600	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	1214	288	0	0	0	0	0	-1
normalized size	1	3.75	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.692	5.944	0.594	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0	-1
normalized size	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.323	0.513	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	-1
normalized size	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.211	0.572	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0	-1
normalized size	1	1.00	6.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.268	0.573	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	-1
normalized size	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.575	0.583	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0	-1
normalized size	1	1.00	8.87	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.986	0.599	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	541	62	1171	0	0	0	0	0	-1
normalized size	1	0.11	2.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.027	2.341	0.395	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	62	908	0	0	1246	0	0	-1
normalized size	1	0.14	1.98	0.00	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.028	1.838	0.412	0.000	79.231	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	62	651	0	0	954	0	0	-1
normalized size	1	0.16	1.66	0.00	0.00	2.44	0.00	0.00	-0.00
time (sec)	N/A	0.028	1.164	0.607	0.000	8.886	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	276	79	0	0	0	0	0	-1
normalized size	1	1.27	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.033	0.572	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	326	153	0	0	0	0	0	-1
normalized size	1	1.22	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.219	0.546	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	167	168	0	0	0	0	0	-1
normalized size	1	0.54	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.380	0.579	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	377	428	428	0	0	0	0	0	-1
normalized size	1	1.14	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.742	3.084	0.592	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	1990	337	0	0	0	0	0	-1
normalized size	1	4.30	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.662	5.878	0.559	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	316	0	0	0	0	0	-1
normalized size	1	1.00	5.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.519	0.568	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	431	0	0	0	0	0	-1
normalized size	1	1.00	7.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.617	0.560	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	442	0	0	0	0	0	-1
normalized size	1	1.00	7.49	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.772	0.590	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0	-1
normalized size	1	1.00	8.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	1.095	0.585	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0	-1
normalized size	1	1.00	8.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	1.616	0.585	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.043	0.505	0.000	77.528	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.031	0.537	0.000	3.007	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.020	0.552	0.000	47.913	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.025	0.535	0.000	2.249	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.062	0.538	0.000	51.161	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.022	0.526	0.000	3.995	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.042	0.561	0.000	64.290	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.050	0.552	0.000	2.820	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.037	0.559	0.000	64.256	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.046	0.496	0.000	2.847	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.036	0.549	0.000	66.283	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
normalized size	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.432	0.556	0.000	1.537	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	106	0	0	0	0	0	-1
normalized size	1	1.05	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.079	0.529	0.000	1.160	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	90	0	0	0	75	0	-1
normalized size	1	1.01	1.07	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.039	0.037	0.396	0.000	0.989	82.864	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.204	0.563	0.000	1.161	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.258	0.580	0.000	0.882	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	137	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	5.077	0.577	0.000	1.324	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	5.043	0.540	0.000	1.226	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	-1
normalized size	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.042	0.046	0.390	0.000	0.759	100.310	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	196	0	0	0	34	0	41
normalized size	1	1.00	4.45	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.010	0.198	0.361	0.000	1.264	15.563	0.000	1.346

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.250	0.566	0.000	0.839	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.257	0.525	0.000	1.202	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.413	0.557	0.000	1.192	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
normalized size	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.019	0.038	0.041	0.000	1.323	0.000	0.000	1.905

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	98	107	98	88
normalized size	1	1.00	1.00	1.03	1.02	1.04	1.14	1.04	0.94
time (sec)	N/A	0.068	0.024	0.040	0.651	0.897	0.090	0.156	1.303

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	74	76	74	66
normalized size	1	1.00	1.00	1.04	1.00	1.06	1.09	1.06	0.94
time (sec)	N/A	0.049	0.017	0.039	0.647	0.962	0.084	0.150	1.242

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	53	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96
time (sec)	N/A	0.029	0.012	0.036	0.605	0.773	0.078	0.161	0.048

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	25
normalized size	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.014	0.009	0.045	0.493	0.839	0.065	0.150	0.036

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	212	639	87	245	720
normalized size	1	1.00	0.88	1.19	0.95	2.87	0.39	1.10	3.23
time (sec)	N/A	0.151	0.179	0.048	1.278	1.318	0.659	0.165	1.480

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	236	711	112	266	740
normalized size	1	1.00	0.87	1.20	0.96	2.90	0.46	1.09	3.02
time (sec)	N/A	0.147	0.202	0.053	1.158	1.186	0.849	0.169	1.520

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	243	314	271	787	151	286	762
normalized size	1	1.00	0.89	1.15	0.99	2.88	0.55	1.05	2.79
time (sec)	N/A	0.174	0.223	0.054	1.211	1.277	1.031	0.194	1.581

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	163	158	173	185	173	146
normalized size	1	1.00	1.00	1.06	1.03	1.12	1.20	1.12	0.95
time (sec)	N/A	0.114	0.033	0.041	0.546	0.606	0.117	0.150	0.067

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	132	139	132	116
normalized size	1	1.00	1.00	1.02	1.02	1.08	1.14	1.08	0.95
time (sec)	N/A	0.077	0.023	0.035	0.544	0.732	0.098	0.151	1.297

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	91	97	91	75
normalized size	1	1.00	1.00	1.06	1.00	1.11	1.18	1.11	0.91
time (sec)	N/A	0.049	0.017	0.041	0.695	0.995	0.090	0.166	0.046

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	53	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96
time (sec)	N/A	0.030	0.008	0.038	0.478	0.809	0.080	0.149	0.045

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	286	1239	187	353	1081
normalized size	1	1.00	0.91	1.72	1.13	4.90	0.74	1.40	4.27
time (sec)	N/A	0.194	0.131	0.048	1.285	1.126	1.119	0.198	1.485

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	298	475	319	1335	219	376	1254
normalized size	1	1.00	1.02	1.63	1.10	4.59	0.75	1.29	4.31
time (sec)	N/A	0.366	0.216	0.056	1.104	1.267	1.979	0.174	1.536

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	319	499	361	1411	264	407	1401
normalized size	1	1.00	0.91	1.43	1.03	4.04	0.76	1.17	4.01
time (sec)	N/A	0.266	0.227	0.058	1.220	1.398	5.850	0.212	1.660

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	322	837	489	2477	435	617	1822
normalized size	1	1.00	0.97	2.52	1.47	7.46	1.31	1.86	5.49
time (sec)	N/A	0.267	0.224	0.047	1.437	1.305	3.585	0.484	1.515

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	271	627	385	1855	303	481	1433
normalized size	1	1.00	0.94	2.18	1.34	6.44	1.05	1.67	4.98
time (sec)	N/A	0.223	0.166	0.046	1.210	1.157	1.696	0.173	1.488

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	287	1240	187	353	1081
normalized size	1	1.00	0.91	1.72	1.13	4.90	0.74	1.40	4.27
time (sec)	N/A	0.190	0.132	0.046	1.093	1.117	1.084	0.177	1.466

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	212	639	87	245	720
normalized size	1	1.00	0.88	1.19	0.95	2.87	0.39	1.10	3.23
time (sec)	N/A	0.138	0.151	0.048	1.126	1.315	0.610	0.165	0.221

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	365	1356	0	437	6153
normalized size	1	1.00	0.76	0.71	0.81	3.02	0.00	0.97	13.70
time (sec)	N/A	0.268	0.143	0.056	1.434	1.361	0.000	0.209	2.755

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	498	550	481	3299	0	667	21975
normalized size	1	1.00	0.97	1.07	0.94	6.43	0.00	1.30	42.84
time (sec)	N/A	0.420	0.335	0.056	1.566	46.363	0.000	0.195	4.004

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	391	1118	644	3222	0	798	2490
normalized size	1	1.00	0.96	2.75	1.58	7.92	0.00	1.96	6.12
time (sec)	N/A	0.396	0.453	0.060	1.446	1.008	0.000	0.179	1.707

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	341	885	521	2580	471	642	2043
normalized size	1	1.00	0.96	2.48	1.46	7.23	1.32	1.80	5.72
time (sec)	N/A	0.367	0.349	0.062	1.454	1.040	47.538	0.174	0.302

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	301	669	405	1938	337	496	1616
normalized size	1	1.00	0.95	2.11	1.28	6.11	1.06	1.56	5.10
time (sec)	N/A	0.317	0.254	0.055	1.309	0.897	6.989	0.179	1.530

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	297	475	319	1335	219	376	1254
normalized size	1	1.00	1.02	1.63	1.10	4.59	0.75	1.29	4.31
time (sec)	N/A	0.377	0.227	0.058	1.261	1.085	2.174	0.168	0.298

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	236	711	112	266	740
normalized size	1	1.00	0.87	1.20	0.96	2.90	0.46	1.09	3.02
time (sec)	N/A	0.153	0.172	0.053	1.348	0.878	0.965	0.166	1.533

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	499	550	470	3299	0	667	21975
normalized size	1	1.00	0.97	1.07	0.92	6.43	0.00	1.30	42.84
time (sec)	N/A	0.428	0.347	0.058	1.457	36.676	0.000	0.217	3.817

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	629	784	670	0	0	967	37266
normalized size	1	1.00	1.06	1.32	1.12	0.00	0.00	1.62	62.53
time (sec)	N/A	0.739	6.192	0.066	1.292	0.000	0.000	0.218	5.617

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	290	408	0	0	0	0	-1
normalized size	1	1.00	0.90	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.820	0.310	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	341	311	0	0	0	0	-1
normalized size	1	1.00	1.23	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.461	0.293	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	155	259	0	0	0	0	-1
normalized size	1	1.00	0.65	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.209	0.314	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	-1
normalized size	1	1.00	0.96	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.101	0.309	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	381	301	0	0	0	0	-1
normalized size	1	1.00	1.36	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.316	0.362	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	422	361	0	0	0	0	-1
normalized size	1	1.00	1.26	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.853	0.320	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	346	322	0	0	0	0	-1
normalized size	1	1.00	0.37	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.655	0.518	0.477	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	881	881	161	273	0	0	0	0	-1
normalized size	1	1.00	0.18	0.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.880	0.186	0.327	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	742	742	161	191	0	0	0	0	-1
normalized size	1	1.00	0.22	0.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	0.032	0.332	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	913	913	331	313	0	0	0	0	-1
normalized size	1	1.00	0.36	0.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.132	0.285	0.286	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	976	976	429	371	0	0	0	0	-1
normalized size	1	1.00	0.44	0.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.683	0.992	0.337	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	477	540	0	0	0	0	-1
normalized size	1	1.00	1.12	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.918	0.292	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	396	412	0	0	0	0	-1
normalized size	1	1.00	1.08	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.669	0.340	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	342	329	0	0	0	0	-1
normalized size	1	1.00	1.11	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.436	0.339	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	233	294	0	0	0	0	-1
normalized size	1	1.00	0.84	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.130	0.306	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	386	322	0	0	0	0	-1
normalized size	1	1.00	1.25	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.280	0.336	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	374	375	0	0	0	0	-1
normalized size	1	1.00	1.03	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.404	0.515	0.351	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0	-1
normalized size	1	1.00	0.87	1.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.880	0.349	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	155	103	0	315	0	0	-1
normalized size	1	1.00	1.50	1.00	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.171	0.208	0.000	2.745	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	155	158	0	339	0	0	-1
normalized size	1	1.00	1.34	1.36	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.167	0.221	0.000	1.917	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	364	0	0	2381	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	11.28	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.629	0.587	0.000	9.033	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	161	0	0	844	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	4.88	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.180	0.575	0.000	1.061	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	84	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.045	0.566	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	256	0	0	0	0	0	-1
normalized size	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.577	0.598	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	621	0	0	0	0	0	-1
normalized size	1	1.00	3.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	2.368	0.603	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	231	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	5.439	0.615	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	294	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.698	0.557	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	346	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.321	0.572	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.166	0.569	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	161	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.039	0.550	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	332	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.273	0.592	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	430	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.401	0.773	0.624	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	560	0	0	3308	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	11.81	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.956	0.430	0.000	45.476	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	358	0	0	1667	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	7.25	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.638	0.646	0.000	3.263	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	78	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.060	0.608	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	99	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.140	0.595	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	625	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	2.265	0.596	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	1216	0	0	0	0	0	-1
normalized size	1	1.00	4.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	5.005	0.609	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	392	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.591	0.410	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	341	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.490	0.589	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	233	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.314	0.657	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	337	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.344	0.675	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	387	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.579	0.661	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	211	0	208	0	0	-1
normalized size	1	1.00	0.83	3.98	0.00	3.92	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.019	3.245	0.000	16.992	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.025	0.619	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
normalized size	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.263	0.553	0.000	1.815	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	119	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.133	0.056	0.532	0.000	1.767	166.026	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	-1
normalized size	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.041	0.033	0.376	0.000	1.453	61.010	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.217	0.560	0.000	1.307	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.242	0.565	0.000	1.191	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	48	0	0	0	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.085	0.020	0.566	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	118	248	164	306	454	0	173
normalized size	1	1.00	0.83	1.73	1.15	2.14	3.17	0.00	1.21
time (sec)	N/A	0.123	0.190	0.063	1.453	1.193	57.222	0.000	2.593

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	84	191	126	208	121	0	99
normalized size	1	1.00	0.85	1.93	1.27	2.10	1.22	0.00	1.00
time (sec)	N/A	0.067	0.158	0.059	1.191	1.008	36.578	0.000	1.901

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	163	106	128	87	0	92
normalized size	1	1.00	0.70	2.20	1.43	1.73	1.18	0.00	1.24
time (sec)	N/A	0.048	0.042	0.057	1.371	1.496	41.308	0.000	1.962

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	74	50	99	42	64	58
normalized size	1	1.00	1.00	1.90	1.28	2.54	1.08	1.64	1.49
time (sec)	N/A	0.019	0.073	0.049	1.201	1.081	2.189	0.200	0.076

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	100	287	0	482	0	0	149
normalized size	1	1.00	0.96	2.76	0.00	4.63	0.00	0.00	1.43
time (sec)	N/A	0.111	0.244	0.108	0.000	0.970	0.000	0.000	1.631

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	122	943	0	801	0	0	1195
normalized size	1	1.00	0.83	6.41	0.00	5.45	0.00	0.00	8.13
time (sec)	N/A	0.209	0.386	0.066	0.000	1.357	0.000	0.000	2.263

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	330	1972	0	1749	0	820	1895
normalized size	1	1.00	1.55	9.26	0.00	8.21	0.00	3.85	8.90
time (sec)	N/A	0.340	0.823	0.068	0.000	2.251	0.000	0.412	3.729

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	353	190	380	1817	0	327
normalized size	1	1.00	0.97	2.15	1.16	2.32	11.08	0.00	1.99
time (sec)	N/A	0.142	0.213	0.060	1.488	1.240	123.483	0.000	3.878

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	106	260	152	268	534	0	197
normalized size	1	1.00	0.84	2.06	1.21	2.13	4.24	0.00	1.56
time (sec)	N/A	0.080	0.260	0.060	1.160	1.087	94.065	0.000	2.579

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	205	132	164	163	0	81
normalized size	1	1.00	0.73	2.05	1.32	1.64	1.63	0.00	0.81
time (sec)	N/A	0.063	0.091	0.057	1.326	1.027	56.146	0.000	2.512

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	100	63	100	92	0	34
normalized size	1	1.00	0.85	1.85	1.17	1.85	1.70	0.00	0.63
time (sec)	N/A	0.025	0.025	0.052	1.202	0.881	2.692	0.000	1.501

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	528	0	519	0	0	556
normalized size	1	1.00	0.96	4.98	0.00	4.90	0.00	0.00	5.25
time (sec)	N/A	0.128	0.236	0.061	0.000	1.081	0.000	0.000	1.684

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	143	834	0	769	0	0	448
normalized size	1	1.00	0.92	5.35	0.00	4.93	0.00	0.00	2.87
time (sec)	N/A	0.222	0.387	0.062	0.000	0.954	0.000	0.000	2.165

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	168	1817	0	1765	0	727	1664
normalized size	1	1.00	0.80	8.69	0.00	8.44	0.00	3.48	7.96
time (sec)	N/A	0.345	0.604	0.067	0.000	1.227	0.000	0.611	3.472

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	201	457	219	494	5513	0	487
normalized size	1	1.00	1.02	2.31	1.11	2.49	27.84	0.00	2.46
time (sec)	N/A	0.158	0.255	0.067	1.207	0.852	158.704	0.000	6.049

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	336	181	350	1841	0	271
normalized size	1	1.00	0.80	2.21	1.19	2.30	12.11	0.00	1.78
time (sec)	N/A	0.102	0.158	0.060	1.251	1.019	112.031	0.000	3.791

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	94	253	161	222	520	0	99
normalized size	1	1.00	0.75	2.02	1.29	1.78	4.16	0.00	0.79
time (sec)	N/A	0.077	0.112	0.060	1.505	0.964	82.773	0.000	3.480

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	120	78	139	99	0	34
normalized size	1	1.00	0.90	1.69	1.10	1.96	1.39	0.00	0.48
time (sec)	N/A	0.034	0.062	0.058	1.249	0.940	4.364	0.000	1.630

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	116	859	0	659	0	0	1427
normalized size	1	1.00	0.87	6.41	0.00	4.92	0.00	0.00	10.65
time (sec)	N/A	0.221	0.270	0.056	0.000	1.306	0.000	0.000	2.156

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	145	1323	0	1001	0	0	1153
normalized size	1	1.00	0.87	7.97	0.00	6.03	0.00	0.00	6.95
time (sec)	N/A	0.234	0.430	0.063	0.000	1.225	0.000	0.000	2.311

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	191	1638	0	1445	0	945	1476
normalized size	1	1.00	0.81	6.91	0.00	6.10	0.00	3.99	6.23
time (sec)	N/A	0.373	0.849	0.071	0.000	1.164	0.000	0.489	3.439

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	535	166	233	386	158	107
normalized size	1	1.00	0.75	4.25	1.32	1.85	3.06	1.25	0.85
time (sec)	N/A	0.090	0.147	0.063	1.327	0.981	89.999	0.204	1.727

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	348	129	158	114	99	63
normalized size	1	1.00	0.90	4.77	1.77	2.16	1.56	1.36	0.86
time (sec)	N/A	0.054	0.087	0.063	1.244	0.931	83.772	0.183	1.621

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	173	109	115	82	78	88
normalized size	1	1.00	1.04	3.39	2.14	2.25	1.61	1.53	1.73
time (sec)	N/A	0.033	0.039	0.059	1.298	0.982	60.252	0.197	1.977

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	98	44	71	66
normalized size	1	1.00	1.00	1.65	1.56	2.28	1.02	1.65	1.53
time (sec)	N/A	0.020	0.018	0.046	1.208	0.927	3.088	0.239	1.441

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	104	228	0	542	0	134	1183
normalized size	1	1.00	0.96	2.11	0.00	5.02	0.00	1.24	10.95
time (sec)	N/A	0.096	0.246	0.064	0.000	0.991	0.000	0.181	1.982

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	150	1135	0	1163	0	300	3813
normalized size	1	1.00	0.87	6.60	0.00	6.76	0.00	1.74	22.17
time (sec)	N/A	0.218	0.775	0.069	0.000	1.065	0.000	0.220	3.536

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	216	2269	0	2307	0	352	2890
normalized size	1	1.00	0.86	9.08	0.00	9.23	0.00	1.41	11.56
time (sec)	N/A	0.400	1.746	0.069	0.000	1.937	0.000	0.288	5.479

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	92	969	200	336	0	222	172
normalized size	1	1.00	0.70	7.34	1.52	2.55	0.00	1.68	1.30
time (sec)	N/A	0.101	0.075	0.067	1.259	0.977	0.000	0.209	1.908

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	90	81	789	164	272	0	160	120
normalized size	1	0.96	0.86	8.39	1.74	2.89	0.00	1.70	1.28
time (sec)	N/A	0.079	0.103	0.064	1.157	0.893	0.000	0.226	1.834

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	48	387	144	210	224	127	71
normalized size	1	1.00	0.63	5.09	1.89	2.76	2.95	1.67	0.93
time (sec)	N/A	0.050	0.024	0.056	1.374	1.052	81.344	0.196	2.438

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	36	198	85	156	71	86	34
normalized size	1	1.02	0.60	3.30	1.42	2.60	1.18	1.43	0.57
time (sec)	N/A	0.031	0.013	0.058	1.183	0.773	4.794	0.165	1.868

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	106	962	0	1075	0	200	3000
normalized size	1	1.00	0.72	6.54	0.00	7.31	0.00	1.36	20.41
time (sec)	N/A	0.194	0.072	0.074	0.000	1.166	0.000	0.206	2.684

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	164	3119	0	2321	0	424	4274
normalized size	1	1.00	0.73	13.92	0.00	10.36	0.00	1.89	19.08
time (sec)	N/A	0.323	0.145	0.073	0.000	2.002	0.000	0.256	6.202

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	239	5158	0	4093	0	516	8936
normalized size	1	1.00	0.75	16.12	0.00	12.79	0.00	1.61	27.92
time (sec)	N/A	0.525	0.321	0.079	0.000	5.028	0.000	0.309	9.488

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	145	1150	228	483	0	203	194
normalized size	1	1.00	1.01	8.04	1.59	3.38	0.00	1.42	1.36
time (sec)	N/A	0.151	0.430	0.061	1.341	0.984	0.000	0.256	2.050

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	118	97	588	190	407	0	163	144
normalized size	1	0.97	0.80	4.82	1.56	3.34	0.00	1.34	1.18
time (sec)	N/A	0.095	0.082	0.065	1.308	1.011	0.000	0.205	2.219

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	60	541	170	331	1479	145	87
normalized size	1	1.00	0.58	5.25	1.65	3.21	14.36	1.41	0.84
time (sec)	N/A	0.065	0.035	0.064	1.251	0.921	155.819	0.220	2.910

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	38	271	101	225	774	98	34
normalized size	1	1.04	0.48	3.43	1.28	2.85	9.80	1.24	0.43
time (sec)	N/A	0.038	0.025	0.068	1.291	0.592	7.925	0.239	1.722

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	118	1767	0	1990	0	247	5387
normalized size	1	1.00	0.59	8.79	0.00	9.90	0.00	1.23	26.80
time (sec)	N/A	0.315	0.084	0.071	0.000	5.603	0.000	0.219	4.622

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	178	4644	0	3887	0	576	5789
normalized size	1	1.00	0.62	16.18	0.00	13.54	0.00	2.01	20.17
time (sec)	N/A	0.448	0.189	0.080	0.000	5.602	0.000	0.326	8.729

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	239	7300	0	6171	0	523	4284
normalized size	1	1.00	0.58	17.85	0.00	15.09	0.00	1.28	10.47
time (sec)	N/A	0.702	0.387	0.079	0.000	14.874	0.000	0.285	8.227

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	167	253	0	890	0	0	4674
normalized size	1	1.00	1.36	2.06	0.00	7.24	0.00	0.00	38.00
time (sec)	N/A	0.094	1.401	0.096	0.000	2.164	0.000	0.000	22.224

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	155	0	247	0	0	478
normalized size	1	1.00	1.00	1.91	0.00	3.05	0.00	0.00	5.90
time (sec)	N/A	0.052	0.068	0.096	0.000	0.931	0.000	0.000	6.583

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	280	0	319	0	0	-1
normalized size	1	1.00	0.71	2.30	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.106	0.080	0.000	1.299	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0	-1
normalized size	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.367	0.297	0.000	1.037	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	33	98	82	33	32
normalized size	1	1.00	1.03	1.15	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.020	0.032	0.046	1.305	0.884	0.330	0.153	0.069

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	205	277	0	0	0	0	-1
normalized size	1	1.00	0.88	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.378	0.142	0.000	0.882	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	86	94	0	0	0	0	-1
normalized size	1	1.00	0.37	0.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.074	0.069	0.000	1.006	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	185	0	0	0	0	-1
normalized size	1	1.00	0.73	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.328	0.172	0.000	0.799	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.149	0.225	0.000	1.328	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	128	390	71	133	123
normalized size	1	1.00	0.89	1.34	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.106	0.102	0.050	1.294	0.663	0.454	0.222	0.274

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	59	47	48	82	49	49
normalized size	1	1.00	0.84	1.20	0.96	0.98	1.67	1.00	1.00
time (sec)	N/A	0.049	0.072	0.052	0.638	0.974	0.299	0.195	0.074

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	24	20	20
normalized size	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.016	0.013	0.040	0.550	0.913	0.196	0.205	0.034

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	23	23	27	24	13
normalized size	1	1.00	1.00	1.41	1.35	1.35	1.59	1.41	0.76
time (sec)	N/A	0.015	0.006	0.048	0.564	0.690	0.266	0.182	1.458

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	22	66	71	71	102	71	90
normalized size	1	1.00	0.21	0.63	0.68	0.68	0.98	0.68	0.87
time (sec)	N/A	0.086	0.007	0.044	1.135	0.823	5.721	0.166	1.497

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	26	23	22
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.019	0.031	0.042	0.558	0.593	0.184	0.152	0.041

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.040	0.767	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.018	0.594	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.043	0.928	0.000	0.981	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	138	186	527	2744	740	131
normalized size	1	1.00	0.83	1.05	1.41	3.99	20.79	5.61	0.99
time (sec)	N/A	0.111	0.284	0.053	0.614	0.682	3.673	0.216	1.638

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	90	104	140	319	1540	450	99
normalized size	1	1.00	0.91	1.05	1.41	3.22	15.56	4.55	1.00
time (sec)	N/A	0.073	0.128	0.055	0.503	1.010	3.346	0.193	1.557

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	74	94	175	726	232	71
normalized size	1	1.00	1.00	1.06	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.045	0.102	0.054	0.440	0.979	1.956	0.188	1.535

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	43	48	69	236	83	38
normalized size	1	1.00	0.92	1.08	1.20	1.72	5.90	2.08	0.95
time (sec)	N/A	0.021	0.075	0.045	0.510	0.979	0.650	0.169	1.483

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	73	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	1.70	0.00	-0.02
time (sec)	N/A	0.017	0.014	0.632	0.000	0.667	3.333	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	0	0	0	592	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	8.11	0.00	-0.01
time (sec)	N/A	0.031	0.046	0.587	0.000	0.956	8.694	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	3706	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	47.51	0.00	-0.01
time (sec)	N/A	0.031	0.056	0.609	0.000	0.956	64.993	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.065	0.585	0.000	0.836	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	149	164	242	667	0	947	157
normalized size	1	1.00	0.94	1.04	1.53	4.22	0.00	5.99	0.99
time (sec)	N/A	0.130	0.211	0.057	0.622	1.002	0.000	0.243	1.705

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	105	117	168	370	1765	539	108
normalized size	1	1.00	0.94	1.04	1.50	3.30	15.76	4.81	0.96
time (sec)	N/A	0.082	0.197	0.055	0.604	0.862	77.474	0.225	1.566

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	74	94	175	726	232	71
normalized size	1	1.00	1.00	1.06	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.047	0.108	0.049	0.633	1.082	2.737	0.272	1.531

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	170	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.094	0.049	0.602	0.000	1.078	6.831	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.155	0.585	0.000	0.966	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.192	0.614	0.000	0.945	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	133	0	0	0	369	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	1.19	0.00	-0.00
time (sec)	N/A	0.501	4.125	0.654	0.000	0.928	9.725	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	104	0	0	0	269	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	1.55	0.00	-0.01
time (sec)	N/A	0.267	1.588	0.634	0.000	1.023	13.012	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	170	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.097	0.361	0.604	0.000	0.669	12.793	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	73	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	1.74	0.00	-0.02
time (sec)	N/A	0.016	0.011	0.614	0.000	0.730	4.112	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.063	0.912	0.000	1.007	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.217	0.928	0.000	1.233	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.264	0.902	0.000	0.838	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	217	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	5.264	0.617	0.000	1.514	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	167	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	5.379	0.676	0.000	0.863	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	666	0	0	0	0	0	-1
normalized size	1	1.00	5.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	2.580	0.608	0.000	0.901	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	0	0	0	592	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	8.22	0.00	-0.01
time (sec)	N/A	0.031	0.044	0.590	0.000	0.645	8.769	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.201	0.910	0.000	0.987	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	147	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.285	0.913	0.000	1.048	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	233	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.438	0.927	0.000	0.990	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.566	0.889	0.000	0.775	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	401	168	0	0	0	199	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.50	0.00	-0.00
time (sec)	N/A	0.578	5.318	0.742	0.000	1.114	93.229	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	197	140	0	0	0	143	0	-1
normalized size	1	0.98	0.69	0.00	0.00	0.00	0.71	0.00	-0.00
time (sec)	N/A	0.269	5.202	0.720	0.000	0.838	42.751	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	89	94	0	0	0	87	0	-1
normalized size	1	0.91	0.96	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.047	0.047	0.604	0.000	0.690	6.154	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	37	0	47
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	1.02
time (sec)	N/A	0.011	0.005	0.036	0.000	0.705	1.902	0.000	2.299

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.309	0.914	0.000	1.071	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.342	0.900	0.000	0.901	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.651	0.930	0.000	0.998	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.088	0.910	0.000	1.095	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	218	0	0	478	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.142	0.828	0.000	0.803	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	0	0	231	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.083	0.797	0.000	1.232	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	82	0	0	85	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	1.47	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.137	0.598	0.000	0.832	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	53	0	31	211	0	75
normalized size	1	1.00	1.00	2.94	0.00	1.72	11.72	0.00	4.17
time (sec)	N/A	0.003	0.036	0.073	0.000	1.005	33.049	0.000	1.756

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.014	0.979	0.000	0.806	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.015	0.925	0.000	1.159	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.013	0.947	0.000	1.286	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	179	1414	0	0	0	0	0	-1
normalized size	1	0.93	7.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	51.213	0.921	0.000	1.136	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	108	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	1.89	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.069	0.904	0.000	1.270	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	136	0	0	400	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.463	0.766	0.000	0.966	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	94	0	0	173	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.170	0.632	0.000	1.347	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	0	0	64
normalized size	1	1.00	1.10	0.00	0.00	1.36	0.00	0.00	1.28
time (sec)	N/A	0.012	0.029	0.605	0.000	1.408	0.000	0.000	1.763

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	6.942	0.836	0.000	1.056	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	1070	0	0	0	0	0	-1
normalized size	1	1.00	8.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	63.260	1.038	0.000	1.237	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	1241	0	0	0	0	0	-1
normalized size	1	1.00	9.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	44.863	1.131	0.000	1.493	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	180.013	0.952	0.000	1.372	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	110	92	178	114	0	621	152
normalized size	1	1.08	0.72	0.61	1.17	0.75	0.00	4.09	1.00
time (sec)	N/A	0.118	0.104	0.043	0.500	1.340	0.000	0.760	1.761

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	118	88	68	124	90	0	495	118
normalized size	1	1.08	0.81	0.62	1.14	0.83	0.00	4.54	1.08
time (sec)	N/A	0.087	0.060	0.046	0.498	0.990	0.000	0.748	1.736

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	62	44	70	66	0	361	83
normalized size	1	1.07	0.93	0.66	1.04	0.99	0.00	5.39	1.24
time (sec)	N/A	0.044	0.046	0.046	0.619	1.141	0.000	0.371	1.641

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	85	174	52	80	0	78	248
normalized size	1	1.00	1.06	2.18	0.65	1.00	0.00	0.98	3.10
time (sec)	N/A	0.078	0.243	0.101	1.300	0.941	0.000	0.320	3.598

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	114	114	182	98	85	0	157	584
normalized size	1	1.19	1.19	1.90	1.02	0.89	0.00	1.64	6.08
time (sec)	N/A	0.084	0.056	0.067	1.462	1.067	0.000	0.421	6.890

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	164	137	226	162	100	0	324	1004
normalized size	1	1.36	1.13	1.87	1.34	0.83	0.00	2.68	8.30
time (sec)	N/A	0.104	0.092	0.066	1.353	1.085	0.000	0.455	15.557

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	161	298	246	138	0	558	2314
normalized size	1	1.00	0.77	1.43	1.18	0.66	0.00	2.68	11.12
time (sec)	N/A	0.149	0.309	0.095	0.548	1.319	0.000	0.627	39.151

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	135	240	192	112	0	432	1681
normalized size	1	1.00	0.85	1.51	1.21	0.70	0.00	2.72	10.57
time (sec)	N/A	0.123	0.191	0.067	0.666	1.500	0.000	0.458	42.568

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	129	182	137	88	0	288	734
normalized size	1	1.00	1.13	1.60	1.20	0.77	0.00	2.53	6.44
time (sec)	N/A	0.047	0.279	0.059	0.543	1.128	0.000	0.350	17.425

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	153	105	83	0	110	243
normalized size	1	1.00	0.97	1.47	1.01	0.80	0.00	1.06	2.34
time (sec)	N/A	0.087	0.089	0.064	1.500	1.216	0.000	0.395	3.486

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	105	153	75	100	0	171	236
normalized size	1	1.00	1.25	1.82	0.89	1.19	0.00	2.04	2.81
time (sec)	N/A	0.080	0.086	0.066	1.472	1.336	0.000	0.383	3.438

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	117	191	153	96	0	172	1154
normalized size	1	1.00	0.94	1.53	1.22	0.77	0.00	1.38	9.23
time (sec)	N/A	0.085	0.143	0.106	0.554	1.247	0.000	0.231	32.625

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	57	95	55	216	108	108
normalized size	1	1.00	0.68	0.55	0.92	0.53	2.10	1.05	1.05
time (sec)	N/A	0.075	0.043	0.046	0.633	1.127	62.839	0.223	2.440

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	98	147	113	77	0	121	720
normalized size	1	1.00	1.13	1.69	1.30	0.89	0.00	1.39	8.28
time (sec)	N/A	0.069	0.105	0.078	0.537	1.010	0.000	0.373	22.496

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	38	54	37	202	59	66
normalized size	1	1.00	0.80	0.58	0.83	0.57	3.11	0.91	1.02
time (sec)	N/A	0.040	0.054	0.044	0.469	1.036	41.910	0.222	2.359

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	101	103	74	55	182	69	293
normalized size	1	1.00	2.15	2.19	1.57	1.17	3.87	1.47	6.23
time (sec)	N/A	0.020	0.212	0.066	0.497	1.099	45.512	0.176	12.685

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	62	29	48	162	45	77
normalized size	1	1.00	1.43	1.35	0.63	1.04	3.52	0.98	1.67
time (sec)	N/A	0.061	0.033	0.072	1.152	0.983	40.135	0.224	3.865

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	73	77	44	56	148	58	61
normalized size	1	1.00	2.21	2.33	1.33	1.70	4.48	1.76	1.85
time (sec)	N/A	0.057	0.030	0.072	1.177	1.178	35.169	0.216	2.593

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	84	45	57	141	114	297
normalized size	1	1.00	1.28	1.40	0.75	0.95	2.35	1.90	4.95
time (sec)	N/A	0.063	0.071	0.082	1.226	1.074	63.618	0.213	8.668

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	37	54	52	146	116	53
normalized size	1	1.00	0.82	0.60	0.87	0.84	2.35	1.87	0.85
time (sec)	N/A	0.062	0.018	0.050	1.233	0.699	61.444	0.206	2.440

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	102	125	85	78	0	268	650
normalized size	1	1.00	1.03	1.26	0.86	0.79	0.00	2.71	6.57
time (sec)	N/A	0.078	0.102	0.074	1.107	1.310	0.000	0.221	21.455

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	148	240	196	115	0	203	1682
normalized size	1	1.00	0.90	1.46	1.20	0.70	0.00	1.24	10.26
time (sec)	N/A	0.120	0.124	0.099	0.568	1.240	0.000	0.280	42.656

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	87	68	124	66	240	124	130
normalized size	1	1.00	0.74	0.58	1.05	0.56	2.03	1.05	1.10
time (sec)	N/A	0.086	0.062	0.046	0.551	1.096	70.843	0.251	2.700

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	121	182	142	90	0	140	1048
normalized size	1	1.00	1.03	1.54	1.20	0.76	0.00	1.19	8.88
time (sec)	N/A	0.097	0.098	0.075	0.639	0.906	0.000	0.233	25.513

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	43	69	42	223	65	76
normalized size	1	1.00	0.85	0.60	0.96	0.58	3.10	0.90	1.06
time (sec)	N/A	0.046	0.038	0.040	0.645	1.143	44.702	0.198	2.663

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	119	124	89	63	199	79	417
normalized size	1	1.00	1.75	1.82	1.31	0.93	2.93	1.16	6.13
time (sec)	N/A	0.032	0.349	0.071	0.459	1.175	41.776	0.265	10.800

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	87	108	37	61	178	55	108
normalized size	1	1.00	1.55	1.93	0.66	1.09	3.18	0.98	1.93
time (sec)	N/A	0.069	0.037	0.077	1.342	0.967	39.158	0.209	3.967

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	90	97	55	68	165	66	77
normalized size	1	1.00	1.58	1.70	0.96	1.19	2.89	1.16	1.35
time (sec)	N/A	0.075	0.041	0.069	1.257	1.290	36.825	0.217	2.945

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	102	158	60	73	162	141	457
normalized size	1	1.00	1.34	2.08	0.79	0.96	2.13	1.86	6.01
time (sec)	N/A	0.073	0.102	0.072	1.292	1.271	69.669	0.494	7.500

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	49	75	67	170	137	79
normalized size	1	1.00	0.88	0.65	1.00	0.89	2.27	1.83	1.05
time (sec)	N/A	0.069	0.031	0.041	1.319	0.670	70.802	0.244	2.765

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	144	227	114	100	0	325	1005
normalized size	1	1.00	1.17	1.85	0.93	0.81	0.00	2.64	8.17
time (sec)	N/A	0.095	0.108	0.070	1.208	1.084	0.000	0.271	19.135

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	119	316	196	190	0	214	-1
normalized size	1	1.00	0.74	1.96	1.22	1.18	0.00	1.33	-0.01
time (sec)	N/A	0.123	0.162	0.087	0.492	1.085	0.000	0.398	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	72	68	123	80	226	200	90
normalized size	1	1.00	0.63	0.59	1.07	0.70	1.97	1.74	0.78
time (sec)	N/A	0.094	0.054	0.049	0.464	1.180	177.265	0.435	2.801

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	90	254	138	159	0	147	-1
normalized size	1	1.00	0.59	1.67	0.91	1.05	0.00	0.97	-0.01
time (sec)	N/A	0.115	0.121	0.077	0.500	0.980	0.000	0.321	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	43	69	56	201	152	67
normalized size	1	1.00	0.59	0.57	0.91	0.74	2.64	2.00	0.88
time (sec)	N/A	0.054	0.067	0.043	0.576	1.137	136.305	0.280	2.747

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	86	160	76	129	182	113	-1
normalized size	1	1.00	1.37	2.54	1.21	2.05	2.89	1.79	-0.02
time (sec)	N/A	0.032	0.244	0.073	0.548	1.407	112.361	0.280	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	84	188	58	101	172	115	-1
normalized size	1	1.00	1.29	2.89	0.89	1.55	2.65	1.77	-0.02
time (sec)	N/A	0.081	0.044	0.076	1.463	1.102	136.445	0.362	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	48	71	103	165	219	73
normalized size	1	1.00	0.76	0.72	1.06	1.54	2.46	3.27	1.09
time (sec)	N/A	0.077	0.027	0.050	1.365	0.996	136.133	0.459	2.866

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	75	315	104	138	0	211	-1
normalized size	1	1.00	0.64	2.69	0.89	1.18	0.00	1.80	-0.01
time (sec)	N/A	0.098	0.032	0.091	1.265	1.065	0.000	0.542	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	77	73	125	132	0	242	104
normalized size	1	1.00	0.65	0.61	1.05	1.11	0.00	2.03	0.87
time (sec)	N/A	0.096	0.031	0.046	1.342	0.877	0.000	0.728	2.897

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	78	387	162	165	0	402	-1
normalized size	1	1.00	0.47	2.33	0.98	0.99	0.00	2.42	-0.01
time (sec)	N/A	0.120	0.033	0.084	1.480	0.689	0.000	0.794	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	53	23	39	148	40	72
normalized size	1	1.00	1.40	1.32	0.58	0.98	3.70	1.00	1.80
time (sec)	N/A	0.054	0.026	0.069	1.351	1.238	30.114	0.171	3.652

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	244	66	79	65	0	0	96
normalized size	1	1.00	4.60	1.25	1.49	1.23	0.00	0.00	1.81
time (sec)	N/A	0.091	0.314	0.053	1.244	1.011	0.000	0.000	3.274

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	69	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.92	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.020	0.857	0.000	0.915	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	50	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.058	0.877	0.000	1.323	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.438	1.602	0.000	1.409	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.049	1.495	0.000	1.308	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0	-1
normalized size	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.429	1.225	0.000	0.998	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	103	0	0	180	0	0	-1
normalized size	1	1.08	1.07	0.00	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.394	2.167	0.000	1.244	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [279] had the largest ratio of [.5294]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
4	A	2	1	1.00	15	0.067
5	A	7	7	1.00	17	0.412
6	A	7	7	1.00	17	0.412
7	A	8	8	1.00	17	0.471
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	8	7	1.00	19	0.368
12	A	9	8	1.00	19	0.421
13	A	8	8	1.00	19	0.421
14	A	8	7	1.00	19	0.368
15	A	8	7	1.00	19	0.368
16	A	8	7	1.00	19	0.368
17	A	7	7	1.00	17	0.412
18	A	13	7	1.00	19	0.368
19	A	14	8	1.00	19	0.421
20	A	9	8	1.00	19	0.421
21	A	9	8	1.00	19	0.421
22	A	9	8	1.00	19	0.421
23	A	9	8	1.00	19	0.421
24	A	7	7	1.00	17	0.412
25	A	14	8	1.00	19	0.421
26	A	15	9	1.00	19	0.474
27	A	3	3	1.00	20	0.150
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
30	A	2	2	1.00	20	0.100
31	A	3	3	1.00	20	0.150
32	A	4	3	1.00	20	0.150
33	A	5	3	1.00	20	0.150
34	C	2	2	0.12	22	0.091
35	C	2	2	0.12	22	0.091
36	C	2	2	0.15	22	0.091
37	C	2	2	0.13	22	0.091
38	C	2	2	0.12	22	0.091
39	C	2	2	0.12	22	0.091
40	A	4	4	1.00	22	0.182
41	A	3	3	1.00	22	0.136
42	A	3	3	1.00	22	0.136
43	A	3	3	1.00	22	0.136
44	A	3	2	1.00	22	0.091
45	A	4	3	1.00	22	0.136
46	A	5	4	1.00	22	0.182
47	A	6	4	1.00	22	0.182
48	A	4	4	1.00	22	0.182
49	A	4	4	1.00	22	0.182
50	A	4	4	1.00	22	0.182
51	A	4	4	1.00	22	0.182
52	A	4	4	1.00	22	0.182
53	A	4	4	1.00	22	0.182
54	A	4	4	1.00	22	0.182
55	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	4	3	1.00	19	0.158
57	A	3	3	1.00	19	0.158
58	A	2	2	1.00	19	0.105
59	A	2	2	1.00	19	0.105
60	A	2	2	1.00	19	0.105
61	A	3	3	1.00	19	0.158
62	A	4	3	1.00	19	0.158
63	A	5	3	1.00	19	0.158
64	A	3	3	1.00	19	0.158
65	A	3	3	1.00	19	0.158
66	A	3	3	1.00	19	0.158
67	A	3	3	1.00	19	0.158
68	A	3	3	1.00	19	0.158
69	A	3	3	1.00	19	0.158
70	A	5	4	1.00	21	0.190
71	A	4	4	1.00	21	0.190
72	A	3	3	1.00	21	0.143
73	A	3	3	1.00	21	0.143
74	A	3	3	1.00	21	0.143
75	A	3	2	1.00	21	0.095
76	A	4	3	1.00	21	0.143
77	A	5	4	1.00	21	0.190
78	A	6	4	1.00	21	0.190
79	A	4	4	1.00	21	0.190
80	A	4	4	1.00	21	0.190
81	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	4	4	1.00	21	0.190
83	A	4	4	1.00	21	0.190
84	A	4	4	1.00	21	0.190
85	A	4	2	1.00	21	0.095
86	C	2	2	0.19	21	0.095
87	C	2	2	0.22	21	0.095
88	C	2	2	0.25	21	0.095
89	A	7	7	1.40	21	0.333
90	A	8	8	1.33	21	0.381
91	C	2	2	2.75	21	0.095
92	C	2	2	4.19	21	0.095
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095
95	A	2	2	1.00	21	0.095
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	C	2	2	0.18	21	0.095
99	C	2	2	0.20	21	0.095
100	A	8	8	1.32	21	0.381
101	A	8	8	1.27	21	0.381
102	C	2	2	2.39	21	0.095
103	C	2	2	3.75	21	0.095
104	A	2	2	1.00	21	0.095
105	A	2	2	1.00	21	0.095
106	A	2	2	1.00	21	0.095
107	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	2	2	1.00	21	0.095
109	C	2	2	0.11	21	0.095
110	C	2	2	0.14	21	0.095
111	C	2	2	0.16	21	0.095
112	A	9	8	1.27	21	0.381
113	A	9	9	1.22	21	0.429
114	C	2	2	0.54	21	0.095
115	C	2	2	1.14	21	0.095
116	C	2	2	4.30	21	0.095
117	A	2	2	1.00	21	0.095
118	A	2	2	1.00	21	0.095
119	A	2	2	1.00	21	0.095
120	A	2	2	1.00	21	0.095
121	A	2	2	1.00	21	0.095
122	A	3	2	1.00	23	0.087
123	A	3	2	1.00	23	0.087
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	23	0.087
126	A	1	1	1.00	23	0.043
127	A	1	1	1.00	23	0.043
128	A	1	1	1.00	23	0.043
129	A	2	2	1.00	23	0.087
130	A	2	2	1.00	23	0.087
131	A	3	2	1.00	23	0.087
132	A	3	2	1.00	23	0.087
133	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
134	A	4	4	1.05	19	0.210
135	A	3	3	1.01	17	0.176
136	A	2	2	1.00	19	0.105
137	A	2	2	1.00	19	0.105
138	A	5	5	1.00	19	0.263
139	A	4	4	1.00	19	0.210
140	A	3	3	0.91	17	0.176
141	A	2	2	1.00	9	0.222
142	A	2	2	1.00	19	0.105
143	A	2	2	1.00	19	0.105
144	A	2	2	1.00	19	0.105
145	A	1	1	1.00	50	0.020
146	A	2	1	1.00	17	0.059
147	A	2	1	1.00	17	0.059
148	A	2	1	1.00	17	0.059
149	A	2	1	1.00	15	0.067
150	A	10	7	1.00	17	0.412
151	A	10	7	1.00	17	0.412
152	A	11	8	1.00	17	0.471
153	A	2	1	1.00	19	0.053
154	A	2	1	1.00	19	0.053
155	A	2	1	1.00	19	0.053
156	A	2	1	1.00	17	0.059
157	A	11	7	1.00	19	0.368
158	A	12	8	1.00	19	0.421
159	A	11	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	11	7	1.00	19	0.368
161	A	11	7	1.00	19	0.368
162	A	11	7	1.00	19	0.368
163	A	10	7	1.00	17	0.412
164	A	19	7	1.00	19	0.368
165	A	20	8	1.00	19	0.421
166	A	12	8	1.00	19	0.421
167	A	12	8	1.00	19	0.421
168	A	12	8	1.00	19	0.421
169	A	12	8	1.00	19	0.421
170	A	10	7	1.00	17	0.412
171	A	20	8	1.00	19	0.421
172	A	21	9	1.00	19	0.474
173	A	10	8	1.00	23	0.348
174	A	9	7	1.00	23	0.304
175	A	8	6	1.00	23	0.261
176	A	5	3	1.00	23	0.130
177	A	9	7	1.00	23	0.304
178	A	10	8	1.00	23	0.348
179	A	10	6	1.00	21	0.286
180	A	9	5	1.00	21	0.238
181	A	7	4	1.00	21	0.190
182	A	10	6	1.00	21	0.286
183	A	11	7	1.00	21	0.333
184	A	11	8	1.00	23	0.348
185	A	10	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
186	A	9	7	1.00	23	0.304
187	A	9	7	1.00	23	0.304
188	A	9	7	1.00	23	0.304
189	A	10	8	1.00	23	0.348
190	A	11	8	1.00	23	0.348
191	A	4	4	1.00	25	0.160
192	A	1	1	1.00	25	0.040
193	A	10	9	1.00	21	0.429
194	A	9	8	1.00	21	0.381
195	A	4	4	1.00	21	0.190
196	A	5	5	1.00	21	0.238
197	A	7	7	1.00	21	0.333
198	A	8	7	1.00	21	0.333
199	A	11	10	1.00	21	0.476
200	A	10	9	1.00	21	0.429
201	A	4	3	1.00	21	0.143
202	A	9	8	1.00	21	0.381
203	A	10	9	1.00	21	0.429
204	A	11	10	1.00	21	0.476
205	A	11	10	1.00	21	0.476
206	A	10	9	1.00	21	0.429
207	A	5	5	1.00	21	0.238
208	A	5	5	1.00	21	0.238
209	A	7	7	1.00	21	0.333
210	A	8	7	1.00	21	0.333
211	A	11	10	1.00	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	10	9	1.00	21	0.429
213	A	10	9	1.00	21	0.429
214	A	10	9	1.00	21	0.429
215	A	11	10	1.00	21	0.476
216	A	4	4	1.00	17	0.235
217	A	4	4	1.00	26	0.154
218	A	3	2	1.00	19	0.105
219	A	4	4	1.00	19	0.210
220	A	3	3	0.91	17	0.176
221	A	2	2	1.00	19	0.105
222	A	2	2	1.00	19	0.105
223	A	7	7	1.00	21	0.333
224	A	6	6	1.00	21	0.286
225	A	6	6	1.00	21	0.286
226	A	5	5	1.00	19	0.263
227	A	4	4	1.00	11	0.364
228	A	7	6	1.00	21	0.286
229	A	8	7	1.00	21	0.333
230	A	9	7	1.00	21	0.333
231	A	7	7	1.00	21	0.333
232	A	7	6	1.00	21	0.286
233	A	6	5	1.00	19	0.263
234	A	5	5	1.00	11	0.454
235	A	7	6	1.00	21	0.286
236	A	8	7	1.00	21	0.333
237	A	9	7	1.00	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
238	A	8	7	1.00	21	0.333
239	A	8	6	1.00	21	0.286
240	A	7	5	1.00	19	0.263
241	A	6	5	1.00	11	0.454
242	A	8	7	1.00	21	0.333
243	A	8	7	1.00	21	0.333
244	A	9	8	1.00	21	0.381
245	A	5	5	1.00	21	0.238
246	A	5	5	1.00	21	0.238
247	A	4	4	1.00	19	0.210
248	A	4	4	1.00	11	0.364
249	A	7	6	1.00	21	0.286
250	A	8	7	1.00	21	0.333
251	A	9	7	1.00	21	0.333
252	A	5	5	1.00	21	0.238
253	A	5	5	0.96	21	0.238
254	A	5	5	1.00	19	0.263
255	A	5	4	1.02	11	0.364
256	A	8	7	1.00	21	0.333
257	A	9	8	1.00	21	0.381
258	A	10	8	1.00	21	0.381
259	A	5	5	1.00	21	0.238
260	A	6	6	0.97	21	0.286
261	A	6	5	1.00	19	0.263
262	A	6	4	1.04	11	0.364
263	A	9	7	1.00	21	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	10	8	1.00	21	0.381
265	A	11	8	1.00	21	0.381
266	A	8	8	1.00	23	0.348
267	A	4	4	1.00	23	0.174
268	A	5	5	1.00	23	0.217
269	A	3	3	1.00	19	0.158
270	A	3	3	1.00	17	0.176
271	A	6	6	1.00	23	0.261
272	A	7	7	1.00	23	0.304
273	A	7	7	1.00	23	0.304
274	A	4	3	1.00	19	0.158
275	A	8	8	1.00	17	0.471
276	A	3	2	1.00	21	0.095
277	A	3	2	1.00	17	0.118
278	A	4	4	1.00	17	0.235
279	A	9	9	1.00	17	0.529
280	A	4	3	1.00	17	0.176
281	A	3	3	1.00	24	0.125
282	A	3	3	1.00	24	0.125
283	A	3	3	1.00	20	0.150
284	A	2	1	1.00	17	0.059
285	A	2	1	1.00	17	0.059
286	A	2	1	1.00	17	0.059
287	A	2	1	1.00	15	0.067
288	A	2	2	1.00	17	0.118
289	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	2	2	1.00	17	0.118
291	A	2	2	1.00	17	0.118
292	A	2	1	1.00	19	0.053
293	A	2	1	1.00	19	0.053
294	A	2	1	1.00	17	0.059
295	A	3	3	1.00	19	0.158
296	A	3	3	1.00	19	0.158
297	A	3	3	1.00	19	0.158
298	A	5	4	1.00	19	0.210
299	A	4	4	1.00	19	0.210
300	A	3	3	1.00	19	0.158
301	A	2	2	1.00	17	0.118
302	A	3	2	1.00	19	0.105
303	A	4	3	1.00	19	0.158
304	A	5	4	1.00	19	0.210
305	A	5	4	1.00	19	0.210
306	A	4	4	1.00	19	0.210
307	A	3	3	1.00	19	0.158
308	A	2	2	1.00	17	0.118
309	A	4	3	1.00	19	0.158
310	A	5	4	1.00	19	0.210
311	A	6	4	1.00	19	0.210
312	A	3	2	1.00	19	0.105
313	A	5	5	1.00	19	0.263
314	A	4	4	0.98	19	0.210
315	A	3	3	0.91	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	2	2	1.00	9	0.222
317	A	2	2	1.00	19	0.105
318	A	2	2	1.00	19	0.105
319	A	2	2	1.00	19	0.105
320	A	1	1	1.00	28	0.036
321	A	4	2	1.00	25	0.080
322	A	3	2	1.00	25	0.080
323	A	2	2	1.00	23	0.087
324	A	1	1	1.00	15	0.067
325	A	1	1	1.00	23	0.043
326	A	1	1	1.00	25	0.040
327	A	1	1	1.00	25	0.040
328	A	2	2	0.93	28	0.071
329	A	1	1	1.00	69	0.014
330	A	5	3	1.00	25	0.120
331	A	3	3	1.00	23	0.130
332	A	2	2	1.00	15	0.133
333	A	2	2	1.00	25	0.080
334	A	2	2	1.00	23	0.087
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080
337	A	6	4	1.08	31	0.129
338	A	4	4	1.08	31	0.129
339	A	2	2	1.07	29	0.069
340	A	5	5	1.00	31	0.161
341	A	5	5	1.19	31	0.161

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	5	4	1.36	31	0.129
343	A	9	8	1.00	31	0.258
344	A	7	7	1.00	31	0.226
345	A	5	5	1.00	28	0.179
346	A	5	5	1.00	31	0.161
347	A	6	6	1.00	31	0.194
348	A	5	5	1.00	29	0.172
349	A	4	4	1.00	29	0.138
350	A	3	3	1.00	29	0.103
351	A	2	2	1.00	27	0.074
352	A	2	2	1.00	26	0.077
353	A	3	3	1.00	29	0.103
354	A	2	2	1.00	29	0.069
355	A	3	3	1.00	29	0.103
356	A	2	2	1.00	29	0.069
357	A	5	5	1.00	29	0.172
358	A	8	7	1.00	31	0.226
359	A	4	4	1.00	31	0.129
360	A	6	6	1.00	31	0.194
361	A	2	2	1.00	29	0.069
362	A	4	4	1.00	28	0.143
363	A	3	3	1.00	31	0.097
364	A	4	4	1.00	31	0.129
365	A	3	3	1.00	31	0.097
366	A	2	2	1.00	31	0.065
367	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	8	8	1.00	31	0.258
369	A	4	4	1.00	31	0.129
370	A	7	7	1.00	31	0.226
371	A	2	2	1.00	29	0.069
372	A	4	4	1.00	28	0.143
373	A	3	3	1.00	31	0.097
374	A	2	2	1.00	31	0.065
375	A	5	5	1.00	31	0.161
376	A	4	4	1.00	31	0.129
377	A	7	7	1.00	31	0.226
378	A	3	3	1.00	31	0.097
379	A	1	1	1.00	57	0.018
380	A	3	3	1.00	32	0.094
381	A	4	4	1.00	41	0.098
382	A	4	3	1.00	31	0.097
383	A	4	4	1.00	35	0.114
384	A	3	3	1.00	31	0.097
385	A	2	2	1.08	76	0.026

Chapter 3

Listing of integrals

3.1 $\int \left(a+bx^3\right)\left(c+dx^3\right)^4 \, dx$

Optimal. Leaf size=94

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

```
[Out] a*c^4*x+1/4*c^3*(4*a*d+b*c)*x^4+2/7*c^2*d*(3*a*d+2*b*c)*x^7+1/5*c*d^2*(2*a*
d+3*b*c)*x^10+1/13*d^3*(a*d+4*b*c)*x^13+1/16*b*d^4*x^16
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{4}c^3x^4(4ad+bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^3)(c+dx^3)^4\,dx = \int \left(ac^4+c^3(bc+4ad)x^3+2c^2d(2bc+3ad)x^6+2cd^2(3bc+2ad)x^9+d^3(4bc+ad)x\right.$$
$$= ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7+\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+a$$

Mathematica [A] time = 0.05, size = 94, normalized size = 1.00

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

fricas [A] time = 0.36, size = 97, normalized size = 1.03

$$\frac{1}{16}x^{16}d^4b+\frac{4}{13}x^{13}d^3cb+\frac{1}{13}x^{13}d^4a+\frac{3}{5}x^{10}d^2c^2b+\frac{2}{5}x^{10}d^3ca+\frac{4}{7}x^7dc^3b+\frac{6}{7}x^7d^2c^2a+\frac{1}{4}x^4c^4b+x^4dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^4*b + 4/13*x^13*d^3*c*b + 1/13*x^13*d^4*a + 3/5*x^10*d^2*c^2*b
+ 2/5*x^10*d^3*c*a + 4/7*x^7*d*c^3*b + 6/7*x^7*d^2*c^2*a + 1/4*x^4*c^4*b +
x^4*d*c^3*a + x*c^4*a
```

giac [A] time = 0.17, size = 97, normalized size = 1.03

$$\frac{1}{16}bd^4x^{16}+\frac{4}{13}bcd^3x^{13}+\frac{1}{13}ad^4x^{13}+\frac{3}{5}bc^2d^2x^{10}+\frac{2}{5}acd^3x^{10}+\frac{4}{7}bc^3dx^7+\frac{6}{7}ac^2d^2x^7+\frac{1}{4}bc^4x^4+ac^3dx^4+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
+ 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4 +
a*c^3*d*x^4 + a*c^4*x
```

maple [A] time = 0.04, size = 97, normalized size = 1.03

$$\frac{b\,d^4x^{16}}{16}+\frac{\left(a\,d^4+4bc\,d^3\right)x^{13}}{13}+\frac{\left(4ac\,d^3+6c^2d^2b\right)x^{10}}{10}+\frac{\left(6a\,c^2d^2+4c^3db\right)x^7}{7}+a\,c^4x+\frac{\left(4a\,c^3d+b\,c^4\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^4,x)
```

```
[Out] 1/16*b*d^4*x^16+1/13*(a*d^4+4*b*c*d^3)*x^13+1/10*(4*a*c*d^3+6*b*c^2*d^2)*x^
10+1/7*(6*a*c^2*d^2+4*b*c^3*d)*x^7+1/4*(4*a*c^3*d+b*c^4)*x^4+a*c^4*x
```

maxima [A] time = 0.47, size = 96, normalized size = 1.02

$$\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\left(bc^4+4\,ac^3d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*
d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*
c^3*d)*x^4
```

mupad [B] time = 0.05, size = 87, normalized size = 0.93

$$x^4\,\left(\frac{b\,c^4}{4}+a\,d\,c^3\right)+x^{13}\,\left(\frac{a\,d^4}{13}+\frac{4\,b\,c\,d^3}{13}\right)+\frac{b\,d^4\,x^{16}}{16}+a\,c^4\,x+\frac{2\,c^2\,d\,x^7\,\left(3\,a\,d+2\,b\,c\right)}{7}+\frac{c\,d^2\,x^{10}\,\left(2\,a\,d+3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^4,x)
```

```
[Out] x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^1
6)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d + 3*
b*c))/5
```

sympy [A] time = 0.13, size = 104, normalized size = 1.11

$$ac^4x+\frac{bd^4x^{16}}{16}+x^{13}\left(\frac{ad^4}{13}+\frac{4bcd^3}{13}\right)+x^{10}\left(\frac{2acd^3}{5}+\frac{3bc^2d^2}{5}\right)+x^7\left(\frac{6ac^2d^2}{7}+\frac{4bc^3d}{7}\right)+x^4\left(ac^3d+\frac{bc^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*a
*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x**4
*(a*c**3*d + b*c**4/4)
```

3.2 $\int \left(a+bx^3\right)\left(c+dx^3\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

```
[Out] a*c^3*x+1/4*c^2*(3*a*d+b*c)*x^4+3/7*c*d*(a*d+b*c)*x^7+1/10*d^2*(a*d+3*b*c)*
x^10+1/13*b*d^3*x^13
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^3+3cd(bc+ad)x^6+d^2(3bc+ad)x^9+bd^3x^{12}\right)dx\\ &= ac^3x+\frac{1}{4}c^2(bc+3ad)x^4+\frac{3}{7}cd(bc+ad)x^7+\frac{1}{10}d^2(3bc+ad)x^{10}+\frac{1}{13}bd^3x^{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.00

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

fricas [A] time = 0.36, size = 74, normalized size = 1.06

$$\frac{1}{13}x^{13}d^3b + \frac{3}{10}x^{10}d^2cb + \frac{1}{10}x^{10}d^3a + \frac{3}{7}x^7dc^2b + \frac{3}{7}x^7d^2ca + \frac{1}{4}x^4c^3b + \frac{3}{4}x^4dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b + 3/10*x^10*d^2*c*b + 1/10*x^10*d^3*a + 3/7*x^7*d*c^2*b + 3
/7*x^7*d^2*c*a + 1/4*x^4*c^3*b + 3/4*x^4*d*c^2*a + x*c^3*a
```

giac [A] time = 0.15, size = 74, normalized size = 1.06

$$\frac{1}{13}\,bd^3x^{13} + \frac{3}{10}\,bcd^2x^{10} + \frac{1}{10}\,ad^3x^{10} + \frac{3}{7}\,bc^2dx^7 + \frac{3}{7}\,acd^2x^7 + \frac{1}{4}\,bc^3x^4 + \frac{3}{4}\,ac^2dx^4 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 + 3
/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

maple [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{b\,d^3x^{13}}{13} + \frac{\left(a\,d^3 + 3bc\,d^2\right)x^{10}}{10} + \frac{\left(3ac\,d^2 + 3b\,c^2d\right)x^7}{7} + a\,c^3x + \frac{\left(3a\,c^2d + b\,c^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^3,x)
```

```
[Out] 1/13*b*d^3*x^13+1/10*(a*d^3+3*b*c*d^2)*x^10+1/7*(3*a*c*d^2+3*b*c^2*d)*x^7+1
/4*(3*a*c^2*d+b*c^3)*x^4+a*c^3*x
```

maxima [A] time = 0.56, size = 70, normalized size = 1.00

$$\frac{1}{13}\,bd^3x^{13} + \frac{1}{10}\left(3\,bcd^2 + ad^3\right)x^{10} + \frac{3}{7}\left(bc^2d + acd^2\right)x^7 + ac^3x + \frac{1}{4}\left(bc^3 + 3\,ac^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

mupad [B] time = 0.03, size = 66, normalized size = 0.94

$$x^4\left(\frac{b\,c^3}{4}+\frac{3\,a\,d\,c^2}{4}\right)+x^{10}\left(\frac{a\,d^3}{10}+\frac{3\,b\,c\,d^2}{10}\right)+\frac{b\,d^3\,x^{13}}{13}+a\,c^3\,x+\frac{3\,c\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^3,x)
```

```
[Out] x^4*((b*c^3)/4 + (3*a*c^2*d)/4) + x^10*((a*d^3)/10 + (3*b*c*d^2)/10) + (b*d
^3*x^13)/13 + a*c^3*x + (3*c*d*x^7*(a*d + b*c))/7
```

sympy [A] time = 0.08, size = 80, normalized size = 1.14

$$ac^3x+\frac{bd^3x^{13}}{13}+x^{10}\left(\frac{ad^3}{10}+\frac{3bcd^2}{10}\right)+x^7\left(\frac{3acd^2}{7}+\frac{3bc^2d}{7}\right)+x^4\left(\frac{3ac^2d}{4}+\frac{bc^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a*
c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

3.3 $\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

```
[Out] a*c^2*x+1/4*c*(2*a*d+b*c)*x^4+1/7*d*(a*d+2*b*c)*x^7+1/10*b*d^2*x^10
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^3+d(2bc+ad)x^6+bd^2x^9\right)dx\\ &= ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

fricas [A] time = 0.36, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}d^2b + \frac{2}{7}x^7dcb + \frac{1}{7}x^7d^2a + \frac{1}{4}x^4c^2b + \frac{1}{2}x^4dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b + 2/7*x^7*d*c*b + 1/7*x^7*d^2*a + 1/4*x^4*c^2*b + 1/2*x^4*d
*c*a + x*c^2*a
```

giac [A] time = 0.19, size = 50, normalized size = 1.00

$$\frac{1}{10}\,bd^2x^{10} + \frac{2}{7}\,bcdx^7 + \frac{1}{7}\,ad^2x^7 + \frac{1}{4}\,bc^2x^4 + \frac{1}{2}\,acdx^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*d
*x^4 + a*c^2*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b\,d^2x^{10}}{10} + \frac{\left(a\,d^2 + 2bcd\right)x^7}{7} + a\,c^2x + \frac{\left(2acd + b\,c^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^2,x)
```

```
[Out] 1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

maxima [A] time = 0.48, size = 48, normalized size = 0.96

$$\frac{1}{10}\,bd^2x^{10} + \frac{1}{7}\left(2\,bcd + ad^2\right)x^7 + \frac{1}{4}\left(bc^2 + 2\,acd\right)x^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^4\left(\frac{b\,c^2}{4}+\frac{a\,d\,c}{2}\right)+x^7\left(\frac{a\,d^2}{7}+\frac{2\,b\,c\,d}{7}\right)+\frac{b\,d^2\,x^{10}}{10}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^2,x)
```

```
[Out] x^4*((b*c^2)/4 + (a*c*d)/2) + x^7*((a*d^2)/7 + (2*b*c*d)/7) + (b*d^2*x^10)/
10 + a*c^2*x
```

sympy [A] time = 0.07, size = 51, normalized size = 1.02

$$ac^2x+\frac{bd^2x^{10}}{10}+x^7\left(\frac{ad^2}{7}+\frac{2bcd}{7}\right)+x^4\left(\frac{acd}{2}+\frac{bc^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 +
b*c**2/4)
```

3.4 $\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

```
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx &= \int \left(ac+(bc+ad)x^3+bdx^6\right)\,dx\\ &= acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

fricas [A] time = 0.35, size = 26, normalized size = 0.93

$$\frac{1}{7}x^7db + \frac{1}{4}x^4cb + \frac{1}{4}x^4da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
[Out] 1/7*x^7*d*b + 1/4*x^4*c*b + 1/4*x^4*d*a + x*c*a
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,bcx^4 + \frac{1}{4}\,adx^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
[Out] 1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bd\,x^7}{7} + \frac{(ad+bc)\,x^4}{4} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c),x)
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

maxima [A] time = 0.54, size = 24, normalized size = 0.86

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,(bc+ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^7}{7} + \left(\frac{a\,d}{4} + \frac{b\,c}{4}\right)\,x^4 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3),x)
```

```
[Out] x^4*((a*d)/4 + (b*c)/4) + a*c*x + (b*d*x^7)/7
```

sympy [**A**] time = 0.07, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c),x)
```

```
[Out] a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

3.5 $\int \frac{a+bx^3}{c+dx^3}\,dx$

Optimal. Leaf size=144

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{bx}{d}$$

```
[Out] b*x/d-1/3*(-a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(4/3)+1/6*(-a*d+b*c)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)+1/3*(-a*d+b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^3}\,dx}{d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}d} - \frac{(bc-ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/}}}{2\sqrt[3]{c}\,d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\,\mathrm{Subst}\big(\int}{c}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x\right.}{6c^{2/3}d^{4/3}}$$

Mathematica [A] time = 0.12, size = 128, normalized size = 0.89

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)-2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)+2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6bc^{2/3}\sqrt[3]{}}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Log[
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

fricas [A] time = 0.45, size = 369, normalized size = 2.56

$$\left[\frac{6\,bc^2dx-3\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\,\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)+\left(c^2d\right)^{\frac{2}{3}}(bc}{6\,c^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)*
log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*
d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^2*d
)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(c
^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*b*c^2
*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (c^2*
d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

giac [A] time = 0.20, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^{2}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^{2}\right)^{\frac{2}{3}}}+\frac{bx}{d}+\frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(-c*d^2)^(2/3) + 1/6*(b*c - a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)
)/(-c*d^2)^(2/3) + b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^
(1/3)))/(c*d)
```

maple [A] time = 0.05, size = 195, normalized size = 1.35

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}-\frac{bc\ln\left(x+ \cdots\right.}{3\left(\frac{c}{d}\right)^{\frac{2}{3}} \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c),x)
```

```
[Out] b*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-1/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^
(1/3))*b*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+1/6/d^2/(c
/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b*c+1/3/d/(c/d)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-1/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b*c
```

maxima [A] time = 1.28, size = 128, normalized size = 0.89

$$\frac{bx}{d}-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/(d^2*(c/d)^(2/3)) + 1/6*(b*c - a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d
)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3*(b*c - a*d)*log(x + (c/d)^(1/3))/(d^2*(c/d
)^(2/3))
```

mupad [B] time = 1.38, size = 123, normalized size = 0.85

$$\frac{b\,x}{d}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3),x)
```

```
[Out] (b*x)/d + (log(d^(1/3)*x + c^(1/3))*(a*d - b*c))/(3*c^(2/3)*d^(4/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(2/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(2/3)*d^(4/3))
```

sympy [A] time = 0.42, size = 71, normalized size = 0.49

$$\frac{bx}{d}+\operatorname{RootSum}\left(27t^3c^2d^4-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3,\left(t\mapsto t\log\left(\frac{3tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2
*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

3.6 $\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}-\frac{x(bc-a}{3cd\left(c+d\right.}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/d/(d*x^3+c)+1/9*(2*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/
3)/d^(4/3)-1/18*(2*a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/
3)/d^(4/3)-1/9*(2*a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2)
)/c^(5/3)/d^(4/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}-\frac{x(bc-a}{3cd\left(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(1/3) - 2*d
^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3)) + ((b*c + 2*a*d)*
Log[c^(1/3) + d^(1/3)*x]])/(9*c^(5/3)*d^(4/3)) - ((b*c + 2*a*d)*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx = -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\int \frac{1}{c+dx^3}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{9c^{5/3}d} + \frac{(bc+2ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{9c^{5/3}d}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{18c^{5/3}d^{4/3}} + \frac{(bc+2a}{}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}} + -$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} - \frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log}{}$$

Mathematica [A] time = 0.10, size = 145, normalized size = 0.86

$$\frac{-(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right) - \frac{6c^{2/3}\sqrt[3]{d}\,x(bc-ad)}{c+dx^3} + 2(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) - 2\sqrt{3}\,(2ad+bc)\tan^{-1}}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*A
rcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3) +
d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/
(18*c^(5/3)*d^(4/3))
```

fricas [A] time = 0.45, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(bc^3d+2\,ac^2d^2+\left(bc^2d^2+2\,acd^3\right)x^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sq
rt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/
3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/
(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(
c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b
*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c
^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^
2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*
(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d + 2
*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x
+ (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2
/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^3*d^3*x^3 +
c^4*d^2)]
```

giac [A] time = 0.19, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}c}-\frac{(bc+2\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(-cd^2\right)^{\frac{2}{3}}c}-\frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\right.\right.\right.}{9\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(
1/3))/((-c*d^2)^(2/3)*c) - 1/18*(b*c + 2*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-
```

```
c/d)^(2/3))/((-c*d^2)^(2/3)*c) - 1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(x -
 (-c/d)^(1/3)))/(c^2*d) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d)
```

maple [A] time = 0.05, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}\, a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}} cd}+\frac{2a\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}} cd}-\frac{a\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}} cd}+\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}} d^2}+\frac{b\ln\left(x+\right.}{9\left(\frac{c}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*(a*d-b*c)/d/c*x/(d*x^3+c)+2/9/c/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/9/d
^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9/c/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))*a-1/18/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/
9/c/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/9/d^2
/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [A] time = 1.16, size = 158, normalized size = 0.93

$$-\frac{(bc-ad)x}{3\left(cd^2x^3+c^2d\right)}+\frac{\sqrt{3}\,(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc+2\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc+2\,ad)\log\left(x\right.}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b*c - a*d)*x/(c*d^2*x^3 + c^2*d) + 1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^2*(c/d)^(2/3)) - 1/18*(b*c
 + 2*a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^2*(c/d)^(2/3)) + 1/9*
(b*c + 2*a*d)*log(x + (c/d)^(1/3))/(c*d^2*(c/d)^(2/3))
```

mupad [B] time = 1.40, size = 143, normalized size = 0.85

$$\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(2\,a\,d+b\,c)}{9\,c^{5/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,a\,d+b\,c)}{9\,c^{5/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1\right.}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3)^2,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) - (log(3^(1/2)
*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*d + b*c))/
(9*c^(5/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) + (x*(a*d - b*c))/(3*
c*d*(c + d*x^3))
```

sympy [A] time = 0.58, size = 97, normalized size = 0.57

$$\frac{x\left(ad-bc\right)}{3c^2d+3cd^2x^3}+\operatorname{RootSum}\left(729t^3c^5d^4-8a^3d^3-12a^2bcd^2-6ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8*
a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*l
og(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

3.7 $\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=197

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}+\frac{x(5ad+}{18c^2d\left(c+\right.}$$

```
[Out] -1/6*(-a*d+b*c)*x/c/d/(d*x^3+c)^2+1/18*(5*a*d+b*c)*x/c^2/d/(d*x^3+c)+1/27*(
5*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/d^(4/3)-1/54*(5*a*d+b*c)*ln(c^(2/3
)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(4/3)-1/27*(5*a*d+b*c)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(4/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}+\frac{x(5ad+}{18c^2d\left(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c + d
*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/
(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x])/(27*
c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(54*c^(8/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx = -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2} + \frac{(bc+5ad)\int \frac{1}{\left(c+dx^3\right)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2} + \frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)} + \frac{(bc+5ad)\int \frac{1}{c+dx^3}\,dx}{9c^2d}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2} + \frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)} + \frac{(bc+5ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{27c^{8/3}d} + \frac{(bc+5ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x}}{27c^{8/3}d}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2} + \frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+}}{54c^{8/3}d^{4/3}}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2} + \frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\right.}{54c^{8/3}d^{4/3}}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2} + \frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)} - \frac{(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{27c^{8/3}d^{4/3}}$$

Mathematica [A] time = 0.13, size = 175, normalized size = 0.89

$$\frac{-(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right) - \frac{9c^{5/3}\sqrt[3]{d}\,x(bc-ad)}{\left(c+dx^3\right)^2} + \frac{3c^{2/3}\sqrt[3]{d}\,x(5ad+bc)}{c+dx^3} + 2(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right) - 2\sqrt{3}}{54c^{8/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*c
 + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)*x
)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 5*a
*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3))
```

fricas [B] time = 0.46, size = 743, normalized size = 3.77

$$\left[\frac{3\left(bc^3d^2+5\,ac^2d^3\right)x^4+3\sqrt{\frac{1}{3}}\left(\left(bc^2d^3+5\,acd^4\right)x^6+bc^4d+5\,ac^3d^2+2\left(bc^3d^2+5\,ac^2d^3\right)x^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cd\ldots}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^
4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(c^
2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c
*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3
+ c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2
*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3
)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(
c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x
^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(
b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d^2 + 5*
a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)
*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*
x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*
d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^
3*x^3 + c^6*d^2)]
```

giac [A] time = 0.19, size = 180, normalized size = 0.91

$$-\frac{\sqrt{3}\,(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\left(-cd^2\right)^{\frac{2}{3}}c^2}-\frac{(bc+5\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(-cd^2\right)^{\frac{2}{3}}c^2}-\frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\ldots\right.\right.\right.}{27\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^
(1/3))/((-c*d^2)^(2/3)*c^2) - 1/54*(b*c + 5*a*d)*log(x^2 + x*(-c/d)^(1/3) +
 (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2) - 1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*log(a
bs(x - (-c/d)^(1/3)))/(c^3*d) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2*x +
 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

maple [A] time = 0.06, size = 249, normalized size = 1.26

$$\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^{2}d}+\frac{5a\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^{2}d}-\frac{5a\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(\frac{c}{d}\right)^{\frac{2}{3}}c^{2}d}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c\,d^{2}}+\frac{b\ln\left(x\right.}{27\left(\frac{c}{d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^3,x)
```

```
[Out] (1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+5/27/c^2/d/(c/
d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/27/c/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-5/5
4/c^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a-1/54/c/d^2/(c/d)^(2
/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+5/27/c^2/d/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/27/c/d^2/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [A] time = 1.43, size = 192, normalized size = 0.97

$$\frac{\left(bcd+5\,ad^{2}\right)x^{4}-2\left(bc^{2}-4\,acd\right)x}{18\left(c^{2}d^{3}x^{6}+2\,c^{3}d^{2}x^{3}+c^{4}d\right)}+\frac{\sqrt{3}\left(bc+5\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(bc+5\,ad\right)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*d
^2*x^3 + c^4*d) + 1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3)) - 1/54*(b*c + 5*a*d)*log(x^2 -
```

```
x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^2*(c/d)^(2/3)) + 1/27*(b*c + 5*a*d)*log
(x + (c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3))
```

mupad [B] time = 1.40, size = 173, normalized size = 0.88

$$\frac{\frac{x^4(5ad+bc)}{18c^2}+\frac{x(4ad-bc)}{9cd}}{c^2+2cdx^3+d^2x^6}+\frac{\ln\left(d^{1/3}x+c^{1/3}\right)(5ad+bc)}{27c^{8/3}d^{4/3}}-\frac{\ln\left(c^{1/3}-2d^{1/3}x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5ad+bc)}{27c^{8/3}d^{4/3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3)^3,x)
```

```
[Out] ((x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d))/(c^2 + d^2*x^6 +
 2*c*d*x^3) + (log(d^(1/3)*x + c^(1/3))*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
 - (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*a*d + b*c))/(27*c^(8/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x
 - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
```

sympy [A] time = 0.78, size = 133, normalized size = 0.68

$$\frac{x^4\left(5ad^2+bcd\right)+x\left(8acd-2bc^2\right)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6}+\text{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3,\left(t\mapsto t\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d**
2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d**3
 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27*_t
*c**3*d/(5*a*d + b*c) + x)))
```

3.8 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}$$

```
[Out] a^2*c^3*x+1/4*a*c^2*(3*a*d+2*b*c)*x^4+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^7+1/10*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^10+1/13*b*d^2*(2*a*d+3*b*c)*x^13+
1/16*b^2*d^3*x^16
```

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)^3\,dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^3+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6abcd+a^2\right.\right.\\ &= a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{10}d\left(3b^2c^2+6abcd\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

fricas [A] time = 0.38, size = 132, normalized size = 1.08

$$\frac{1}{16}x^{16}d^3b^2+\frac{3}{13}x^{13}d^2cb^2+\frac{2}{13}x^{13}d^3ba+\frac{3}{10}x^{10}dc^2b^2+\frac{3}{5}x^{10}d^2cba+\frac{1}{10}x^{10}d^3a^2+\frac{1}{7}x^7c^3b^2+\frac{6}{7}x^7dc^2ba+\frac{3}{7}x^7d^2ca^2+\frac{1}{2}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/10*x^10*d*c
^2*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/7*x^7*c^3*b^2 + 6/7*x^7
*d*c^2*b*a + 3/7*x^7*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*x^4*d*c^2*a^2 + x*c^
3*a^2
```

giac [A] time = 0.16, size = 132, normalized size = 1.08

$$\frac{1}{16}b^2d^3x^{16}+\frac{3}{13}b^2cd^2x^{13}+\frac{2}{13}abd^3x^{13}+\frac{3}{10}b^2c^2dx^{10}+\frac{3}{5}abcd^2x^{10}+\frac{1}{10}a^2d^3x^{10}+\frac{1}{7}b^2c^3x^7+\frac{6}{7}abc^2dx^7+\frac{3}{7}a^2cd^2x^7+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2*
d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a*b
*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a^2*
c^3*x
```

maple [A] time = 0.05, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{16}}{16}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{10}}{10}+\frac{\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^7}{7}+a^2c^3x+\frac{\left(3a^2c^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^3,x)
```

```
[Out] 1/16*b^2*d^3*x^16+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/4*(3*a^2*c
^2*d+2*a*b*c^3)*x^4+a^2*c^3*x
```

maxima [A] time = 0.55, size = 124, normalized size = 1.02

$$\frac{1}{16}\,b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+a^2c^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

mupad [B] time = 1.20, size = 116, normalized size = 0.95

$$x^7\left(\frac{3\,a^2\,c\,d^2}{7}+\frac{6\,a\,b\,c^2\,d}{7}+\frac{b^2\,c^3}{7}\right)+x^{10}\left(\frac{a^2\,d^3}{10}+\frac{3\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{10}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{16}}{16}+\frac{a\,c^2\,x^4\,\left(3\,a\,d+2\,b\,c\right)}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^3,x)
```

```
[Out] x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^10*((a^2*d^3)/10
+ (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^16)/16 + (a*
c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

sympy [A] time = 0.09, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{16}}{16}+x^{13}\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^7\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^4\left(\frac{3a^2c}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13)
 + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c**3
/2)
```

3.9 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] a^2*c^2*x+1/2*a*c*(a*d+b*c)*x^4+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+1/5*b*d
*(a*d+b*c)*x^10+1/13*b^2*d^2*x^13
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2\,dx=\int \left(a^2c^2+2ac(bc+ad)x^3+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^9+b^2d^2x^{12}$$
$$=a^2c^2x+\frac{1}{2}ac(bc+ad)x^4+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{13}b^2d$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

fricas [A] time = 0.37, size = 91, normalized size = 1.11

$$\frac{1}{13}x^{13}d^2b^2+\frac{1}{5}x^{10}dcb^2+\frac{1}{5}x^{10}d^2ba+\frac{1}{7}x^7c^2b^2+\frac{4}{7}x^7dcba+\frac{1}{7}x^7d^2a^2+\frac{1}{2}x^4c^2ba+\frac{1}{2}x^4dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/7*x^7*c^2*b^2 +
 4/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 1/2*x^4*c^2*b*a + 1/2*x^4*d*c*a^2 + x*
c^2*a^2
```

giac [A] time = 0.21, size = 91, normalized size = 1.11

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{1}{5}\,b^2cdx^{10}+\frac{1}{5}\,abd^2x^{10}+\frac{1}{7}\,b^2c^2x^7+\frac{4}{7}\,abcdx^7+\frac{1}{7}\,a^2d^2x^7+\frac{1}{2}\,abc^2x^4+\frac{1}{2}\,a^2cdx^4+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7 +
 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + a^
2*c^2*x
```

maple [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{b^2d^2x^{13}}{13}+\frac{\left(2ab\,d^2+2b^2cd\right)x^{10}}{10}+\frac{\left(a^2d^2+4abcd+b^2c^2\right)x^7}{7}+a^2c^2x+\frac{\left(2a^2cd+2ab\,c^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/7*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^7+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+a^2*c^2*x
```

maxima [A] time = 0.71, size = 82, normalized size = 1.00

$$\frac{1}{13}\,b^2d^2x^{13}+\frac{1}{5}\left(b^2cd+abd^2\right)x^{10}+\frac{1}{7}\left(b^2c^2+4\,abcd+a^2d^2\right)x^7+a^2c^2x+\frac{1}{2}\left(abc^2+a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

mupad [B] time = 0.04, size = 75, normalized size = 0.91

$$x^7\left(\frac{a^2\,d^2}{7}+\frac{4\,a\,b\,c\,d}{7}+\frac{b^2\,c^2}{7}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^{13}}{13}+\frac{a\,c\,x^4\,\left(a\,d+b\,c\right)}{2}+\frac{b\,d\,x^{10}\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^2,x)
```

```
[Out] x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + a^2*c^2*x + (b^2*d^2*x^13
)/13 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^10*(a*d + b*c))/5
```

sympy [A] time = 0.08, size = 90, normalized size = 1.10

$$a^2c^2x+\frac{b^2d^2x^{13}}{13}+x^{10}\left(\frac{abd^2}{5}+\frac{b^2cd}{5}\right)+x^7\left(\frac{a^2d^2}{7}+\frac{4abcd}{7}+\frac{b^2c^2}{7}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*(
a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

3.10 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

```
[Out] a^2*c*x+1/4*a*(a*d+2*b*c)*x^4+1/7*b*(2*a*d+b*c)*x^7+1/10*b^2*d*x^10
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx &= \int \left(a^2c+a(2bc+ad)x^3+b(bc+2ad)x^6+b^2dx^9\right) \, dx\\ &= a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

fricas [A] time = 0.36, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}db^2 + \frac{1}{7}x^7cb^2 + \frac{2}{7}x^7dba + \frac{1}{2}x^4cba + \frac{1}{4}x^4da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 1/2*x^4*c*b*a + 1/4*x^4*d
*a^2 + x*c*a^2
```

giac [A] time = 0.16, size = 50, normalized size = 1.00

$$\frac{1}{10}b^2dx^{10} + \frac{1}{7}b^2cx^7 + \frac{2}{7}abdx^7 + \frac{1}{2}abcx^4 + \frac{1}{4}a^2dx^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*d
*x^4 + a^2*c*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b^2d\,x^{10}}{10} + \frac{\left(2abd + b^2c\right)x^7}{7} + a^2cx + \frac{\left(a^2d + 2abc\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

maxima [A] time = 0.57, size = 48, normalized size = 0.96

$$\frac{1}{10}b^2dx^{10} + \frac{1}{7}\left(b^2c + 2\,abd\right)x^7 + \frac{1}{4}\left(2\,abc + a^2d\right)x^4 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{b^2\,d\,x^{10}}{10}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3),x)
```

```
[Out] x^4*((a^2*d)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (b^2*d*x^10)/
10 + a^2*c*x
```

sympy [A] time = 0.07, size = 51, normalized size = 1.02

$$a^2cx+\frac{b^2dx^{10}}{10}+x^7\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4 +
 a*b*c/2)
```

3.11 $\int \frac{\left(a+bx^3\right)^2}{c+dx^3}\,dx$

Optimal. Leaf size=173

$$-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}\,+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{7/3}}-\frac{bx(bc-2}{d^2}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/4*b^2*x^4/d+1/3*(-a*d+b*c)^2*ln(c^(1/3)+d^(1/3)*x)/
c^(2/3)/d^(7/3)-1/6*(-a*d+b*c)^2*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/
c^(2/3)/d^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3
^(1/2))/c^(2/3)/d^(7/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}\,+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{7/3}}-\frac{bx(bc-2}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1/
3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c - a
*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2}{c+dx^3}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^3}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^3\right)}\right)dx$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^3}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\int\frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}d^2}+\frac{(bc-ad)^2\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\int\frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{7/3}}+ \cdots$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}\cdots\right.}{6c^{2/3}d^{7/3}}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-\cdots}{\cdots}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)-12bc^{2/3}\sqrt[3]{d}\,x(bc-2ad)+4(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)+4\sqrt{3}\,(bc-\cdots}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] (-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqrt
[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*(b
*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

fricas [A] time = 0.48, size = 505, normalized size = 2.92

$$\left[\frac{3\,b^2c^2d^2x^4+6\,\sqrt{\frac{1}{3}}\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\,\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d
^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*s
qrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3
)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*
d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^
2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d - 2*a*b*
c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3
)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3))
 - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

giac [A] time = 0.19, size = 211, normalized size = 1.22

$$-\frac{\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d}-\frac{\left(b^2c^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) - 1/3*(b^
```

```
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/
(c*d^4) + 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

maple [B] time = 0.05, size = 334, normalized size = 1.93

$$\frac{b^2x^4}{4d}+\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a^2\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{2\sqrt{3}\,abc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] 1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*c*x+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2
-2/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b*c+1/3/d^3/(c/d)^(2/3)*ln(x+(c/d)
^(1/3))*b^2*c^2-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2+1/3
/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a*b*c-1/6/d^3/(c/d)^(2/3
)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^2*c^2+1/3/d/(c/d)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^2-2/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b*c+1/3/d^3/(c/d)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^2*c^2
```

maxima [A] time = 1.23, size = 189, normalized size = 1.09

$$\frac{b^2dx^4-4\left(b^2c-2\,abd\right)x}{4\,d^2}+\frac{\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(x^2-x\left(\right.\right.}{6\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d^3*(c/
d)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(c/d)^(1/3) + (
c/d)^(2/3))/(d^3*(c/d)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (c/d)^(1/3))/(d^3*(c/d)^(2/3))
```

mupad [B] time = 1.39, size = 152, normalized size = 0.88

$$\frac{b^2\,x^4}{4\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,c^{2/3}\,d^{7/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^2}{c^{2/3}\,d^{7/3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3),x)
```

```
[Out] (b^2*x^4)/(4*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (log(d^(1/3)*x + c^(1/3))*(
a*d - b*c)^2)/(3*c^(2/3)*d^(7/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x -
 c^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(c^(2/3)*d^(7/3)) - (log(3^
(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*c^(2/3)*d^(7/3))
```

sympy [A] time = 0.68, size = 156, normalized size = 0.90

$$\frac{b^2x^4}{4d}+x\left(\frac{2ab}{d}-\frac{b^2c}{d^2}\right)+\operatorname{RootSum}\left(27t^3c^2d^7-a^6d^6+6a^5bcd^5-15a^4b^2c^2d^4+20a^3b^3c^3d^3-15a^2b^4c^4d^2+6ab^5c^5d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(27*_t**3*c**2*d**7 -
a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d*
*3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*lo
g(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2} \, dx$

Optimal. Leaf size=203

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)}{3\sqrt{3}\,c^{5/3}}$$

```
[Out] b^2*x/d^2+1/3*(-a*d+b*c)^2*x/c/d^2/(d*x^3+c)-2/9*(-a*d+b*c)*(a*d+2*b*c)*ln(
c^(1/3)+d^(1/3)*x)/c^(5/3)/d^(7/3)+1/9*(-a*d+b*c)*(a*d+2*b*c)*ln(c^(2/3)-c^
(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/d^(7/3)+2/9*(-a*d+b*c)*(a*d+2*b*c)*arc
tan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/d^(7/3)*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)}{3\sqrt{3}\,c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b
*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c
^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \int \left(\frac{b^2}{d^2} - \frac{b^2c^2 - a^2d^2 + 2bd(bc-ad)x^3}{d^2(c+dx^3)^2}\right)dx$$

$$= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{(c+dx^3)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{(2(bc-ad)(2bc+ad))\int \frac{1}{c+dx^3}\,dx}{3cd^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{(2(bc-ad)(2bc+ad))\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{9c^{5/3}d^2} - \frac{(2(bc-ad)(2bc+ad))\int \frac{}{c^{2/3}-}}{9c^{5/3}d^2}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}} + \frac{((bc-ad)(2bc+ad))\int \frac{-}{c^{2/3}}}{9c^{5/3}d^{7/3}}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)\log\left(c^2\right.}{9c^{5/3}d}$$

$$= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} + \frac{2(bc-ad)(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{7/3}} - \frac{2(bc-ad)(2bc+ad)\log\left(\right.}{9c^{5/3}d^{7/3}}$$

Mathematica [A] time = 0.22, size = 210, normalized size = 1.03

$$\frac{-\frac{2(-a^2d^2-abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{5/3}} + \frac{2\sqrt{3}(-a^2d^2-abcd+2b^2c^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} + \frac{(-a^2d^2-abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{5/3}} + \frac{3\sqrt[3]{}}{}}{9d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3]
*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]
])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/c
^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

fricas [B] time = 0.46, size = 771, normalized size = 3.80

$$\left[\frac{9\,b^2c^3d^2x^4-3\,\sqrt{\frac{1}{3}}\left(2\,b^2c^4d-abc^3d^2-a^2c^2d^3+\left(2\,b^2c^3d^2-abc^2d^3-a^2cd^4\right)x^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\,\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*
d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (2*b^
2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3
 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)
^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt(1/3)*(2*
b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*
c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^
2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2*d - a^2*c*d
^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (
c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 +
(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^
(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^
4*d^3)]
```

giac [A] time = 0.19, size = 233, normalized size = 1.15

$$\frac{b^2x}{d^2}+\frac{2\,\sqrt{3}\left(2\,b^2c^2-abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,\left(-cd^2\right)^{\frac{2}{3}}cd}+\frac{\left(2\,b^2c^2-abcd-a^2d^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,\left(-cd^2\right)^{\frac{2}{3}}cd}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 + 2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d) + 1/9*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)
*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)
*c*d^2)
```

maple [B] time = 0.06, size = 367, normalized size = 1.81

$$\frac{a^2 x}{3\left(d x^3+c\right)c}-\frac{2abx}{3\left(d x^3+c\right)d}+\frac{b^2 c x}{3\left(d x^3+c\right)d^2}+\frac{2\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}+\frac{2a^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}-\frac{a^2\ln\left(x^2-\left(\frac{c}{d}\right)\right.}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] b^2*x/d^2+1/3/c*x/(d*x^3+c)*a^2-2/3/d*x/(d*x^3+c)*a*b+1/3/d^2*c*x/(d*x^3+c)
*b^2+2/9/d/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2+2/9/d^2/(c/d)^(2/3)*ln(x+(c/
d)^(1/3))*a*b-4/9/d^3*c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b^2-1/9/d/c/(c/d)^(2/
3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2-1/9/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(
1/3)*x+(c/d)^(2/3))*a*b+2/9/d^3*c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2
/3))*b^2+2/9/d/c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)
)*a^2+2/9/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a
*b-4/9/d^3*c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^
2
```

maxima [A] time = 1.28, size = 226, normalized size = 1.11

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x}{3\left(cd^3x^3+c^2d^2\right)}+\frac{b^2x}{d^2}-\frac{2\sqrt{3}\left(2\,b^2c^2-abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(2\,b^2c^2-abcd-a^2d^2\right)\log\left(x\right.}{9\,cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 - 2
/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/(c*d^3*(c/d)^(2/3)) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2*d^
2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^3*(c/d)^(2/3)) - 2/9*(2*b^2*
c^2 - a*b*c*d - a^2*d^2)*log(x + (c/d)^(1/3))/(c*d^3*(c/d)^(2/3))
```

mupad [B] time = 1.41, size = 191, normalized size = 0.94

$$\frac{b^2\,x}{d^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c\,\left(d^3\,x^3+c\,d^2\right)}+\frac{2\,\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}+\frac{2\,\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\right.}{9\,c^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*c*(c*d^2 + d^3*x^3)) +
 (2*log(d^(1/3)*x + c^(1/3))*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3))
 + (2*log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3)) - (2*log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(a*d + 2*b*c)
)/(9*c^(5/3)*d^(7/3))
```

sympy [A] time = 1.14, size = 189, normalized size = 0.93

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{3c^2d^2+3cd^3x^3}+\mathrm{RootSum}\left(729t^3c^5d^7-8a^6d^6-24a^5bcd^5+24a^4b^2c^2d^4+88a^3b^3c^3d^3-48a^2b^4c^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**3
**x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 24*
a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 - 96*
a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*d**2
 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=258

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{7/3}}-\frac{(5a^2d^2+\cdots}{}$$

```
[Out] -1/6*(-a*d+b*c)*x*(b*x^3+a)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+4*b*c)*x
/c^2/d^2/(d*x^3+c)+1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(1/3)+d^(1/3)*
x)/c^(8/3)/d^(7/3)-1/54*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*ln(c^(2/3)-c^(1/3)*
d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(7/3)-1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)
*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/d^(7/3)*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {413, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{7/3}}-\frac{(5a^2d^2+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c +
5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*A
rcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(7/3
)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(
8/3)*d^(7/3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}+\frac{\int \frac{a(bc+5ad)+2b(2bc+ad)x^3}{\left(c+dx^3\right)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\int \frac{1}{c+dx^3}\,dx}{9c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{27c^{8/3}d^2}+$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}-\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}}$$

Mathematica [A] time = 0.28, size = 234, normalized size = 0.91

$$\frac{2\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-2\sqrt{3}\left(5a^2d^2+2abcd+2b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)-\frac{3c^{2/3}\sqrt[3]{d}x\left(-a^2d^2\left(8c+5\right.\right.}{}}{54c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] ((-3*c^(2/3)*d^(1/3)*x*(2*a*b*c*d*(2*c - d*x^3) - a^2*d^2*(8*c + 5*d*x^3) +
 b^2*c^2*(4*c + 7*d*x^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d
 + 5*a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 +
```

```
2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d +
5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7
/3))
```

fricas [B] time = 0.45, size = 1067, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3)
*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*
d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*
x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*
sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/
3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^
2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^
2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c
) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^
3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^
2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3*(7*b^2*c^4*
d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*c^5*d + 2*a*b
*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5*a^2*c*d^5)*x^
6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*sqrt((c^2*d)^(1/
3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(
1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3
)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
(2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5
*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(
2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^2*c^3*d
^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3)]
```

giac [A] time = 0.20, size = 264, normalized size = 1.02

$$-\frac{\sqrt{3}\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,\left(-cd^2\right)^{\frac{2}{3}}\,c^2d}-\frac{\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,\left(-cd^2\right)^{\frac{2}{3}}\,c^2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x +
 (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c^2*d) - 1/54*(2*b^2*c^2 + 2*a
*b*c*d + 5*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3
)*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(abs(x
- (-c/d)^(1/3)))/(c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^
2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2
*d^2)
```

maple [F(-1)]   time = 180.00, size = 0, normalized size = 0.00

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

maxima [A]   time = 1.32, size = 267, normalized size = 1.03

$$-\frac{\left(7\,b^2c^2d-2\,abcd^2-5\,a^2d^3\right)x^4+4\left(b^2c^3+abc^2d-2\,a^2cd^2\right)x}{18\left(c^2d^4x^6+2\,c^3d^3x^3+c^4d^2\right)}+\frac{\sqrt{3}\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right.\right.}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*d
 - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*sqrt(3)*(
2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(
c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3)) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2
)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^3*(c/d)^(2/3)) + 1/27*(2*b^
2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*log(x + (c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3))
```

mupad [B]   time = 1.43, size = 249, normalized size = 0.97

$$\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}-\frac{\frac{2\,x\left(-2\,a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{9\,c\,d^2}-\frac{x^4\left(5\,a^2\,d^2+2\,a\,b\,c\,d-7\,b^2\,c^2\right)}{18\,c^2\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3)^3,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*
d^(7/3)) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2*d
^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (log
(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a^2*
d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3)) - (log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*c^2 +
2*a*b*c*d))/(27*c^(8/3)*d^(7/3))
```

sympy [A] time = 1.62, size = 233, normalized size = 0.90

$$\frac{x^4\left(5a^2d^3+2abcd^2-7b^2c^2d\right)+x\left(8a^2cd^2-4abc^2d-4b^2c^3\right)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}+\text{RootSum}\left(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*a
*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**4*
x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**5 -
 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*d**2
 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(5*a**
2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

3.14 $\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$

Optimal. Leaf size=252

$$-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{13/3}}+\frac{dx(2bc-$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/4*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^4/b^3+1/7*d^3*(-a*d+4*b*c)*x^7/b^2+1/10*d^4*x^10/b+1/3*(-a*
d+b*c)^4*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/6*(-a*d+b*c)^4*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/3)-1/3*(-a*d+b*c)^4*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(13/3)*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d
^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^4}{a+bx^3}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{b^3} + \frac{d^3(4bc-ad)x^6}{b^2} + \right.$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2} + \frac{d^4}{1}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2} + \frac{d^4}{1}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2} + \frac{d^4}{1}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2} + \frac{d^4}{1}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2} + \frac{d^4}{1}$$

Mathematica [A] time = 0.13, size = 253, normalized size = 1.00

$$\frac{-\frac{70(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{140(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{140\sqrt{3}(bc-ad)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + 105b^{4/3}d^2x^4\left(a^2d^2-4ab\right.}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 10
5*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b*c
 - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(-a^
(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

fricas [A] time = 0.47, size = 873, normalized size = 3.46

$$\left[\frac{42\,a^2b^4d^4x^{10} + 60\left(4\,a^2b^4cd^3 - a^3b^3d^4\right)x^7 + 105\left(6\,a^2b^4c^2d^2 - 4\,a^3b^3cd^3 + a^4b^2d^4\right)x^4 + 210\,\sqrt{\frac{1}{3}}\left(ab^5c^4 - 4\,a^2b^4c^3 \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*
(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4
)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqr
t(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1
/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d -
 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1/420*(42*a
^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(6*a^2*b^4*c
^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*b^5*c^4 - 4*
a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^
2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d
 - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5)]
```

giac [A] time = 0.20, size = 391, normalized size = 1.55

$$-\frac{\sqrt{3}\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^3} - \frac{\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a \cdots\right.}{6 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(
2/3)*b^3) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^3)
- 1/3*(b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4
*b^6*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*
d^4*x^10 + 80*b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4 - 140*
a*b^8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*c^2*d^2*
x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

maple [B] time = 0.05, size = 661, normalized size = 2.62

$$\frac{d^4x^{10}}{10b}-\frac{a\,d^4x^7}{7b^2}+\frac{4c\,d^3x^7}{7b}+\frac{a^2d^4x^4}{4b^3}-\frac{ac\,d^3x^4}{b^2}+\frac{3c^2d^2x^4}{2b}+\frac{\sqrt{3}\,a^4d^4\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}+\frac{a^4d^4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}-\frac{a^4d^4\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a),x)
```

```
[Out] -4/3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*c*
d^3+2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*c
^2*d^2-4/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*
a*c^3*d-1/7*d^4/b^2*x^7*a+4/7*d^3/b*x^7*c-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))*c^4+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^4+4*d/b*c^3*x+
1/4*d^4/b^3*x^4*a^2+3/2*d^2/b*x^4*c^2-d^4/b^4*a^3*x-4/3/b^4/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*a^3*c*d^3+2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*c^2*d^2-4/
3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c^3*d+2/3/b^4/(a/b)^(2/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))*a^3*c*d^3-1/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))*a^2*c^2*d^2+2/3/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*a*c^3*d+1/3/b^5/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*a^4*d^4-6*d^2/b^2*a*c^2*x-d^3/b^2*x^4*a*c+4*d^3/b^3*a^2*c*x+1/3/b/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^4+1/3/b^5/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))*a^4*d^4-1/6/b^5/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*a^4*d^4+1/10*d^4*x^10/b
```

maxima [A] time = 1.19, size = 364, normalized size = 1.44

$$\frac{14\,b^3d^4x^{10}+20\left(4\,b^3cd^3-ab^2d^4\right)x^7+35\left(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4\right)x^4+140\left(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2bcd^3-\right.}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*d
^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b^5*(a/b)^(2/3)) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

mupad [B] time = 1.43, size = 250, normalized size = 0.99

$$x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^7\left(\frac{a\,d^4}{7\,b^2}-\frac{4\,c\,d^3}{7\,b}\right)+x^4\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{4\,b}+\frac{3\,c^2\,d^2}{2\,b}\right)+\frac{d^4\,x^{10}}{10\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right.}{3\,a^{2/3}\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^4/(a + b*x^3),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(4*b) + (3*c^2*d^2)/(2*b)) + (d^4*x^10)/(10*b) + (log(b^(1/3)*x + a^
(1/3))*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^4)/(a^(2/3)*b^(13/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3))
```

sympy [A] time = 1.31, size = 371, normalized size = 1.47

$$x^7\left(-\frac{ad^4}{7b^2}+\frac{4cd^3}{7b}\right)+x^4\left(\frac{a^2d^4}{4b^3}-\frac{acd^3}{b^2}+\frac{3c^2d^2}{2b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)+\mathrm{RootSum}\left(27t^3a^2b^{13}-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*d
**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 - 6
*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(27*_t**3*a**2*b**13 - a**12*d**12
 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9
- 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d*
*6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**9*c**9
*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12, Lambda
(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)
```

3.15 $\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$

Optimal. Leaf size=208

$$-\frac{(bc-ad)^3 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3 \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{dx\left(a^2d^2-\right.}{}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/4*d^2*(-a*d+3*b*c)*x^4/b^2+1/7*d^3*
x^7/b+1/3*(-a*d+b*c)^3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3)-1/6*(-a*d+b*c
)^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/3)-1/3*(-a*d+b*
c)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(10/3)*3^(
1/2)
```

Rubi [A] time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3 \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3 \tan}{\sqrt{3}a^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^
2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^3}{a+bx^3}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^3}{b^2}+\frac{d^3x^6}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^3\right)}\right) d$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^3}\,dx}{b^3}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b^3}+\frac{(bc-ad)^3}{}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-a}{}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-a}{}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{(bc-a}{}$$

Mathematica [A] time = 0.11, size = 203, normalized size = 0.98

$$\frac{\frac{14(ad-bc)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{28(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}}+\frac{28\sqrt{3}\,(bc-ad)^3\tan^{-1}\left(\frac{2\sqrt[3]{b}\,x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+84\sqrt[3]{b}\,dx\left(a^2d^2-3abcd+3b^2\right.}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c -
 a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1/3)
 + b^(1/3)*x]])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]])/a^(2/3))/(84*b^(10/3))
```

fricas [A] time = 0.44, size = 700, normalized size = 3.37

$$\left[\frac{12\,a^{2}b^{3}d^{3}x^{7} + 21\left(3\,a^{2}b^{3}cd^{2} - a^{3}b^{2}d^{3}\right)x^{4} - 42\,\sqrt{\frac{1}{3}}\left(ab^{4}c^{3} - 3\,a^{2}b^{3}c^{2}d + 3\,a^{3}b^{2}cd^{2} - a^{4}bd^{3}\right)\sqrt{\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^{3}+}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sqr
t(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((-a
^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2
*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)
^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)
^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^2*b^4),
1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 + 84*sqrt
(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt(-(-a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(
a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3
)*x)/(a^2*b^4)]
```

giac [A] time = 0.18, size = 296, normalized size = 1.42

$$-\frac{\sqrt{3}\left(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}} - \frac{\left(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3}\right)\log\left(x^{2} + \right.}{6\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b
^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1
/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 + 84*b^6*c^2*d*x -
84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

maple [B] time = 0.05, size = 486, normalized size = 2.34

$$\frac{d^3x^7}{7b}-\frac{a\,d^3x^4}{4b^2}+\frac{3c\,d^2x^4}{4b}-\frac{\sqrt{3}\;a^3d^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{a^3d^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{a^3d^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{\sqrt{3}\;a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^3*d^3+1/b^3/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))*a^2*c*d^2-1/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c^2*d
+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3+1/6/b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*a^3*d^3-1/2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*a^2*c*d^2+1/2/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*c^
2*d-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3-1/3/b^4/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*d^3+1/b^3/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*c*d^2-1/b^2/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*c^2*d+1/3/b/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3
```

maxima [A] time = 1.32, size = 273, normalized size = 1.31

$$\frac{4\,b^2d^3x^7+7\left(3\,b^2cd^2-abd^3\right)x^4+28\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{28\,b^3}+\frac{\sqrt{3}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*
(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

mupad [B] time = 1.40, size = 192, normalized size = 0.92

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^4\,\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)+\frac{d^3\,x^7}{7\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)^3}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a\ldots\right.}{3\,a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^4*((a*d^3)/(4*b^2)
- (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c
)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^3)/
(a^(2/3)*b^(10/3))
```

sympy [A] time = 1.00, size = 257, normalized size = 1.24

$$x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\mathrm{RootSum}\left(27t^3a^2b^{10}+a^9d^9-9a^8bcd^8+36a^7b^2c^2d^7-84a^6b^3c^3\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*d*
*8 + 36*a**7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**4*d
**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c**7*
d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*d*
*3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**7/(7*b
)
```

3.16 $\int \frac{(c+dx^3)^2}{a+bx^3}\, dx$

Optimal. Leaf size=173

$$-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-a}{b^2}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/4*d^2*x^4/b+1/3*(-a*d+b*c)^2*ln(a^(1/3)+b^(1/3)*x)/a
^(2/3)/b^(7/3)-1/6*(-a*d+b*c)^2*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(2/3)/b^(7/3)-1/3*(-a*d+b*c)^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^
(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-a}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*d)
^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{a+bx^3}\,dx = \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^3\right)}\right)dx$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^3}\,dx}{b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b^2}+\frac{(bc-ad)^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{7/3}}+\frac{(bc}{}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad}{}$$

Mathematica [A] time = 0.15, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+3a^{2/3}b^{4/3}d^2x^4-12a^{2/3}\sqrt[3]{b}\,dx(ad-2bc)+4(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4*(
b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

fricas [A] time = 0.45, size = 507, normalized size = 2.93

$$\left[\frac{3\,a^{2}b^{2}d^{2}x^{4}+6\,\sqrt{\frac{1}{3}}\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^{3}-3\,\left(a^{2}b\right)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}\right.}{}}}{bx^{3}+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^
2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]
```

giac [A] time = 0.32, size = 211, normalized size = 1.22

$$-\frac{\sqrt{3}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(b^{4}c^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b^
```

```
4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^4) + 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

maple [B] time = 0.04, size = 334, normalized size = 1.93

$$\frac{d^2x^4}{4b}+\frac{\sqrt{3}\,a^2d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{a^2d^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{a^2d^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{2\sqrt{3}\,acd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a),x)
```

```
[Out] 1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*c*x+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a
^2*d^2-2/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c*d+1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c^2-1/6/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*d
^2+1/3/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*c*d-1/6/b/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*d^2-2/3/b^2/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*c*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2
```

maxima [A] time = 1.32, size = 190, normalized size = 1.10

$$\frac{bd^2x^4+4\left(2\,bcd-ad^2\right)x}{4\,b^2}+\frac{\sqrt{3}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(x^2-x\left(\frac{a}{b}\right)\right.}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/
b)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 1.38, size = 152, normalized size = 0.88

$$\frac{d^2 x^4}{4b} - x\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,(a\,d - b\,c)^2}{3\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d - b\,c)^2}{a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3),x)
```

```
[Out] (d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (log(b^(1/3)*x + a^(1/3))*(
a*d - b*c)^2)/(3*a^(2/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(a^(2/3)*b^(7/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*a^(2/3)*b^(7/3))
```

sympy [A] time = 0.69, size = 156, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2} + \frac{2cd}{b}\right) + \operatorname{RootSum}\left(27t^3a^2b^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**
5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4
*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/(a*
*2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)
```

3.17 $\int \frac{c+dx^3}{a+bx^3} dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

```
[Out] d*x/b+1/3*(-a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(-a*d+b*c)*l
n(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(-a*d+b*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^3}\,dx}{b}$$
$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b} + \frac{(bc-ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b}$$
$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}}{2\sqrt[3]{a}\,b}$$
$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int\right.}{a^{2}}$$
$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\right.}{6a^{2/3}b^{4/3}}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}\,dx+2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{}}{\sqrt[3]{}}}{\sqrt{3}}\right.}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

fricas [A] time = 0.43, size = 390, normalized size = 2.69

$$\left[\frac{6\,a^2bdx-3\sqrt{\frac{1}{3}}\left(ab^2c-a^2bd\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(-a^2b\right)^{\frac{2}{3}}}{6\,a^2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2)]

giac [A] time = 0.19, size = 133, normalized size = 0.92

$$-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}}+\frac{dx}{b}-\frac{(bc-ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\right.\right.\right.}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")

[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)

maple [A] time = 0.05, size = 195, normalized size = 1.34

$$-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{ad\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{c\ln\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)/(b*x^3+a),x)

```
[Out] d*x/b-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d+1/3/b/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*c+1/6/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*d-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/3/b^2/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 1.08, size = 128, normalized size = 0.88

$$\frac{dx}{b}+\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c - a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

mupad [B] time = 1.38, size = 123, normalized size = 0.85

$$\frac{d\,x}{b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3),x)
```

```
[Out] (d*x)/b - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c))/(3*a^(2/3)*b^(4/3)) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*a^(2/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*a^(2/3)*b^(4/3))
```

sympy [A] time = 0.44, size = 71, normalized size = 0.49

$$\operatorname{RootSum}\left(27t^3a^2b^4+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(-\frac{3tab}{ad-bc}+x\right)\right)\right)+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2}\right.}{6c^{2/3}(bc-ad)}$$

```
[Out] 1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2}\right.}{6c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}}{3c^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)} + \frac{b\int -}{2}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log}{3c^{2/3}}$$

Mathematica [A] time = 0.11, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{2/3}}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

fricas [A] time = 0.50, size = 254, normalized size = 0.88

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right) + 2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right) - \left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

giac [A] time = 0.27, size = 278, normalized size = 0.97

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^{2}d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^{2}-acd\right)}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,abc-\sqrt{3}\,a^{2}d}-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\cdots\right.}{3\cdots}\right.}{\sqrt{3}\,bc^{2}-\sqrt{3}\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

maple [A] time = 0.05, size = 222, normalized size = 0.77

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)\cdots\right.}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d-b*c)/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(a*d-b*c)/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/
6/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/(a*d-b*c)/(c/
d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.24, size = 293, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)
^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) -
a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2
/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

mupad [B] time = 7.70, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
```

```
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

sympy [A] time = 79.72, size = 447, normalized size = 1.55

$$\mathrm{RootSum}\left(t^3\left(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3\right) + b^2, \left(t \mapsto t\log\left(x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 16}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a
**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243
*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b*
*3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*
a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2
*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c*
*3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (8
1*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c
**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*
_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c*
*3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

3.19 $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{\ldots}\right.}{18c^{5/3}(bc-a\ldots}$$

```
[Out] -1/3*d*x/c/(-a*d+b*c)/(d*x^3+c)+1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(
-a*d+b*c)^2-1/9*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+
b*c)^2-1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+
b*c)^2+1/18*d^(2/3)*(-2*a*d+5*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(5/3)/(-a*d+b*c)^2-1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/a^(2/3)/(-a*d+b*c)^2*3^(1/2)+1/9*d^(2/3)*(-2*a*d+5*b*c)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^2*3^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{\ldots}\right.}{18c^{5/3}(bc-a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c -
 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/
3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*
d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*c
- a*d)^2) - (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx = -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{\int \frac{3bc-2ad-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^2\int \frac{1}{a+bx^3}\,dx}{(bc-ad)^2} - \frac{(d(5bc-2ad))\int \frac{1}{c+dx^3}\,dx}{3c(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}(bc-ad)^2} + \frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)^2} - \frac{(d(5bc-2\ldots}{9c^{5\ldots}}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}(bc-ad)^2}\ldots$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}(bc-ad)^2}\ldots$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} - \frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^2}\ldots$$

Mathematica [A] time = 0.24, size = 336, normalized size = 0.97

$$-3b^{5/3}c^{5/3}\left(c+dx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+a^{2/3}d^{2/3}\left(c+dx^3\right)(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] (6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x^
3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] +
```

```
6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5/3)*(
c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d^(2/3)
*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]
)/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

fricas [A] time = 7.44, size = 432, normalized size = 1.25

$$\frac{6\sqrt{3}\left(bcdx^3+bc^2\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}\left(\left(5bcd-2ad^2\right)x^3+5bc^2-2acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}c\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a
*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 + 5
*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3)
 - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*
x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c
^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^
2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*l
og(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

giac [A] time = 0.20, size = 443, normalized size = 1.28

$$-\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{(5bcd\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d +
 a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/6*(
-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
```

```
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3)*b*c
- 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) - 1/18*(5*
(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d
)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3 + c)*(b*c^
2 - a*c*d))
```

maple [A] time = 0.05, size = 406, normalized size = 1.17

$$\frac{a d^2 x}{3(ad-bc)^2\left(dx^3+c\right)c}-\frac{bdx}{3(ad-bc)^2\left(dx^3+c\right)}+\frac{2\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9(ad-bc)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c}+\frac{2ad\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9(ad-bc)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c}-\frac{ad\ln\left(x^2-\right.}{9(ad-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b/(a*d-b*c)^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*b/(a*d-b*c)^2/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b/(a*d-b*c)^2/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d^2/(a*d-b*c)^2/c*x/(d*x^3+c)*a
-1/3*d/(a*d-b*c)^2*x/(d*x^3+c)*b+2/9*d/(a*d-b*c)^2/c/(c/d)^(2/3)*ln(x+(c/d)
^(1/3))*a-5/9/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9*d/(a*d-b*c)^2
/c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+5/18/(a*d-b*c)^2/(c/d)^(
2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/9*d/(a*d-b*c)^2/c/(c/d)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-5/9/(a*d-b*c)^2/(c/d)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [A] time = 1.30, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abc^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{d}{3\left(bc^3-ac^2d+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^2
*(a/b)^(1/3) - 2*a*b*c*d*(a/b)^(1/3) + a^2*d^2*(a/b)^(1/3))*(a/b)^(1/3)) -
```

```
1/9*sqrt(3)*(5*b*c - 2*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1
/3))/((b^2*c^3*(c/d)^(1/3) - 2*a*b*c^2*d*(c/d)^(1/3) + a^2*c*d^2*(c/d)^(1/3
))*(c/d)^(1/3)) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3) - 1/6
*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*b*c*d*
(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) + 1/18*(5*b*c - 2*a*d)*log(x^2 - x*(c/d)
^(1/3) + (c/d)^(2/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*
c*d^2*(c/d)^(2/3)) + 1/3*b*log(x + (a/b)^(1/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*
b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) - 1/9*(5*b*c - 2*a*d)*log(x + (c/d
)^(1/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*c*d^2*(c/d)^(
2/3))
```

mupad [B] time = 16.81, size = 2589, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^2),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c
+ (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5
*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(2/3))/81 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d
 - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(1/3))/9 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^
2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*((8*a^3*d^5 - 125*b^3*c^3*d^2
 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 - 43
74*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^
4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*
c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*(a*d + b*c)
*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*(b^5/(a^
2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^
2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/
3))/3 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*
d^2))/(9*c^3*(a*d - b*c)^4))*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^
5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 -
 162*a^7*b*c*d^5))^(1/3) + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((27
*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (27*a*b^3
*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3
)/(c^5*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^
3)/(c^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*
a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)
^3)/(c^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 +
 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*
((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^
6*c^11 + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 145
```

```
80*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(
3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c - (27*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(1/
3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(2
/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d
^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(
1/3))/18 - (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b
*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*a^3*d^5 - 125*b^3*c^3*
d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 -
 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935
*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*((
(3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*
a*b*c*d))/c + (81*a*b^3*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*
(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*
c)^6))^(2/3))/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a
^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 + (2
*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c
^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162
*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^
2*d^4 - 162*a^7*b*c*d^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i +
 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c -
 (81*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*
d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*c)^6))^(2/3))
/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/
(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 - (2*b^6*d^5*x*(4
*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c
)^4))*(3^(1/2)*1i + 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*
d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a
^7*b*c*d^5))^(1/3))/2 + (d*x)/(3*c*(c + d*x^3)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.20 $\int \frac{\left(c+dx^3\right)^5}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=320

$$-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad}{3}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/4*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^4/b^4+1/7*d^4*(-2*a*d+5*b*c)*x^7/b^3+1/10*
d^5*x^10/b^2+1/3*(-a*d+b*c)^5*x/a/b^5/(b*x^3+a)+1/9*(-a*d+b*c)^4*(13*a*d+2*
b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(16/3)-1/18*(-a*d+b*c)^4*(13*a*d+2*b*c
)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(16/3)-1/9*(-a*d+b*c)
^4*(13*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)
/b^(16/3)*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)}{4b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}\right.}{18a^{5/3}b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d
)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^
3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c +
13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^5}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{b^4}+\frac{d^4(5bc\ldots}{b\ldots}\right.$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+\frac{d^4(5bc-\ldots}{7b^3\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+\frac{d^4(5bc-\ldots}{7b^3\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+\frac{d^4(5bc-\ldots}{7b^3\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+\frac{d^4(5bc-\ldots}{7b^3\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+\frac{d^4(5bc-\ldots}{7b^3\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+\frac{d^4(5bc-\ldots}{7b^3\ldots}$$

Mathematica [A] time = 0.30, size = 313, normalized size = 0.98

$$\frac{-\frac{70(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{140(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{140\sqrt{3}(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3
)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(7/
```

```
3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c - a*
d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a*d)^4
*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126
0*b^(16/3))
```

fricas [B] time = 0.47, size = 1619, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10
 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(4
0*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)
*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2
+ 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a
^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 +
 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d -
40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*
b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*
b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^
2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5
*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^
5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 -
 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d
^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(126*a^3*b^5*d^5*x^13
 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*a^3*b^5*c^2*d^3 - 5
0*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*b^5*c^3*d^2 - 70*a^4*b^
4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)*x^4 + 420*sqrt(1/3)*(2*a^2*b
^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6
*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3
*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt((a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
(a^2*b)^(1/3)/b)/a^2) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*
d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b
^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*
a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*
c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2
```

```
*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(
a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*
d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d
^5)*x)/(a^3*b^7*x^3 + a^4*b^6)]
```

giac [A] time = 0.19, size = 529, normalized size = 1.65

$$-\frac{\sqrt{3}\left(2\,b^{5}c^{5}+5\,ab^{4}c^{4}d-40\,a^{2}b^{3}c^{3}d^{2}+70\,a^{3}b^{2}c^{2}d^{3}-50\,a^{4}bcd^{4}+13\,a^{5}d^{5}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{4}}-\frac{\left(2\,b^{5}c^{5}+5\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c
^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^4) - 1/18*(2*b^5*c^5 + 5*a*b^4*c^4*d -
 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^4) - 1/9*(2*b^5*c
^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d
^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^5) + 1/3*(b
^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x +
5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/140*(14*b^18*d^5*x^10
+ 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 - 350*a*b^1
7*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*a*b^17*c^2*
d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

maple [B] time = 0.06, size = 905, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^5/(b*x^3+a)^2,x)
```

```
[Out] -50/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
*d^4+70/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*c^2*d^3-40/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*c^3*d^2-2/7*d^5/b^3*x^7*a+5/7*d^4/b^2*x^7*c+3/4*d^5/b^4*x^4*a^2+5/2*
d^3/b^2*x^4*c^2-4*d^5/b^5*a^3*x+10*d^2/b^2*c^3*x+1/3/a*x/(b*x^3+a)*c^5-35/9
/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d^3+20/9/b^3*a/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3*d^2-50/9/b^5*a^3/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))*c*d^4+70/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2*d
```

```
^3-40/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3*d^2+25/9/b^5*a^3/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c*d^4-10/3/b^3*a^2*x/(b*x^3+a)*c^2*d^3+
10/3/b^2*a*x/(b*x^3+a)*c^3*d^2+13/9/b^6*a^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^5+5/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*c^4*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*c^5+5/3/b^4*a^3*x/(b*x^3+a)*c*d^4-13/18/b^6*a^4/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^5-5/18/b^2/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))*c^4*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))*c^5+13/9/b^6*a^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^5+5/9/b^2/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*c^4*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^5-
5/2*d^4/b^3*x^4*a*c+15*d^4/b^4*a^2*c*x-1/3/b^5*a^4*x/(b*x^3+a)*d^5-5/3/b*x/
(b*x^3+a)*c^4*d-20*d^3/b^3*a*c^2*x+1/10*d^5*x^10/b^2
```

maxima [A] time = 1.35, size = 509, normalized size = 1.59

$$\frac{\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)x}{3\left(ab^6x^3 + a^2b^5\right)} + \frac{14\,b^3d^5x^{10} + 20\left(5\,b^3cd^4 - 2\,ab^2d^5\right)x^7 + 35\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + 1/140*(14*b^3*d^5*x^10 + 2
0*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3
*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
 4*a^3*d^5)*x)/b^5 + 1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^
3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^6*(a/b)^(2/3)) - 1/18*(2*b^5*c^5 +
5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 +
13*a^5*d^5)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^6*(a/b)^(2/3)) + 1/
9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50
*a^4*b*c*d^4 + 13*a^5*d^5)*log(x + (a/b)^(1/3))/(a*b^6*(a/b)^(2/3))
```

mupad [B] time = 0.39, size = 416, normalized size = 1.30

$$x\left(\frac{10\,c^3\,d^2}{b^2} - \frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b} - \frac{a^2\,d^5}{b^4} + \frac{10\,c^2\,d^3}{b^2}\right)}{b} + \frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right) - x^7\left(\frac{2\,a\,d^5}{7\,b^3} - \frac{5\,c\,d^4}{7\,b^2}\right) + x^4\left(\frac{a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right.}{2\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^5/(a + b*x^3)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3 -
(5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5*x^1
0)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d
^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) - (log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*
c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b
^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d + 2*b*c))/
(9*a^(5/3)*b^(16/3))
```

sympy [A] time = 12.43, size = 546, normalized size = 1.71

$$x^7\left(-\frac{2ad^5}{7b^3}+\frac{5cd^4}{7b^2}\right)+x^4\left(\frac{3a^2d^5}{4b^4}-\frac{5acd^4}{2b^3}+\frac{5c^2d^3}{2b^2}\right)+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)+\frac{x\left(-a^5d^5+5a^4b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4)
- 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a
**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5
+ 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b
**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t**3*a
**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b**2*c*
*2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**11 + 12
71886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*a**8*b**
7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d**6 + 7194
*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*b**12*c**1
2*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**15*c**15, La
mbda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4 + 70*a**3*b
**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b**5*c**5) + x
))) + d**5*x**10/(10*b**2)
```

3.21 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2} dx$

Optimal. Leaf size=267

$$-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{2(bc-ad)^3(5ad}{3\sqrt{3}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+1/2*d^3*(-a*d+2*b*c)*x^4/b^3+1/7*
d^4*x^7/b^2+1/3*(-a*d+b*c)^4*x/a/b^4/(b*x^3+a)+2/9*(-a*d+b*c)^3*(5*a*d+b*c)
*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(13/3)-1/9*(-a*d+b*c)^3*(5*a*d+b*c)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(13/3)-2/9*(-a*d+b*c)^3*(5*a*
d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(13/3)*3
^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{\right.}}{9a^{5/3}b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(
2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(
b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3
) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^4}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)}{b^4} + \frac{2d^3(2bc-ad)x^3}{b^3} + \frac{d^4x^6}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd(bc}{b^4\left(a+bx^3\right)^2}\right.$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^3}{\left(a+bx^3\right)^2}\,dx}{b^4}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)} + \frac{\left(2(bc-ad)^3(bc\right.}{3}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)} + \frac{\left(2(bc-ad)^3(bc\right.}{9}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)} + \frac{2(bc-ad)^3(bc+}{9}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)} + \frac{2(bc-ad)^3(bc+}{9}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)} - \frac{2(bc-ad)^3(bc+}{3\sqrt{}}$$

Mathematica [A] time = 0.24, size = 260, normalized size = 0.97

$$\frac{\frac{14(ad-bc)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}} + \frac{28(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} + \frac{28\sqrt{3}(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} + 126\sqrt[3]{b}}{126b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2*
b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a +
b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^
```

```
(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

fricas [B] time = 0.49, size = 1316, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63
*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/3)
*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5
*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3
*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^
2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)
^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a^2*b^3*
c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^
4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b
)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4
+ 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*
c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*
x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^
6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*
b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x
^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*
a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2
*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arcta
n(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)
/a^2) - 14*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d
^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2
*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
(-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 +
14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2
+ 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^
(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^
2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^6*x^3 + a^4*b^5)]
```

giac [A] time = 0.19, size = 412, normalized size = 1.54

$$-\frac{2\sqrt{3}\left(b^4c^4 + 2\,ab^3c^3d - 12\,a^2b^2c^2d^2 + 14\,a^3bcd^3 - 5\,a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3} - \frac{\left(b^4c^4 + 2\,ab^3c^3d - 12\,a^2b^2c^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3
 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^
3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*
c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/3*
(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*
d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^4 - 7*a
*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^10*d^4*x)
/b^14
```

maple [B] time = 0.06, size = 708, normalized size = 2.65

$$\frac{d^4x^7}{7b^2}-\frac{a\,d^4x^4}{2b^3}+\frac{c\,d^3x^4}{b^2}+\frac{a^3d^4x}{3\left(b\,x^3+a\right)b^4}-\frac{4a^2c\,d^3x}{3\left(b\,x^3+a\right)b^3}+\frac{2a\,c^2d^2x}{\left(b\,x^3+a\right)b^2}+\frac{c^4x}{3\left(b\,x^3+a\right)a}-\frac{4c^3dx}{3\left(b\,x^3+a\right)b}-\frac{10\sqrt{3}\,a^3d^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a)^2,x)
```

```
[Out] -8/3/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*
d^2+28/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*c*d^3+6*d^2/b^2*c^2*x-1/2*d^4/b^3*x^4*a+d^3/b^2*x^4*c+1/3/a*x/(b*x^3+a)*c
^4+3*d^4/b^4*a^2*x-10/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*d^4+4/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*c^3*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*c^4-4/3/b^3*a^2*x/(b*x^3+a)*c*d^3+2/b^2*a*x/(b*x^3+a)*c^2*d^2+
28/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*d^3-8/3/b^3*a/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))*c^2*d^2-14/9/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))*c*d^3+4/3/b^3*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d^2
+5/9/b^5*a^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^4-10/9/b^5*a^3
/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^4-2/9/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))*c^3*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c
^4+1/3/b^4*a^3*x/(b*x^3+a)*d^4-4/3/b*x/(b*x^3+a)*c^3*d-8*d^3/b^3*a*c*x+4/9/
b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*c^4+1/7*d^4*x^7/b^2
```

maxima [A] time = 1.16, size = 397, normalized size = 1.49

$$\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)x}{3\left(ab^5x^3+a^2b^4\right)}+\frac{2\,b^2d^4x^7+7\left(2\,b^2cd^3-abd^4\right)x^4+14\left(6\,b^2c^2d^2-8\,abcd^3+3\,a\right.}{14\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4)*
x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*sqrt(3)*(b^
4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^5*(a/b)^(2/3)) - 1/9
*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 2/9*(b^4*c^4
 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*log(x +
 (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

mupad [B] time = 1.49, size = 302, normalized size = 1.13

$$x\left(\frac{2\,a\,\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^4\,\left(\frac{a\,d^4}{2\,b^3}-\frac{c\,d^3}{b^2}\right)+\frac{d^4\,x^7}{7\,b^2}+\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d\right.}{3\,a\,\left(b^5\,x^3+a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^4/(a + b*x^3)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*b^4
 + b^5*x^3)) - (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3*(5*a*d + b*c))/(9*
a^(5/3)*b^(13/3)) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3)) - (2*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d
- b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
```

sympy [A] time = 8.54, size = 405, normalized size = 1.52

$$x^4\left(-\frac{ad^4}{2b^3}+\frac{cd^3}{b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)+\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{3a^2b^4+3ab^5x^3}+\text{RootSum}\left(72\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b*
*3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*
d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + RootSum
(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720*a**
10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*d**8
- 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b**7*c**
7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a**2*b**1
0*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, Lambda(_t, _t*log(-9*_t*
a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)
```

3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+}{3\sqrt{3}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/4*d^3*x^4/b^2+1/3*(-a*d+b*c)^3*x/a/b^3/(b*x^3+a)
+1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(10/3)-1/18
*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(10/3)-1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)
/a^(1/3)*3^(1/2))/a^(5/3)/b^(10/3)*3^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^
3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2*b*
c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*
(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)
*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^3}{\left(a+bx^3\right)^2}\,dx=\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^3}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{b^3\left(a+bx^3\right)^2}\right)dx$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{\left(a+bx^3\right)^2}\,dx}{b^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{a+bx^3}\,dx}{3ab^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b^3}+\frac{\left((bc-ad\right.}{}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}-\frac{\left((bc-\right.}{}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-a}{}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}-\frac{(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{10/3}}+\frac{(bc-a}{}$$

Mathematica [A] time = 0.16, size = 227, normalized size = 0.97

$$\frac{-\frac{2(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{4(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}+\frac{4\sqrt{3}(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}\,x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+36\sqrt[3]{b}\,d}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c -
a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan
[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2*(2
*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b*c +
 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/
3))
```

fricas [B] time = 0.46, size = 1027, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sqr
t(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 +
(2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqr
t(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3
)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(
b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3
 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b
)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 +
 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d
- 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*
x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 -
 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^
4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d
^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/
3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2
 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)
*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^
3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

giac [A] time = 0.20, size = 319, normalized size = 1.36

$$-\frac{\sqrt{3}\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^2}-\frac{\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\mathrm{l}}{18\left(-ab^2\right)^{\frac{2}{3}}ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1
```

```
/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c^3 + 3*a*b^
2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^2*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3
*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^5*d^3*x
)/b^8
```

maple [B] time = 0.05, size = 529, normalized size = 2.26

$$\frac{d^3x^4}{4b^2}-\frac{a^2d^3x}{3\left(b\,x^3+a\right)b^3}+\frac{ac\,d^2x}{\left(b\,x^3+a\right)b^2}+\frac{c^3x}{3\left(b\,x^3+a\right)a}-\frac{c^2dx}{\left(b\,x^3+a\right)b}+\frac{7\sqrt{3}\;a^2d^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{7a^2d^3\ln\left(x+\right.}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/3/b^3*a^2*x/(b*x^3+a)*d^3+1/b
^2*a*x/(b*x^3+a)*c*d^2-1/b*x/(b*x^3+a)*c^2*d+1/3/a*x/(b*x^3+a)*c^3+7/9/b^4*
a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^3-4/3/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*c*d^2+1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2*d+2/9/b/a/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*c^3-7/18/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*d^3+2/3/b^3*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c*d^2-1/6/b
^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d-1/9/b/a/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3+7/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^3-4/3/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c*d^2+1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3
```

maxima [A] time = 1.22, size = 306, normalized size = 1.31

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x}{3\left(ab^4x^3+a^2b^3\right)}+\frac{bd^3x^4+4\left(3\,bcd^2-2\,ad^3\right)x}{4\,b^3}+\frac{\sqrt{3}\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)a}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2*
b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*sqrt(3)*(2*b^3
*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^(2/3)) - 1/18*(2*b^3*c^3 + 3*a*b^2
*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*
a^3*d^3)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

mupad [B] time = 0.30, size = 240, normalized size = 1.03

$$\frac{d^3\,x^4}{4\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right.}{9\,a^{5/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3)^2,x)
```

```
[Out] (d^3*x^4)/(4*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*
a^(5/3)*b^(10/3))
```

sympy [A] time = 4.33, size = 291, normalized size = 1.24

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x\left(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3\right)}{3a^2b^3+3ab^4x^3}+\mathrm{RootSum}\left(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*
a**5*b**10 - 343*a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d**7
+ 2946*a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c**5*d
**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*b**8*c**8*d -
 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*d**3 - 12*a**2*b*c*d
**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))) + d**3*x**4/(4*b**2)
```

3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)}{3\sqrt{3}\,a^{5/}}$$

```
[Out] d^2*x/b^2+1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)+2/9*(-a*d+b*c)*(2*a*d+b*c)*ln(
a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/9*(-a*d+b*c)*(2*a*d+b*c)*ln(a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-2/9*(-a*d+b*c)*(2*a*d+b*c)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)}{3\sqrt{3}\,a^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c
 + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a
^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^2}\,dx = \int\left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{b^2\left(a+bx^3\right)^2}\right)dx$$

$$= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{\left(a+bx^3\right)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b^2}+\frac{(2(bc-ad)(bc+2ad))\int \frac{2}{a^{2/3}-}}{9a^{5/3}b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{((bc-ad)(bc+2ad))\int \frac{-}{a^{2/3}-}}{9a^{5/3}b^{7/3}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}\right.}{9a^{5/3}b^{7/}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}-\frac{2(bc-ad)(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{\right.}}{9a^{5/3}b^{7/3}}$$

Mathematica [A] time = 0.20, size = 205, normalized size = 1.01

$$\frac{\frac{2\left(-2a^2d^2+abcd+b^2c^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}-\frac{2\sqrt{3}\left(-2a^2d^2+abcd+b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-\frac{\left(-2a^2d^2+abcd+b^2c^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{3\sqrt[3]{b}\,x(}{a(a-}}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3]
*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x])/a
^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

fricas [B] time = 0.47, size = 768, normalized size = 3.78

$$\left[\frac{9\,a^3b^2d^2x^4-3\sqrt{\frac{1}{3}}\left(a^2b^3c^2+a^3b^2cd-2\,a^4bd^2+\left(ab^4c^2+a^2b^3cd-2\,a^3b^2d^2\right)x^3\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax \cdots}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*
d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a
^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a
*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)
*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b
^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(
-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4 + 6*sqrt(
1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*b^3*c*d -
2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2*c^2 + a^2*b
*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*
log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2*c^2 + a^2*b*c
*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*lo
g(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + 4*a^4*b*d^2)*x
)/(a^3*b^4*x^3 + a^4*b^3)]
```

giac [A] time = 0.21, size = 227, normalized size = 1.12

$$\frac{d^2x}{b^2}-\frac{2\sqrt{3}\left(b^2c^2+abcd-2\,a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(b^2c^2+abcd-2\,a^2d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 - 2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/9*(b^2*c^2 + a*
b*c*d - 2*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)
*a*b) - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3 + a)
*a*b^2)
```

maple [B] time = 0.05, size = 367, normalized size = 1.81

$$\frac{a\,d^2x}{3\left(b\,x^3+a\right)b^2}+\frac{c^2x}{3\left(b\,x^3+a\right)a}-\frac{2cdx}{3\left(b\,x^3+a\right)b}-\frac{4\sqrt{3}\;a\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4a\,d^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2a\,d^2\ln\left(x^2-\right.}{9\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^2,x)
```

```
[Out] d^2*x/b^2+1/3/b^2*a*x/(b*x^3+a)*d^2-2/3/b*x/(b*x^3+a)*c*d+1/3/a*x/(b*x^3+a)
*c^2-4/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^2+2/9/b^2/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2+2/9/b^3*a/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^2-1/9/b^2/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*c*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*c^2-4/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*d^2+2/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
*c*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^
2
```

maxima [A] time = 1.39, size = 220, normalized size = 1.08

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x}{3\left(ab^3x^3+a^2b^2\right)}+\frac{d^2x}{b^2}+\frac{2\sqrt{3}\left(b^2c^2+abcd-2\,a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2c^2+abcd-2\,a^2d^2\right)\log\left(x^2\right.}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 + 2
/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^
2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2/9*(b^2*c^
2 + a*b*c*d - 2*a^2*d^2)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

mupad [B] time = 1.47, size = 191, normalized size = 0.94

$$\frac{d^2\,x}{b^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a\,\left(b^3\,x^3+a\,b^2\right)}-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)}{9\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*a*(a*b^2 + b^3*x^3)) -
 (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3))
 - (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3)) + (2*log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(2*a*d + b*c)
)/(9*a^(5/3)*b^(7/3))
```

sympy [A] time = 2.56, size = 189, normalized size = 0.93

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{3a^2b^2+3ab^3x^3}+\mathrm{RootSum}\left(729t^3a^5b^7+64a^6d^6-96a^5bcd^5-48a^4b^2c^2d^4+88a^3b^3c^3d^3+24a^2b^4c^4d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + RootS
um(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c**
2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**5*d
 - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*c*d
- 2*b**2*c**2) + x))) + d**2*x/b**2
```

3.24 $\int \frac{c+dx^3}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{x(bc-a}{3ab\left(a+\right.}$$

```
[Out] 1/3*(-a*d+b*c)*x/a/b/(b*x^3+a)+1/9*(a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3
)/b^(4/3)-1/18*(a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3
)/b^(4/3)-1/9*(a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))
/a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{x(bc-a}{3ab\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*b*c + a*d)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*b*c + a*d)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^2}\,dx = \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b} + \frac{(2bc+ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} + \frac{(2bc+ad}{}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(2}{}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} - \frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log}{}$$

Mathematica [A] time = 0.09, size = 145, normalized size = 0.86

$$\frac{-(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) - \frac{6a^{2/3}\sqrt[3]{b}\,x(ad-bc)}{a+bx^3} + 2(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right) - 2\sqrt{3}\,(ad+2bc)\tan^{-1}}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/3)
 + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

fricas [A] time = 0.47, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(2\,a^2b^2c + a^3bd + \left(2\,ab^3c + a^2b^2d\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\left(a^2b\right)^{\frac{1}{3}}ax - a^2 + 3\sqrt{\frac{1}{3}}\left(2\,abx^2 + \left(a^2b\right)^{\frac{2}{3}}x - \left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2
*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a
^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*
d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((2*b^2*c +
 a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

giac [A] time = 0.17, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(2\,bc + ad)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{(2\,bc + ad)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{(2\,bc + ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\right.\right.\right.}{9\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + a*d)*log(x^2 + x*(-a/b)^(1/3) + (-
```

```
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a*b)
```

maple [A] time = 0.05, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{d\ln\left(x+\right.}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+2/9/
b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1
/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+2/9/b/
a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 1.44, size = 158, normalized size = 0.93

$$\frac{(bc-ad)x}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{\sqrt{3}\,(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,bc+ad)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,bc+ad)\log\left(x+\right.}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(2*b*
c + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*(
2*b*c + a*d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 1.43, size = 143, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d + 2*b*c))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (x*(a*d - b*c))/(3*
a*b*(a + b*x^3))
```

sympy [A] time = 1.42, size = 97, normalized size = 0.57

$$\frac{x\left(-ad+bc\right)}{3a^{2}b+3ab^{2}x^{3}}+\operatorname{RootSum}\left(729t^{3}a^{5}b^{4}-a^{3}d^{3}-6a^{2}bcd^{2}-12ab^{2}c^{2}d-8b^{3}c^{3},\left(t\mapsto t\log\left(\frac{9ta^{2}b}{ad+2bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - a
**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _t*
log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

3.25 $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=346

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}\,a^{5/3}(bc-ad)^2}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)+1/9*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(1/3)+b^(1/3
)*x)/a^(5/3)/(-a*d+b*c)^2+1/3*d^(5/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d+b
*c)^2-1/18*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(5/3)/(-a*d+b*c)^2-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/c^(2/3)/(-a*d+b*c)^2-1/9*b^(2/3)*(-5*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)^2*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}\,a^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^2) -
(d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(5/3)*(b*c - a*d)^2) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)^2) - (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{\int \frac{-2bc+3ad-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{d^2\int \frac{1}{c+dx^3}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{a+bx^3}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{d^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{(b(2bc-5ad}{9a^{5/3}(}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)^2} -$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)^2} -$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^2} +$$

Mathematica [A] time = 0.20, size = 337, normalized size = 0.97

$$-b^{2/3}c^{2/3}\left(a+bx^3\right)(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-3a^{5/3}d^{5/3}\left(a+bx^3\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*a
*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a^(
5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*b^
```

```
(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*b*c -
 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 3*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18
*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

fricas [A] time = 7.38, size = 440, normalized size = 1.27

$$-\frac{2\sqrt{3}\left(\left(2b^2c-5abd\right)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^(
1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(3)
*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a
^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 3*
(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2
/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2
*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^3)
```

giac [A] time = 0.20, size = 443, normalized size = 1.28

$$-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2abc^2d+a^2cd^2\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2abc^2d+a^2cd^2\right)}-\frac{\left(2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/6*(
-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
```

```
^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3)*b*c
- 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) + 1/18*(2*
(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3 + a)*(a*b*
c - a^2*d))
```

maple [A] time = 0.06, size = 406, normalized size = 1.17

$$\frac{b^2 c x}{3\left(ad-bc\right)^2\left(b\,x^3+a\right)a}-\frac{bdx}{3\left(ad-bc\right)^2\left(b\,x^3+a\right)}+\frac{2\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{2bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{bc\ln\left(x^2-\left(\right.\right.}{9\left(ad-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] -1/3*b/(a*d-b*c)^2*x/(b*x^3+a)*d+1/3*b^2/(a*d-b*c)^2/a*x/(b*x^3+a)*c-5/9/(a
*d-b*c)^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+2/9*b/(a*d-b*c)^2/a/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))*c+5/18/(a*d-b*c)^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*d-1/9*b/(a*d-b*c)^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
c-5/9/(a*d-b*c)^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*d+2/9*b/(a*d-b*c)^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*c+1/3*d/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*d/(a*d-b*c)
^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*d/(a*d-b*c)^2/(c/d)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.29, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}\left(2\,bc-5\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b}{3\left(a^2bc-a^3d+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*b*c - 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/((a*b^2*c^2*(a/b)^(1/3) - 2*a^2*b*c*d*(a/b)^(1/3) + a^3*d^2*(a/b)^(1/3
```

```
))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/((b^2*c^2*(c/d)^(1/3) - 2*a*b*c*d*(c/d)^(1/3) + a^2*d^2*(c/d)^(1/3
))*(c/d)^(1/3)) + 1/3*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3) - 1/1
8*(2*b*c - 5*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*c^2*(a/b)^(
2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(
c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^
2*d^2*(c/d)^(2/3)) + 1/9*(2*b*c - 5*a*d)*log(x + (a/b)^(1/3))/(a*b^2*c^2*(a
/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) + 1/3*d*log(x +
(c/d)^(1/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^2*d^2*(c/d)^(
2/3))
```

mupad [B] time = 15.93, size = 2492, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a
+ 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(
a*d - b*c)^6))^(1/3))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))
/81 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/9 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^
2))/(9*a^3*(a*d - b*c)^4))*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^
2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d +
10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 437
4*a^10*b*c*d^5))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2
*b^2*c^2 + 3*a*b*c*d))/a + 81*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c
^2*(a*d - b*c)^6))^(1/3))*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(27
*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b
*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/3 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*
c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(d^5/(27*b^6
*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b
^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3) + (log(((3^(1/2)
*1i - 1)*(((3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b
^2*c^2 + 3*a*b*c*d))/a + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d
- b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/2)*(-(b^2*(5
*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(27*a^3*d^3 - 8
*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(-(b^2*
(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^5*d^6*x*(85*a^3*d^
3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3
^(1/2)*1i - 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^2 - 60*a*b^4
*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d + 10935*a^7*b^
4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 4374*a^10*b*c*d
```

```
^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*
(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (27*a*b^3*c*d^3*(3^(
1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d
- b*c)^6))^(1/3))/2)*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/
324 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/18 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d
^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3
+ 150*a^2*b^3*c*d^2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 437
4*a^6*b^5*c^5*d + 10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9
*b^2*c^2*d^4 - 4374*a^10*b*c*d^5))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(
1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*
c*d))/a + (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(
c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^
4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*
a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/6 + (2*b^5*d^6*x*(85*a^3*d^3 - 4
*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2
)*1i - 1)*(d^5/(27*b^6*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b
^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d
- b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (81*a*b^3*c*d^3*(3^(1/2)*
1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/
(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^
2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))
^(1/3))/6 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*
b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^6*c^8 + 27*a^6
*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 +
405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3))/2 - (b*x)/(3*a*(a + b*x^3)*(
a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=419

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}\,a^{5/3}(bc-ad)^3}$$

```
[Out] 1/3*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^3+c)+1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)
/(d*x^3+c)+2/9*b^(5/3)*(-4*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c
)^3+2/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^3-1/9
*b^(5/3)*(-4*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a
*d+b*c)^3-1/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/c^(5/3)/(-a*d+b*c)^3-2/9*b^(5/3)*(-4*a*d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)^3*3^(1/2)-2/9*d^(5/3)*(-a*d+4*b*c
)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^3*3^
(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {414, 527, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}\,a^{5/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d
)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3
)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[
3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*
x])/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1
/3)*x])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c
 - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*(b*c - a
*d)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-2bc+3ad-5bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx}{3a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-6\left(b^2c^2-3abcd+a^2d^2\right)-6b}{\left(a+bx^3\right)\left(c+dx^3\right.}}{9ac(bc-ad}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{1}{a+bx}}{3a(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{}{\sqrt[3]{a}-}}{9a^{5/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{}\right.}{9a^{5/3}(bc-ad}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{}\right.}{9a^{5/3}(bc-ad}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\right.}{3\sqrt{3}\,a^{5/3}(bc-a}$$

Mathematica [A] time = 0.63, size = 381, normalized size = 0.91

$$\frac{1}{9}\left(\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}(ad-bc)^3} + \frac{2b^{5/3}(4ad-bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}(ad-bc)^3} + \frac{2\sqrt{3}\,b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{1}{-}\right.}{a^{5/3}(ad-bc)^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c +
d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
```

```
))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a*d)
*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) + (2*
b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) + a*d)^
3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c - a*d
)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^3))/9
```

fricas [B] time = 85.30, size = 897, normalized size = 2.14

$$3\left(b^3c^2d - a^2bd^3\right)x^4 + 2\sqrt{3}\left(\left(b^3c^2d - 4\,ab^2cd^2\right)x^6 + ab^2c^3 - 4\,a^2bc^2d + \left(b^3c^3 - 3\,ab^2c^2d - 4\,a^2bcd^2\right)x^3\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b^3*c^2*d - a^2*b*d^3)*x^4 + 2*sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2)
*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2
)*x^3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*
b)/b) + 2*sqrt(3)*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*
d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*arctan
(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - ((b^3*c^2*d - 4*a*b^2
*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*
b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^
2/a^2)^(2/3)) - ((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^
2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d^2*
x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*((b^3*c^2*d - 4*a*b^
2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2
*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*((4*a*b^2*c
*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 3*(b
^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c
^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3
*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^6 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*
b*c^2*d^3 - a^5*c*d^4)*x^3)
```

giac [A] time = 0.22, size = 664, normalized size = 1.58

$$-\frac{2\left(b^{3}c-4\,ab^{2}d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\left(a^{2}b^{3}c^{3}-3\,a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-a^{5}d^{3}\right)}-\frac{2\left(4\,bcd^{2}-ad^{3}\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)}+\frac{2\left(\left(-ab^{2}\right)^{\frac{1}{3}}b^{2}c-4\,\left(-ab^{2}\right)\right.}{3\left(\sqrt{3}\,a^{2}b^{3}c^{3}-3\,\sqrt{3}\,a^{3}b^{2}c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*c
^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)*(
-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^3*c^3
 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3) + 2/3
*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*(2*x +
(-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*d + 3*sq
rt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b^2*c - 4*
(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3*c^3
 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^(1/3)*b*c*d
 - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*d*x^4 + a*b*d
^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3 + a*c)*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

maple [A] time = 0.06, size = 606, normalized size = 1.45

$$\frac{a\,d^{3}x}{3\left(ad-bc\right)^{3}\left(d\,x^{3}+c\right)c}-\frac{b^{3}cx}{3\left(ad-bc\right)^{3}\left(b\,x^{3}+a\right)a}+\frac{b^{2}dx}{3\left(ad-bc\right)^{3}\left(b\,x^{3}+a\right)}-\frac{b\,d^{2}x}{3\left(ad-bc\right)^{3}\left(d\,x^{3}+c\right)}+\frac{2\sqrt{3}\,a\,d^{2}\,\mathrm{arcta}}{9\left(ad-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b^2/(a*d-b*c)^3*x/(b*x^3+a)*d-1/3*b^3/(a*d-b*c)^3/a*x/(b*x^3+a)*c+8/9*b
/(a*d-b*c)^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^(2
```

```
/3)*ln(x+(a/b)^(1/3))*c-4/9*b/(a*d-b*c)^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))*d+1/9*b^2/(a*d-b*c)^3/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*c+8/9*b/(a*d-b*c)^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c+1/3*d^3/(a*d-b*c)^3/c*x/(d*x^3+c)*a-1/3*d^2/(a*d-b*
c)^3*x/(d*x^3+c)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-8/
9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9*d^2/(a*d-b*c)^3/c/(c/d)
^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+4/9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln
(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-8/9*d/(a*d-b*c)^3/(c/d)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [B] time = 1.26, size = 784, normalized size = 1.87

$$\frac{2\sqrt{3}\left(b^2c-4\,abd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}+3\,a^3bcd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\left(4\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{3}\left(2\right.}{3}\right.}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-3\,ab^2c^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}+3\,a^2bc^2d^2\left(\frac{c}{d}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(b^2*c - 4*a*b*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/((a*b^3*c^3*(a/b)^(1/3) - 3*a^2*b^2*c^2*d*(a/b)^(1/3) + 3*a^3*b*c*d^
2*(a/b)^(1/3) - a^4*d^3*(a/b)^(1/3))*(a/b)^(1/3)) + 2/9*sqrt(3)*(4*b*c*d -
a*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b^3*c^4*(c/d)^
(1/3) - 3*a*b^2*c^3*d*(c/d)^(1/3) + 3*a^2*b*c^2*d^2*(c/d)^(1/3) - a^3*c*d^3
*(c/d)^(1/3))*(c/d)^(1/3)) - 1/9*(b^2*c - 4*a*b*d)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/b)^(2/3) + 3*a^3
*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) - 1/9*(4*b*c*d - a*d^2)*log(x^2
 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^
(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 2/9*(b^2*c -
 4*a*b*d)*log(x + (a/b)^(1/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/
b)^(2/3) + 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) + 2/9*(4*b*c*d
- a*d^2)*log(x + (c/d)^(1/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^(2
/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 1/3*((b^2*c*d
+ a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^6 + (a*b^3*c^4
- a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)
```

mupad [B] time = 24.31, size = 3637, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4
*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d + b*
c) + b*d*x^6) + log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d -
 b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b
*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*
a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d
- 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(
a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^
4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49
*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 +
 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561
*a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*
b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*
b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^(1/3) + log((2*((4*((54*b^3*d^3*x*(a*d -
b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^
3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9
))^(1/3))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^
4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2
*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d
^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^
6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2
*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*a
^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^1
4 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a
^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^
6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3) + (log(((
3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3
+ b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)
*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^
9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d
^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^
2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((
b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d
^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^
2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1
/2)*1i - 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c
^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7*
c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^
4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8
))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a
```

```
*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27
*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)
^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))
^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^
3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c
^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 + (
16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3
*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*
(a*d - b*c)^8))*(3^(1/2)*1i + 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b
^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c
^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^
4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^
7 - 6561*a^13*b*c*d^8))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1
)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^
4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*((d^5*(a*d - 4*b*c)^3)
/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^
4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a
^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d -
b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*
d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b
*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i - 1)*(-(8*a^3*d^8 - 512*b^
3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^14 - 729*a^9*c^5
*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a^3*b^6*c^11*d^3
 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^6*b^3*c^8*d^6 -
 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2 - (log(((3^(1/2)*1i +
 1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*(
(d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6
+ b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 -
11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4
*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 + (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c
^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a
*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i + 1)*
(-(8*a^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b
^9*c^14 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 6
1236*a^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61
236*a^6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.27 $\int \left(a-bx^3\right) \left(a+bx^3\right)^{2/3} \, dx$

Optimal. Leaf size=112

$$-\frac{7a^2 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{7a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

```
[Out] 7/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(b*x^3+a)^(5/3)-7/18*a^2*ln(-b^(1/3)*x+(b*x^
3+a)^(1/3))/b^(1/3)+7/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {388, 195, 239}

$$-\frac{7a^2 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{7a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (7*a*x*(a + b*x^3)^(2/3))/18 - (x*(a + b*x^3)^(5/3))/6 + (7*a^2*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx=-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{6}(7a)\int \left(a+bx^3\right)^{2/3}\,dx$$

$$=\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{9}\left(7a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$=\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{7a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{7a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{\right.}}{18\sqrt[3]{b}}$$

Mathematica [C] time = 0.11, size = 62, normalized size = 0.55

$$\frac{1}{6}x\left(a+bx^3\right)^{2/3}\left(\frac{7a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}-a-bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(-a - b*x^3 + (7*a*Hypergeometric2F1[-2/3, 1/3, 4/3, -
((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/6
```

fricas [B] time = 0.44, size = 399, normalized size = 3.56

$$\left[\frac{21\sqrt{\frac{1}{3}}\,a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*log((
(-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) -
 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^2*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))
/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

maxima [B] time = 1.22, size = 322, normalized size = 2.88

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(bx^3+\cdots\right.}{\left(b-\frac{bx^3\cdots}{x^3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a - 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(a-b\,x^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3), x)
```

sympy [C] time = 4.98, size = 80, normalized size = 0.71

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.28 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] -1/3*x*(b*x^3+a)^(2/3)-2/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+4/9*a*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {388, 239}

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] -(x*(a + b*x^3)^(2/3))/3 + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3)
)/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
])/(3*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{3}x\left(a+bx^3\right)^{2/3} + \frac{1}{3}(4a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= -\frac{1}{3}x\left(a+bx^3\right)^{2/3} + \frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} - \frac{2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 1.47

$$\frac{2a\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}-4a\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+4\sqrt{3}a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]] - 4*a*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + 2*a
*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/
(9*b^(1/3))
```

fricas [B] time = 0.44, size = 363, normalized size = 3.99

$$\left[\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\right.}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
```

```
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 + a)
^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*a
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt
(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 3*(b*x^
3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{-b\,x^3 + a}{\left(b\,x^3 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
[Out] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.22, size = 244, normalized size = 2.68

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)a-\frac{1}{18}\left(\frac{2\sqrt{3}\,a}{\vphantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a - 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(1/3),x)
```

```
[Out] int((a - b*x^3)/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 5.53, size = 76, normalized size = 0.84

$$\frac{a^{\frac{2}{3}} x \Gamma\left(\frac{1}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 \Gamma\left(\frac{4}{3}\right)} - \frac{b x^{4} \Gamma\left(\frac{4}{3}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}}{3 \sqrt[3]{a} \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.29 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=85

$$\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] 2*x/(b*x^3+a)^(1/3)+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)-1/3*arctan(1
/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {385, 239}

$$\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{2x}{\sqrt[3]{a+bx^3}} - \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2x}{\sqrt[3]{a+bx^3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} + \frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [C] time = 0.04, size = 62, normalized size = 0.73

$$\frac{4ax - bx^4\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (4*a*x - b*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((b*
x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

fricas [B] time = 0.44, size = 372, normalized size = 4.38

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(b^2x^3+ab\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\cdots}\right.}{6\left(b^2x^3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
- 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a)^(
2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) -
```

```
(b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2*(b*x^
3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)*b^(2/3
)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(
1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{-b\,x^3 + a}{\left(b\,x^3 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

maxima [A] time = 1.25, size = 130, normalized size = 1.53

$$\frac{1}{6}\,b\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2\,\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{\left(bx^3 + a\right)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,\log\left(-b^{\frac{1}{3}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b
^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1
/3)/x)/b^(4/3)) + x/(b*x^3 + a)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(4/3),x)
```

```
[Out] int((a - b*x^3)/(a + b*x^3)^(4/3), x)
```

sympy [C] time = 16.14, size = 70, normalized size = 0.82

$$\frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

```
[Out] x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(4
/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(7
/3))
```

3.30 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}} \, dx$

Optimal. Leaf size=47

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

```
[Out] 1/4*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+3/4*x/a/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {378, 191}

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}} + \frac{3}{4}\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx$$
$$= \frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}} + \frac{3x}{4a\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.60

$$\frac{x\left(2a+bx^3\right)}{2a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]
[Out] (x*(2*a + b*x^3))/(2*a*(a + b*x^3)^(4/3))
```

fricas [A] time = 0.42, size = 44, normalized size = 0.94

$$\frac{\left(bx^4+2\,ax\right)\left(bx^3+a\right)^{\frac{2}{3}}}{2\left(ab^2x^6+2\,a^2bx^3+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")
[Out] 1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

maple [A] time = 0.05, size = 25, normalized size = 0.53

$$\frac{\left(b\,x^{3}+2a\right)x}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(7/3),x)
```

```
[Out] 1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

maxima [A] time = 0.52, size = 50, normalized size = 1.06

$$-\frac{\left(b-\frac{4\left(bx^{3}+a\right)}{x^{3}}\right)x^{4}}{4\left(bx^{3}+a\right)^{\frac{4}{3}}a}-\frac{bx^{4}}{4\left(bx^{3}+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
+ a)^(4/3)*a)
```

mupad [B] time = 1.35, size = 27, normalized size = 0.57

$$\frac{x\,\left(b\,x^3+a\right)+a\,x}{2\,a\,{\left(b\,x^3+a\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(7/3),x)
```

```
[Out] (x*(a + b*x^3) + a*x)/(2*a*(a + b*x^3)^(4/3))
```

sympy [B] time = 91.68, size = 190, normalized size = 4.04

$$a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^{3}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^{4}\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^{3}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)}\right)-\frac{}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
```

```
[Out] a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

3.31 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=55

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

```
[Out] 2/7*x/(b*x^3+a)^(7/3)+5/28*x/a/(b*x^3+a)^(4/3)+15/28*x/a^2/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {385, 192, 191}

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (2*x)/(7*(a + b*x^3)^(7/3)) + (5*x)/(28*a*(a + b*x^3)^(4/3)) + (15*x)/(28*a
^2*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx = \frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5}{7}\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx$$

$$= \frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a}$$

$$= \frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.73

$$\frac{x\left(28a^2+35abx^3+15b^2x^6\right)}{28a^2\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(28*a^2 + 35*a*b*x^3 + 15*b^2*x^6))/(28*a^2*(a + b*x^3)^(7/3))
```

fricas [A] time = 0.42, size = 69, normalized size = 1.25

$$\frac{\left(15\,b^2x^7+35\,abx^4+28\,a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{28\left(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.04, size = 37, normalized size = 0.67

$$\frac{\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)x}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

maxima [A] time = 0.62, size = 85, normalized size = 1.55

$$\frac{\left(4\,b-\frac{7\left(bx^{3}+a\right)}{x^{3}}\right)bx^{7}}{28\left(bx^{3}+a\right)^{\frac{7}{3}}a^{2}}+\frac{\left(2\,b^{2}-\frac{7\left(bx^{3}+a\right)b}{x^{3}}+\frac{14\left(bx^{3}+a\right)^{2}}{x^{6}}\right)x^{7}}{14\left(bx^{3}+a\right)^{\frac{7}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
- 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

mupad [B] time = 1.42, size = 44, normalized size = 0.80

$$\frac{15\,x\left(b\,x^{3}+a\right)^{2}+8\,a^{2}\,x+5\,a\,x\,\left(b\,x^{3}+a\right)}{28\,a^{2}\left(b\,x^{3}+a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/3
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

3.32 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=74

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

```
[Out] 1/5*x/(b*x^3+a)^(10/3)+4/35*x/a/(b*x^3+a)^(7/3)+6/35*x/a^2/(b*x^3+a)^(4/3)+
18/35*x/a^3/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {385, 192, 191}

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3)) + (6*x)/(35*a^2*(
a + b*x^3)^(4/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4}{5}\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx$$
$$=\frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{24\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{35a}$$
$$=\frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{35a^2}$$
$$=\frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.69

$$\frac{x\left(35a^3+70a^2bx^3+60ab^2x^6+18b^3x^9\right)}{35a^3\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^3 + 60*a*b^2*x^6 + 18*b^3*x^9))/(35*a^3*(a + b*x^3)
^(10/3))
```

fricas [A] time = 0.43, size = 91, normalized size = 1.23

$$\frac{\left(18\,b^3x^{10}+60\,ab^2x^7+70\,a^2bx^4+35\,a^3x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{35\left(a^3b^4x^{12}+4\,a^4b^3x^9+6\,a^5b^2x^6+4\,a^6bx^3+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2/
3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{13}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.65

$$\frac{\left(18b^3x^9 + 60a\,b^2x^6 + 70a^2b\,x^3 + 35a^3\right)x}{35\left(b\,x^3 + a\right)^{\frac{10}{3}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

maxima [B] time = 0.58, size = 119, normalized size = 1.61

$$-\frac{\left(14\,b^2 - \frac{40\left(bx^3+a\right)b}{x^3} + \frac{35\left(bx^3+a\right)^2}{x^6}\right)bx^{10}}{140\left(bx^3 + a\right)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3 - \frac{60\left(bx^3+a\right)b^2}{x^3} + \frac{105\left(bx^3+a\right)^2 b}{x^6} - \frac{140\left(bx^3+a\right)^3}{x^9}\right)x^{10}}{140\left(bx^3 + a\right)^{\frac{10}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

mupad [B] time = 1.39, size = 58, normalized size = 0.78

$$\frac{x}{5\left(b\,x^3 + a\right)^{10/3}} + \frac{18\,x}{35\,a^3\left(b\,x^3 + a\right)^{1/3}} + \frac{6\,x}{35\,a^2\left(b\,x^3 + a\right)^{4/3}} + \frac{4\,x}{35\,a\left(b\,x^3 + a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3)) + (6*x)/(35*a^
2*(a + b*x^3)^(4/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=93

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

```
[Out] 2/13*x/(b*x^3+a)^(13/3)+11/130*x/a/(b*x^3+a)^(10/3)+99/910*x/a^2/(b*x^3+a)^
(7/3)+297/1820*x/a^3/(b*x^3+a)^(4/3)+891/1820*x/a^4/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {385, 192, 191}

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3)) + (99*x)/
(910*a^2*(a + b*x^3)^(7/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (891*x
)/(1820*a^4*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11}{13}\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{297\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a^2}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{891\int}{}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{}{1820}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.67

$$\frac{x\left(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12}\right)}{1820a^4\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^4
*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

fricas [A] time = 0.43, size = 113, normalized size = 1.22

$$\frac{\left(891\,b^4x^{13}+3861\,ab^3x^{10}+6435\,a^2b^2x^7+5005\,a^3bx^4+1820\,a^4x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{1820\left(a^4b^5x^{15}+5\,a^5b^4x^{12}+10\,a^6b^3x^9+10\,a^7b^2x^6+5\,a^8bx^3+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4
+ 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b^3
*x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 59, normalized size = 0.63

$$\frac{\left(891b^4x^{12} + 3861b^3x^9a + 6435b^2x^6a^2 + 5005b\,x^3a^3 + 1820a^4\right)x}{1820\left(b\,x^3 + a\right)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820*
a^4)/(b*x^3+a)^(13/3)/a^4
```

maxima [B] time = 0.54, size = 153, normalized size = 1.65

$$\frac{\left(140\,b^3 - \frac{546\left(bx^3+a\right)b^2}{x^3} + \frac{780\left(bx^3+a\right)^2b}{x^6} - \frac{455\left(bx^3+a\right)^3}{x^9}\right)bx^{13}}{1820\left(bx^3 + a\right)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4 - \frac{182\left(bx^3+a\right)b^3}{x^3} + \frac{390\left(bx^3+a\right)^2b^2}{x^6} - \frac{455\left(bx^3+a\right)^3b}{x^9} + \frac{455\left(b\right.}{x}\right.}{455\left(bx^3 + a\right)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*(
b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b*
x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 45
5*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

mupad [B] time = 1.37, size = 73, normalized size = 0.78

$$\frac{2\,x}{13\,{\left(b\,x^3+a\right)}^{13/3}}+\frac{891\,x}{1820\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}+\frac{297\,x}{1820\,a^3\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{99\,x}{910\,a^2\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{11\,x}{130\,a\,{\left(b\,x^3+a\right)}^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (297
*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (11
*x)/(130*a*(a + b*x^3)^(10/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx$

Optimal. Leaf size=483

$$-\frac{2\sqrt[3]{2}\,a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\,a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}-\frac{4\sqrt[3]{2}\,a^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right.}{3\sqrt[3]{b}}$$

```
[Out] -7/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(b*x^3+a)^(4/3)-7/5*a^2*x*(1+b*x^3/a)^(2/3)*
hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2/3*2^(1/3)*a^(5/3)*ln
(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+2/3*2^(1/3)*a^(5/3)*
ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b*x^3+a)^(1/3))/b^(1/3)-4/3*2^(1/3)*a^(5/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(5/3)*ln(2*2^(1/3)+(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^
(1/3)-4/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)-2/3*2^(1/3)*a^(5/3)*arctan(1/3*(1+2^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{7}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -7/3, 4/3, (b*x^3)/a, -((b*x^3)/a)]
)/(1 + (b*x^3)/a)^(1/3)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx=\frac{\left(a^2\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{7/3}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$=\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{7}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.38, size = 232, normalized size = 0.48

$$\frac{4x\left(\frac{52a^4F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}-8a^2-9abx^3-b^2x^6\right)+27abx^4\left(\frac{bx^3}{a}\right.}{20\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (27*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
(b*x^3)/a] + 4*x*(-8*a^2 - 9*a*b*x^3 - b^2*x^6 + (52*a^4*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^
3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))
)))/(20*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3 + a\right)^{\frac{7}{3}}}{bx^3 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{7}{3}}}{-b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3 + a\right)^{\frac{7}{3}}}{bx^3 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3 + a\right)^{7/3}}{a - b\,x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/3)/(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(7/3)/(a - b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{b^2x^6 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{2abx^3 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)
```

```
[Out] -Integral(a**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b**2*x**6*(
a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(2*a*b*x**3*(a + b*x**3)**(1
/3)/(-a + b*x**3), x)
```

3.35 $\int \frac{(a+bx^3)^{4/3}}{a-bx^3} dx$

Optimal. Leaf size=464

$$-\frac{\sqrt[3]{2}\,a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{\sqrt[3]{2}\,a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\,a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}-$$

```
[Out] -1/2*x*(b*x^3+a)^(1/3)-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3]
,-b*x^3/a)/(b*x^3+a)^(2/3)-1/3*2^(1/3)*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)
*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/
3)-2/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^
(1/3)+1/6*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(1/3)-2/3*2^(1/3)*a^(2/3)*
arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/
3)*3^(1/2)-1/3*2^(1/3)*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{4}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -4/3, 4/3, (b*x^3)/a, -((b*x^3)/a)])/
(1 + (b*x^3)/a)^(1/3)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{4}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.12, size = 217, normalized size = 0.47

$$\frac{x\left(\frac{48a^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+5bx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\right.\right.}{8\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] (x*(-4*(a + b*x^3) + 5*b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/
a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
(b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] -
 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(8*(a + b*x^3)^
(2/3))
```

fricas [F] time = 34.20, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{bx^3-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="fricas")
[Out] integral(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{-b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
[Out] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{a-b\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(a - b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx-\int \frac{bx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)
```

```
[Out] -Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b*
x**3)**(1/3)/(-a + b*x**3), x)
```

3.36 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

Optimal. Leaf size=398

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\cdots\right)}{6\ 2^{2/3}\cdots}$$

```
[Out] -1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/
3)+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*ln(1+2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)+1/12*ln(2*2^(1/3)+(a
^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(
1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)*3^(1/2)-1/6*arctan(1/3
*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(1/3)/b
^(1/3)*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (b*x^3)/a, -((b*x^3)/a)])/
(a*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.13, size = 151, normalized size = 0.38

$$\frac{4ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/((a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a]
+ b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

fricas [B] time = 29.48, size = 644, normalized size = 1.62

$$-\frac{1}{18}\sqrt{3}\,2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\arctan\left(\frac{6\sqrt{3}\,2^{\frac{2}{3}}\left(ab^6x^{16}+33\,a^2b^5x^{13}+110\,a^3b^4x^{10}+110\,a^4b^3x^7+33\,a^5b^2x^4+a^6bx\right)\left(bx^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^6
*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2*x
^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a*b^
5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b*x^3
 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417*a^2*
b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(b^6*x^
18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b^2*x^6
+ 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(2/3)*(a*
b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^(2/3) - 2
^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 + a^4)*(-1/
(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x)*(b*x^3 +
a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3 + a^4)) + 1
/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 + 2^(2/3)*(b^2*
x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 + a*x)*(b*x^3 +
a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{-b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{a-b\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(a - b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

3.37 $\int \frac{1}{\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=452

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}(\ldots}{\sqrt[3]{\ldots}}\right.}{12\,2^{2/3}a^{4/3}\ldots}$$

```
[Out] 1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2
/3)-1/12*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b
^(1/3)+1/12*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*ln(1+2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)+1/24*ln(2*2^(1/
3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/b^(1/3)*3^(1/2)-1/12*arct
an(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(
4/3)/b^(1/3)*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 1, 2/3, 4/3, (b*x^3)/a, -((b*x^3)/a)
])/(a*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a(a+bx^3)^{2/3}}$$

Mathematica [C] time = 0.13, size = 153, normalized size = 0.34

$$\frac{4axF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)(a+bx^3)^{2/3}\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (4*a*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(a
 + b*x^3)^(2/3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] +
b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4
/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^3+a\right)\left(b\,x^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,\left(a-b\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(a - b*x^3)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a\left(a+bx^{3}\right)^{\frac{2}{3}}+bx^{3}\left(a+bx^{3}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(2/3),x)
[Out] -Integral(1/(-a*(a + b*x**3)**(2/3) + b*x**3*(a + b*x**3)**(2/3)), x)
```

3.38 $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\, dx$

Optimal. Leaf size=473

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\ 2^{2/3}a^{7/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}(\ldots}{\sqrt[3]{\ldots}}\ldots\right)}{24\ 2^{2/3}a^{7/3}\ldots}$$

```
[Out] 1/4*x/a^2/(b*x^3+a)^(2/3)+1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3
],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)-1/24*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3
+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)+1/24*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/
(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3
)/b^(1/3)-1/12*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^
(7/3)/b^(1/3)+1/48*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)-1/12*arctan
(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(
7/3)/b^(1/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/3))*3^(1/2))*2^(1/3)/a^(7/3)/b^(1/3)*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};1,\frac{5}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 1, 5/3, 4/3, (b*x^3)/a, -((b*x^3)/a)
])/(a^2*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{a(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{5}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^2(a+bx^3)^{2/3}}$$

Mathematica [C] time = 0.14, size = 213, normalized size = 0.45

$$\frac{x\left(-\frac{bx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{4}{a^2}+\frac{48F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}\right)}{16(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] (x*(4/a^2 - (b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a])/a^3 + (48*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^
3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*Appell
F1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(16*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^3+a\right)\left(b\,x^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{5/3}\,\left(a-b\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(a - b*x^3)),x)
[Out] int(1/((a + b*x^3)^(5/3)*(a - b*x^3)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a^2\left(a+bx^3\right)^{\frac{2}{3}}+b^2x^6\left(a+bx^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)
```

```
[Out] -Integral(1/(-a**2*(a + b*x**3)**(2/3) + b**2*x**6*(a + b*x**3)**(2/3)), x)
```

3.39 $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

Optimal. Leaf size=492

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{24\ 2^{2/3}a^{10/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\ 2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}(\ldots}{\sqrt[3]{\ldots}}\right.}{48\ 2^{2/3}a^{10/3}\ldots}$$

```
[Out] 1/10*x/a^2/(b*x^3+a)^(5/3)+13/40*x/a^3/(b*x^3+a)^(2/3)+9/20*x*(1+b*x^3/a)^(
2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)-1/48*ln(2^(2/
3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/48*ln(1
+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)-1/24*ln(1+2^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/96*ln(2*2^(1/3)+(a^(1/3)+b
^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^
(1/3)/a^(10/3)/b^(1/3)-1/24*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1/3)*3^(1/2)-1/48*arctan(1/3*(1+
2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(10/3)/b^(1
/3)*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};1,\frac{8}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^3\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 1, 8/3, 4/3, (b*x^3)/a, -((b*x^3)/a)
])/(a^3*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a-bx^3\right)\left(a+bx^3\right)^{8/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(a-bx^3\right)\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{a^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{8}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^3\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 0.18, size = 240, normalized size = 0.49

$$\frac{x\left(\frac{368a^3\left(a+bx^3\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+16a^2-13bx^3\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^{2/3} F\right.}{160a^4\left(a+bx^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] (x*(16*a^2 + 52*a*(a + b*x^3) - 13*b*x^3*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*
AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (368*a^3*(a + b*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), (b*x^3)/a])))))/(160*a^4*(a + b*x^3)^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-b\,x^3+a\right)\left(b\,x^3+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{8/3}\,\left(a-b\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(a - b*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(8/3)*(a - b*x^3)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)
```

```
[Out] Timed out
```

3.40 $\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3}\, dx$

Optimal. Leaf size=139

$$-\frac{38a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81\sqrt[3]{b}}+\frac{76a^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}+\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+\right.$$

```
[Out] 38/81*a^2*x*(b*x^3+a)^(2/3)-8/27*a*x*(b*x^3+a)^(5/3)-1/9*x*(-b*x^3+a)*(b*x^
3+a)^(5/3)-38/81*a^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+76/243*a^3*arct
an(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 195, 239}

$$\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{38a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81\sqrt[3]{b}}+\frac{76a^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (38*a^2*x*(a + b*x^3)^(2/3))/81 - (8*a*x*(a + b*x^3)^(5/3))/27 - (x*(a - b*
x^3)*(a + b*x^3)^(5/3))/9 + (76*a^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(1/3)) - (38*a^3*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)])/(81*b^(1/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a-bx^3\right)^2\left(a+bx^3\right)^{2/3}\,dx = -\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{\int \left(a+bx^3\right)^{2/3}\left(10a^2b-16ab^2x^3\right)dx}{9b}$$
$$= -\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{1}{27}\left(38a^2\right)\int \left(a+bx^3\right)^{2/3}\,dx$$
$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{1}{81}\left(76a^3\right)$$
$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}+\frac{76a^3\tan^{-}}{81\sqrt{}}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 1.09

$$\frac{1}{243}\left(\frac{38a^3\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{b}}+3\left(a+bx^3\right)^{2/3}\left(5a^2x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + (38*a^3*(2*Sqrt[3
]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3
)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3
)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/243
```

fricas [A] time = 0.88, size = 421, normalized size = 3.03

$$\left[\frac{114\sqrt{\frac{1}{3}}\,a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^
(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a^3*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1/243*(
228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2
*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(((-b)^(1
/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*x^7 - 24*a
*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

maxima [B] time = 1.53, size = 552, normalized size = 3.97

$$-\frac{1}{9}\left(\frac{2\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(bx^{3}\right.}{\left(b-\frac{bx}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a^2 - 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*a*b - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{2/3}\,{\left(a-b\,x^3\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3)^2, x)
```

sympy [C] time = 9.17, size = 126, normalized size = 0.91

$$\frac{a^{\frac{8}{3}} x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a} \right)}{3 \Gamma\left(\frac{4}{3}\right)} - \frac{2 a^{\frac{5}{3}} b x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a} \right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} b^{2} x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{b x^{3} e^{i \pi}}{a} \right)}{3 \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*hype
r((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.41 $\int \frac{\left(a-bx^3\right)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=120

$$-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

```
[Out] -13/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(-b*x^3+a)*(b*x^3+a)^(2/3)-17/18*a^2*ln(-b
^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+17/27*a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {416, 388, 239}

$$-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (-13*a*x*(a + b*x^3)^(2/3))/18 - (x*(a - b*x^3)*(a + b*x^3)^(2/3))/6 + (17*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/
3)) - (17*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{\int \frac{7a^2b-13ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$
$$= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{1}{9}\left(17a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$
$$= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}+\frac{17a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{17a^2\log\left(-\sqrt[3]{b}\,x+\right.}{18\sqrt[3]{b}}$$

Mathematica [A] time = 0.08, size = 141, normalized size = 1.18

$$\frac{17a^2\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{54\sqrt[3]{b}}+\left(a+bx^3\right)^{2/3}\left(\frac{bx^4}{6}-\frac{8ax}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(2/3)*((-8*a*x)/9 + (b*x^4)/6) + (17*a^2*(2*Sqrt[3]*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x
^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x
^3)^(1/3)]))/(54*b^(1/3))
```

fricas [A] time = 0.81, size = 399, normalized size = 3.32

$$\left[\frac{51\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*log(
((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)*a^2*b
*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))
*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)
^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(
2/3))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3+a\right)^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.40, size = 436, normalized size = 3.63

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)a^2-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a^2 - 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*a*b - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^3\right)}^2}{{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 7.60, size = 121, normalized size = 1.01

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.42 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{4/3}} \, dx$

Optimal. Leaf size=113

$$\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}-\frac{10a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] 2*x*(-b*x^3+a)/(b*x^3+a)^(1/3)+7/3*x*(b*x^3+a)^(2/3)+5/3*a*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(1/3)-10/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {413, 388, 239}

$$\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}-\frac{10a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) + (
5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx &= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{\int \frac{-a^2b+7ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}\\
&= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{7}{3}x\left(a+bx^3\right)^{2/3}-\frac{1}{3}(10a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\
&= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{7}{3}x\left(a+bx^3\right)^{2/3}-\frac{10a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}+\frac{5a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 1.21

$$\frac{x\left(13a+bx^3\right)}{3\sqrt[3]{a+bx^3}}-\frac{5a\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(13*a + b*x^3))/(3*(a + b*x^3)^(1/3)) - (5*a*(2*Sqrt[3]*ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(
1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(
1/3)]))/(9*b^(1/3))
```

fricas [B] time = 0.84, size = 412, normalized size = 3.65

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^2x^3+a^2b\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*
x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b
*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*x^3
 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 + a^2)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1/9*(10
*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x
^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3)*(b^(1/3
)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13*a*b*x)*(b
*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3+a\right)^2}{\left(b\,x^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

maxima [B] time = 1.37, size = 296, normalized size = 2.62

$$\frac{1}{9}b^2\left(\frac{4\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\left(bx^3+a\right)a}{x^3}\right)}{\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^3}{x}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) + 1/3*a*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + a*x/(b*x^3 + a)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{\left(b\,x^3+a\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(4/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a + bx^3\right)^2}{\left(a + bx^3\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

3.43 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx$

Optimal. Leaf size=110

$$-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

```
[Out] 1/2*x*(-b*x^3+a)/(b*x^3+a)^(4/3)-1/2*x/(b*x^3+a)^(1/3)-1/2*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/b^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/
2))/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {413, 385, 239}

$$-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) - x/(2*(a + b*x^3)^(1/3)) + ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx &= \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}+\frac{\int \frac{2a^2b+4ab^2x^3}{(a+bx^3)^{4/3}}\,dx}{4ab}\\ &= \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 1.19

$$\frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}}+\log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log
[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b
^(1/3))
```

fricas [B] time = 0.91, size = 521, normalized size = 4.74

$$\left[-\frac{6\left(bx^3+a\right)^{\frac{2}{3}}b^2x^4-3\sqrt{\frac{1}{3}}\left(b^3x^6+2\,ab^2x^3+a^2b\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a
^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 -
 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2
/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^
3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b*x^3 +
a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt(-(-b)^(
1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(
1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (
b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/
3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6
 + 2*a*b^2*x^3 + a^2*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3+a\right)^2}{\left(b\,x^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)

[Out] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)

maxima [B] time = 1.35, size = 180, normalized size = 1.64

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}}-\frac{bx^4}{2\left(bx^3+a\right)^{\frac{4}{3}}}-\frac{1}{12}\left(\frac{3\left(b+\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{\left(bx^3+a\right)^{\frac{4}{3}}b^2}+\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)}{x}\right.}{b^{\frac{7}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")

[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/(b*x^3 + a)^(4/3) - 1/2*b*x^4/(b*x^3 + a)^(4/3) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*b^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{\left(b\,x^3+a\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b*x^3)^2/(a + b*x^3)^(7/3),x)

[Out] int((a - b*x^3)^2/(a + b*x^3)^(7/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a+bx^3\right)^2}{\left(a+bx^3\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

3.44 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=76

$$\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

```
[Out] 1/7*x*(-b*x^3+a)^2/a/(b*x^3+a)^(7/3)+3/14*x*(-b*x^3+a)/a/(b*x^3+a)^(4/3)+9/
14*x/a/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {378, 191}

$$\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (3*x*(a - b*x^3))/(14*a*(a + b*
x^3)^(4/3)) + (9*x)/(14*a*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx=\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{6}{7}\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx$$
$$=\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9}{14}\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx$$
$$=\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.53

$$\frac{x\left(7a^2+7abx^3+4b^2x^6\right)}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(7*a^2 + 7*a*b*x^3 + 4*b^2*x^6))/(7*a*(a + b*x^3)^(7/3))
```

fricas [A] time = 0.86, size = 67, normalized size = 0.88

$$\frac{\left(4\,b^2x^7+7\,abx^4+7\,a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{7\left(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2*
b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")

[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)

maple [A] time = 0.04, size = 37, normalized size = 0.49

$$\frac{\left(4b^2x^6 + 7ab\,x^3 + 7a^2\right)x}{7\left(b\,x^3 + a\right)^{\frac{7}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x)

[Out] 1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a

maxima [A] time = 0.50, size = 105, normalized size = 1.38

$$\frac{\left(4\,b - \frac{7\left(bx^3+a\right)}{x^3}\right)bx^7}{14\left(bx^3 + a\right)^{\frac{7}{3}}a} + \frac{b^2x^7}{7\left(bx^3 + a\right)^{\frac{7}{3}}a} + \frac{\left(2\,b^2 - \frac{7\left(bx^3+a\right)b}{x^3} + \frac{14\left(bx^3+a\right)^2}{x^6}\right)x^7}{14\left(bx^3 + a\right)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")

[Out] 1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a)

mupad [B] time = 1.43, size = 44, normalized size = 0.58

$$\frac{4\,x\left(b\,x^3 + a\right)^2 + 4\,a^2\,x - a\,x\,\left(b\,x^3 + a\right)}{7\,a\left(b\,x^3 + a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)

[Out] (4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)

[Out] Timed out
```

3.45 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal. Leaf size=105

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

```
[Out] 1/20*x*(-b*x^3+a)^3/a^2/(b*x^3+a)^(10/3)+19/140*x*(-b*x^3+a)^2/a^2/(b*x^3+a
)^(7/3)+57/280*x*(-b*x^3+a)/a^2/(b*x^3+a)^(4/3)+171/280*x/a^2/(b*x^3+a)^(1/
3)
```

Rubi [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {382, 378, 191}

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*x*(a - b*x^3)^2)/(140*a
^2*(a + b*x^3)^(7/3)) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^(4/3)) + (1
71*x)/(280*a^2*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx &= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx}{20a}\\
&= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{70a}\\
&= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}}+\frac{171\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{280a}\\
&= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.49

$$\frac{x\left(140a^3+245a^2bx^3+230ab^2x^6+69b^3x^9\right)}{140a^2\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3 + 245*a^2*b*x^3 + 230*a*b^2*x^6 + 69*b^3*x^9))/(140*a^2*(a + b*
x^3)^(10/3))
```

fricas [A] time = 0.91, size = 91, normalized size = 0.87

$$\frac{\left(69\,b^3x^{10}+230\,ab^2x^7+245\,a^2bx^4+140\,a^3x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{140\left(a^2b^4x^{12}+4\,a^3b^3x^9+6\,a^4b^2x^6+4\,a^5bx^3+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a)
^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.46

$$\frac{\left(69b^3x^9 + 230a\,b^2x^6 + 245a^2b\,x^3 + 140a^3\right)x}{140\left(b\,x^3 + a\right)^{\frac{10}{3}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/a
^2
```

maxima [A] time = 0.59, size = 155, normalized size = 1.48

$$-\frac{\left(7\,b - \frac{10\left(bx^3+a\right)}{x^3}\right)b^2x^{10}}{70\left(bx^3 + a\right)^{\frac{10}{3}}a^2} - \frac{\left(14\,b^2 - \frac{40\left(bx^3+a\right)b}{x^3} + \frac{35\left(bx^3+a\right)^2}{x^6}\right)bx^{10}}{70\left(bx^3 + a\right)^{\frac{10}{3}}a^2} - \frac{\left(14\,b^3 - \frac{60\left(bx^3+a\right)b^2}{x^3} + \frac{105\left(bx^3+a\right)^2b}{x^6} - \frac{140\left(bx^3+a\right)^3}{x^9}\right)x^{1}}{140\left(bx^3 + a\right)^{\frac{10}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)^(
10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b/x
^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

mupad [B] time = 1.39, size = 56, normalized size = 0.53

$$\frac{69\,x}{140\,a^2\,{\left(b\,x^3+a\right)}^{1/3}}-\frac{2\,x}{35\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{23\,x}{140\,a\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{2\,a\,x}{5\,{\left(b\,x^3+a\right)}^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)
```

```
[Out] (69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)/
(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.46 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx$

Optimal. Leaf size=98

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

```
[Out] 2/13*x*(-b*x^3+a)/(b*x^3+a)^(13/3)+8/65*x/(b*x^3+a)^(10/3)+47/455*x/a/(b*x^
3+a)^(7/3)+141/910*x/a^2/(b*x^3+a)^(4/3)+423/910*x/a^3/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 192, 191}

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + (8*x)/(65*(a + b*x^3)^(10/3)) +
 (47*x)/(455*a*(a + b*x^3)^(7/3)) + (141*x)/(910*a^2*(a + b*x^3)^(4/3)) + (
423*x)/(910*a^3*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{\int \frac{11a^2b-5ab^2x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47}{65}\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47x}{455a\left(a+bx^3\right)^{7/3}} + \frac{282\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a}$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47x}{455a\left(a+bx^3\right)^{7/3}} + \frac{141x}{910a^2\left(a+bx^3\right)^{4/3}} + \frac{423\int \cdots}{910\cdots}$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47x}{455a\left(a+bx^3\right)^{7/3}} + \frac{141x}{910a^2\left(a+bx^3\right)^{4/3}} + \frac{423x\cdots}{910a^3\sqrt[3]{a}\cdots}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.63

$$\frac{x\left(910a^4+2275a^3bx^3+3055a^2b^2x^6+1833ab^3x^9+423b^4x^{12}\right)}{910a^3\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

[Out] (x*(910*a^4 + 2275*a^3*b*x^3 + 3055*a^2*b^2*x^6 + 1833*a*b^3*x^9 + 423*b^4*
x^12))/(910*a^3*(a + b*x^3)^(13/3))

fricas [A]   time = 0.91, size = 113, normalized size = 1.15

$$\frac{\left(423\,b^{4}x^{13} + 1833\,ab^{3}x^{10} + 3055\,a^{2}b^{2}x^{7} + 2275\,a^{3}bx^{4} + 910\,a^{4}x\right)\left(bx^{3} + a\right)^{\frac{2}{3}}}{910\left(a^{3}b^{5}x^{15} + 5\,a^{4}b^{4}x^{12} + 10\,a^{5}b^{3}x^{9} + 10\,a^{6}b^{2}x^{6} + 5\,a^{7}bx^{3} + a^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")

[Out] 1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4 +
 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3*x
^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3} - a\right)^{2}}{\left(bx^{3} + a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")

[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)

maple [A]   time = 0.05, size = 59, normalized size = 0.60

$$\frac{\left(423b^{4}x^{12} + 1833b^{3}x^{9}a + 3055b^{2}x^{6}a^{2} + 2275b\,x^{3}a^{3} + 910a^{4}\right)x}{910\left(b\,x^{3} + a\right)^{\frac{13}{3}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x)

[Out] 1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a^
4)/(b*x^3+a)^(13/3)/a^3

maxima [B]   time = 0.57, size = 206, normalized size = 2.10

$$\frac{\left(35\,b^{2} - \frac{91\left(bx^{3}+a\right)b}{x^{3}} + \frac{65\left(bx^{3}+a\right)^{2}}{x^{6}}\right)b^{2}x^{13}}{455\left(bx^{3} + a\right)^{\frac{13}{3}}a^{3}} + \frac{\left(140\,b^{3} - \frac{546\left(bx^{3}+a\right)b^{2}}{x^{3}} + \frac{780\left(bx^{3}+a\right)^{2}b}{x^{6}} - \frac{455\left(bx^{3}+a\right)^{3}}{x^{9}}\right)bx^{13}}{910\left(bx^{3} + a\right)^{\frac{13}{3}}a^{3}} + \frac{\left(35\,b^{4} - \frac{182\left(bx^{3}+a\right.}{x^{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*x
^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) + 1/
455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*
x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^3)
```

mupad [B] time = 1.44, size = 71, normalized size = 0.72

$$\frac{423\,x}{910\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}-\frac{2\,x}{65\,{\left(b\,x^3+a\right)}^{10/3}}+\frac{141\,x}{910\,a^2\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{47\,x}{455\,a\,{\left(b\,x^3+a\right)}^{7/3}}+\frac{4\,a\,x}{13\,{\left(b\,x^3+a\right)}^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (423*x)/(910*a^3*(a + b*x^3)^(1/3)) - (2*x)/(65*(a + b*x^3)^(10/3)) + (141*
x)/(910*a^2*(a + b*x^3)^(4/3)) + (47*x)/(455*a*(a + b*x^3)^(7/3)) + (4*a*x)
/(13*(a + b*x^3)^(13/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.47 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{19/3}}\,dx$

Optimal. Leaf size=117

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3\left(a+bx^3\right)^{4/3}}+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

```
[Out] 1/8*x*(-b*x^3+a)/(b*x^3+a)^(16/3)+11/104*x/(b*x^3+a)^(13/3)+1/13*x/a/(b*x^3
+a)^(10/3)+9/91*x/a^2/(b*x^3+a)^(7/3)+27/182*x/a^3/(b*x^3+a)^(4/3)+81/182*x
/a^4/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 192, 191}

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3\left(a+bx^3\right)^{4/3}}+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + (11*x)/(104*(a + b*x^3)^(13/3)) +
x/(13*a*(a + b*x^3)^(10/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (81*x)/(182*a^4*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{\int \frac{14a^2b-8ab^2x^3}{(a+bx^3)^{16/3}}\,dx}{16ab}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{10}{13}\int \frac{1}{(a+bx^3)^{13/3}}\,dx$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{13a}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{54\int \frac{1}{(a+bx^3)}}{91a^2}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 0.62

$$\frac{x\left(364a^5+1183a^4bx^3+2080a^3b^2x^6+1872a^2b^3x^9+864ab^4x^{12}+162b^5x^{15}\right)}{364a^4\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(364*a^5 + 1183*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1872*a^2*b^3*x^9 + 864*a*
b^4*x^12 + 162*b^5*x^15))/(364*a^4*(a + b*x^3)^(16/3))
```

fricas [A] time = 0.99, size = 135, normalized size = 1.15

$$\frac{\left(162\, b^{5} x^{16} + 864\, a b^{4} x^{13} + 1872\, a^{2} b^{3} x^{10} + 2080\, a^{3} b^{2} x^{7} + 1183\, a^{4} b x^{4} + 364\, a^{5} x\right) \left(b x^{3} + a\right)^{\frac{2}{3}}}{364 \left(a^{4} b^{6} x^{18} + 6\, a^{5} b^{5} x^{15} + 15\, a^{6} b^{4} x^{12} + 20\, a^{7} b^{3} x^{9} + 15\, a^{8} b^{2} x^{6} + 6\, a^{9} b x^{3} + a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^7
 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^5*
x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 + a^
10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b x^{3} - a\right)^{2}}{\left(b x^{3} + a\right)^{\frac{19}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

maple [A] time = 0.05, size = 70, normalized size = 0.60

$$\frac{\left(162 b^{5} x^{15} + 864 a\, b^{4} x^{12} + 1872 a^{2} b^{3} x^{9} + 2080 a^{3} b^{2} x^{6} + 1183 a^{4} b\, x^{3} + 364 a^{5}\right) x}{364 \left(b\, x^{3} + a\right)^{\frac{16}{3}} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x)
```

```
[Out] 1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+1183
*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

maxima [B] time = 0.51, size = 257, normalized size = 2.20

$$-\frac{\left(455\,b^3-\frac{1680\,(bx^3+a)b^2}{x^3}+\frac{2184\,(bx^3+a)^2b}{x^6}-\frac{1040\,(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4}-\frac{\left(455\,b^4-\frac{2240\,(bx^3+a)b^3}{x^3}+\frac{4368\,(bx^3+a)^2b^2}{x^6}-\frac{4160\,(bx^3+a}{x^9}\right.}{3640\,(bx^3+a)^{\frac{16}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1456*
(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*x^
3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*x^1
6/((b*x^3 + a)^(16/3)*a^4)
```

mupad [B] time = 1.43, size = 86, normalized size = 0.74

$$\frac{81\,x}{182\,a^4\,\left(b\,x^3+a\right)^{1/3}}-\frac{x}{52\,\left(b\,x^3+a\right)^{13/3}}+\frac{27\,x}{182\,a^3\,\left(b\,x^3+a\right)^{4/3}}+\frac{9\,x}{91\,a^2\,\left(b\,x^3+a\right)^{7/3}}+\frac{x}{13\,a\,\left(b\,x^3+a\right)^{10/3}}+\frac{a\,x}{4\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (81*x)/(182*a^4*(a + b*x^3)^(1/3)) - x/(52*(a + b*x^3)^(13/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + x/(13*a*(a +
b*x^3)^(10/3)) + (a*x)/(4*(a + b*x^3)^(16/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

3.48 $\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx$

Optimal. Leaf size=94

$$\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

```
[Out] -9/44*a*x*(b*x^3+a)^(7/3)-1/11*x*(-b*x^3+a)*(b*x^3+a)^(7/3)+57/44*a^3*x*(b*
x^3+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 246, 245}

$$\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (-9*a*x*(a + b*x^3)^(7/3))/44 - (x*(a - b*x^3)*(a + b*x^3)^(7/3))/11 + (57*
a^3*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4
4*(1 + (b*x^3)/a)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx = -\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\int \left(a+bx^3\right)^{4/3}\left(12a^2b-18ab^2x^3\right)dx}{11b}$$

$$= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{1}{44}\left(57a^2\right)\int \left(a+bx^3\right)^{4/3} dx$$

$$= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\left(57a^3\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{4}}{44\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3}\right.}{44\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.05, size = 97, normalized size = 1.03

$$\frac{x\left(114a^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+106a^4+53a^3bx^3-78a^2b^2x^6-5ab^3x^9+20b^4x^{12}\right)}{220\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(106*a^4 + 53*a^3*b*x^3 - 78*a^2*b^2*x^6 - 5*a*b^3*x^9 + 20*b^4*x^12 + 1
14*a^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]
))/(220*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3x^9 - ab^2x^6 - a^2bx^3 + a^3\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="fricas")
[Out] integral((b^3*x^9 - a*b^2*x^6 - a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{4}{3}}\left(bx^3 - a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(-b\,x^3 + a\right)^2 \left(b\,x^3 + a\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{4}{3}}\left(bx^3 - a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3 + a\right)^{4/3} \left(a - b\,x^3\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(a - b*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(4/3)*(a - b*x^3)^2, x)
```

sympy [C] time = 8.41, size = 168, normalized size = 1.79

$$\frac{a^{\frac{10}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{7}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{a^{\frac{4}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}\,b^3x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\right.}{3\Gamma\left(\frac{1}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)
```

```
[Out] a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hyper
((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)
*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

3.49 $\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=94

$$\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

```
[Out] -3/8*a*x*(b*x^3+a)^(4/3)-1/8*x*(-b*x^3+a)*(b*x^3+a)^(4/3)+3/2*a^2*x*(b*x^3+
a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 246, 245}

$$\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*a*x*(a + b*x^3)^(4/3))/8 - (x*(a - b*x^3)*(a + b*x^3)^(4/3))/8 + (3*a^2
*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(2*(1
 + (b*x^3)/a)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx &= -\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\int \sqrt[3]{a+bx^3}\left(9a^2b-15ab^2x^3\right)dx}{8b}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{1}{2}\left(3a^2\right)\int \sqrt[3]{a+bx^3}\,dx\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\left(3a^2\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{2\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.90

$$\frac{x\left(6a^3\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+2a^3-a^2bx^3-2ab^2x^6+b^3x^9\right)}{8\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(2*a^3 - a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9 + 6*a^3*(1 + (b*x^3)/a)^(2/3)
*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(8*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{1}{3}}\left(bx^3 - a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(-b\,x^3 + a\right)^2 \left(b\,x^3 + a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{1}{3}}\left(bx^3 - a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3 + a\right)^{1/3} \left(a - b\,x^3\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(a - b*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(a - b*x^3)^2, x)
```

sympy **[C]** time = 5.59, size = 126, normalized size = 1.34

$$\frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{4}{3}}bx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,b^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(7/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(4/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b**2*x**7*gamma(7/3)*hype
r((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.50 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=94

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}$$

```
[Out] -6/5*a*x*(b*x^3+a)^(1/3)-1/5*x*(-b*x^3+a)*(b*x^3+a)^(1/3)+12/5*a^2*x*(1+b*x
^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 246, 245}

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (-6*a*x*(a + b*x^3)^(1/3))/5 - (x*(a - b*x^3)*(a + b*x^3)^(1/3))/5 + (12*a^
2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
5*(a + b*x^3)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{2/3}}\,dx &= -\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{\int \frac{6a^2b-12ab^2x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{5b}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{1}{5}\left(12a^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{\left(12a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.80

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-7a^2x-6abx^4+b^2x^7}{5\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (-7*a^2*x - 6*a*b*x^4 + b^2*x^7 + 12*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeome
tric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2x^6 - 2\,abx^3 + a^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3 + a\right)^2}{\left(b\,x^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a - b\,x^3\right)^2}{\left(b\,x^3 + a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(2/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 5.76, size = 121, normalized size = 1.29

$$\frac{a^{\frac{4}{3}} x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{4}{3}\right)} - \frac{2 \sqrt[3]{a} b x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{b^{2} x^{7} \Gamma\left(\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{b x^{3} e^{i\pi}}{a}\right)}{3 a^{\frac{2}{3}} \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

3.51 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=74

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(-b*x^3+a)/(b*x^3+a)^(2/3)+3/4*b*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/
3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 12, 365, 364}

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{5/3}}\,dx &= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}+\frac{\int \frac{6ab^2x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}\\
&= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}+(3b)\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}+\frac{\left(3b\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}\\
&= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}+\frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.84

$$\frac{-3ax\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+5ax+bx^4}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (5*a*x + b*x^4 - 3*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/
3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2}x^{6}-2\,abx^{3}+a^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{b^{2}x^{6}+2\,abx^{3}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{\left(b\,x^3+a\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(5/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a+bx^3\right)^2}{\left(a+bx^3\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)
```

3.52 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{8/3}} \, dx$

Optimal. Leaf size=74

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}$$

```
[Out] 2/5*x*(-b*x^3+a)/(b*x^3+a)^(5/3)+3/5*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/
3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 21, 246, 245}

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperg
eometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{8/3}}\,dx &= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{\int \frac{3a^2b+3ab^2x^3}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}\\
&= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{3}{5}\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}}\\
&= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.95

$$\frac{3x\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*x*(a - b*x^3) + 3*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(5/3))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2}x^{6}-2\,abx^{3}+a^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{b^{3}x^{9}+3\,ab^{2}x^{6}+3\,a^{2}bx^{3}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a - b\,x^3\right)^2}{\left(b\,x^3 + a\right)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(8/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(8/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{11/3}}\,dx$

Optimal. Leaf size=77

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}$$

```
[Out] 1/4*x*(-b*x^3+a)/(b*x^3+a)^(8/3)+3/4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 8/
3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 12, 246, 245}

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{11/3}}\,dx &= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{\int \frac{6a^2b}{\left(a+bx^3\right)^{8/3}}\,dx}{8ab}\\
&= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{1}{4}(3a)\int \frac{1}{\left(a+bx^3\right)^{8/3}}\,dx\\
&= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{4a\left(a+bx^3\right)^{2/3}}\\
&= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 85, normalized size = 1.10

$$\frac{7a^2x+3x\left(a+bx^3\right)^2\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+5abx^4+3b^2x^7}{10a\left(a+bx^3\right)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (7*a^2*x + 5*a*b*x^4 + 3*b^2*x^7 + 3*x*(a + b*x^3)^2*(1 + (b*x^3)/a)^(2/3)*
Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*a*(a + b*x^3)^(8/3))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{b^4x^{12} + 4\,ab^3x^9 + 6\,a^2b^2x^6 + 4\,a^3bx^3 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^4*x^12 + 4*a*b^3*
x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3 + a\right)^2}{\left(b\,x^3 + a\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{\left(b\,x^3+a\right)^{11/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(11/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(11/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)
[Out] Timed out
```

3.54 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx$

Optimal. Leaf size=93

$$\frac{15x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\left(a+bx^3\right)^{2/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}$$

```
[Out] 2/11*x*(-b*x^3+a)/(b*x^3+a)^(11/3)+3/22*x/(b*x^3+a)^(8/3)+15/22*x*(1+b*x^3/
a)^(2/3)*hypergeom([1/3, 8/3],[4/3],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 246, 245}

$$\frac{15x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\left(a+bx^3\right)^{2/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*x)/(22*(a + b*x^3)^(8/3)) +
(15*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(22*a^2*(a + b*x^3)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{14/3}}\,dx &= \frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}+\frac{\int \frac{9a^2b-3ab^2x^3}{\left(a+bx^3\right)^{11/3}}\,dx}{11ab}\\
&= \frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{15}{22}\int \frac{1}{\left(a+bx^3\right)^{8/3}}\,dx\\
&= \frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{\left(15\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{22a^2\left(a+bx^3\right)^{2/3}}\\
&= \frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\left(a+bx^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 95, normalized size = 1.02

$$
\frac{x\left(16a^3+23a^2bx^3+21ab^2x^6+6\left(a+bx^3\right)^3\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+6b^3x^9\right)}{22a^2\left(a+bx^3\right)^{11/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (x*(16*a^3 + 23*a^2*b*x^3 + 21*a*b^2*x^6 + 6*b^3*x^9 + 6*(a + b*x^3)^3*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(22*a^2*
(a + b*x^3)^(11/3))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^{2}x^{6}-2\,abx^{3}+a^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{b^{5}x^{15}+5\,ab^{4}x^{12}+10\,a^{2}b^{3}x^{9}+10\,a^{3}b^{2}x^{6}+5\,a^{4}bx^{3}+a^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^5*x^15 + 5*a*b^4*
x^12 + 10*a^2*b^3*x^9 + 10*a^3*b^2*x^6 + 5*a^4*b*x^3 + a^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int\frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="maxima")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a - b\,x^3\right)^2}{\left(b\,x^3 + a\right)^{14/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(14/3),x)
[Out] int((a - b*x^3)^2/(a + b*x^3)^(14/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(14/3),x)
[Out] Timed out
```

3.55 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx$

Optimal. Leaf size=93

$$\frac{57x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}$$

```
[Out] 1/7*x*(-b*x^3+a)/(b*x^3+a)^(14/3)+9/77*x/(b*x^3+a)^(11/3)+57/77*x*(1+b*x^3/
a)^(2/3)*hypergeom([1/3, 11/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 246, 245}

$$\frac{57x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(a - b*x^3))/(7*(a + b*x^3)^(14/3)) + (9*x)/(77*(a + b*x^3)^(11/3)) + (5
7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, -((b*x^3)/a)])/
(77*a^3*(a + b*x^3)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{17/3}}\,dx &= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{\int \frac{12a^2b-6ab^2x^3}{\left(a+bx^3\right)^{14/3}}\,dx}{14ab}\\ &= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{57}{77}\int \frac{1}{\left(a+bx^3\right)^{11/3}}\,dx\\ &= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{\left(57\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{11/3}}\,dx}{77a^3\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 106, normalized size = 1.14

$$\frac{x\left(2282a^4+4879a^3bx^3+6270a^2b^2x^6+3591ab^3x^9+798\left(a+bx^3\right)^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+798b^4x^{12}\right)}{3080a^3\left(a+bx^3\right)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(2282*a^4 + 4879*a^3*b*x^3 + 6270*a^2*b^2*x^6 + 3591*a*b^3*x^9 + 798*b^4
*x^12 + 798*(a + b*x^3)^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, -((b*x^3)/a)]))/(3080*a^3*(a + b*x^3)^(14/3))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(b^{2} x^{6} - 2\, a b x^{3} + a^{2}\right)\left(b x^{3} + a\right)^{\frac{1}{3}}}{b^{6} x^{18} + 6\, a b^{5} x^{15} + 15\, a^{2} b^{4} x^{12} + 20\, a^{3} b^{3} x^{9} + 15\, a^{4} b^{2} x^{6} + 6\, a^{5} b x^{3} + a^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^6*x^18 + 6*a*b^5*
x^15 + 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^
6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b x^{3} - a\right)^{2}}{\left(b x^{3} + a\right)^{\frac{17}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\, x^{3} + a\right)^{2}}{\left(b\, x^{3} + a\right)^{\frac{17}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b x^{3} - a\right)^{2}}{\left(b x^{3} + a\right)^{\frac{17}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a-b\,x^3\right)^2}{\left(b\,x^3+a\right)^{17/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(17/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(17/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(17/3),x)
```

```
[Out] Timed out
```

3.56 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=174

$$-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{162b^{4/3}}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}}+\frac{x\left(a+bx^3\right)^{5/3}(9bc-ad)}{54b}+\frac{5ax\left(a+bx^3\right.}{16}$$

```
[Out] 5/162*a*(-a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b+1/54*(-a*d+9*b*c)*x*(b*x^3+a)^(5/3
)/b+1/9*d*x*(b*x^3+a)^(8/3)/b-5/162*a^2*(-a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a
)^(1/3))/b^(4/3)+5/243*a^2*(-a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 239}

$$-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{162b^{4/3}}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}}+\frac{x\left(a+bx^3\right)^{5/3}(9bc-ad)}{54b}+\frac{5ax\left(a+bx^3\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (5*a*(9*b*c - a*d)*x*(a + b*x^3)^(2/3))/(162*b) + ((9*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(54*b) + (d*x*(a + b*x^3)^(8/3))/(9*b) + (5*a^2*(9*b*c - a*d)*Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(4/3)) -
 (5*a^2*(9*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(162*b^(4/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx = \frac{dx\left(a+bx^3\right)^{8/3}}{9b} - \frac{(-9bc+ad)\int \left(a+bx^3\right)^{5/3}\,dx}{9b}$$

$$= \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{(5a(9bc-ad))\int \left(a+bx^3\right)^{2/3}\,dx}{54b}$$

$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{\left(5a^2(9\right.}{}$$

$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{5a^2(9b}{}$$

Mathematica [C] time = 0.07, size = 75, normalized size = 0.43

$$\frac{x\left(a+bx^3\right)^{2/3}\left(d\left(a+bx^3\right)^2 - \frac{a(ad-9bc)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d*(a + b*x^3)^2 - (a*(-9*b*c + a*d)*Hypergeometric2F1
[-5/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(9*b)
```

fricas [A] time = 0.71, size = 482, normalized size = 2.77

$$\left[-\frac{15\sqrt{\frac{1}{3}}\left(9\,a^2b^2c-a^3bd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3
- 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^
(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(
9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a
^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*
(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^2*b*c -
 a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*b*c - a^
3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3)*(b^(1/3)
*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7 + 3*(9*b^3
*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int\left(b\,x^3+a\right)^{\frac{5}{3}}\left(d\,x^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

maxima [B] time = 1.68, size = 406, normalized size = 2.33

$$-\frac{1}{54}\left(\frac{10\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c + 1/486*(10*sqrt(3)*a^3*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 1
0*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3)*
a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x^8
)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/
x^9))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3), x)
```

sympy [C] time = 10.42, size = 170, normalized size = 0.98

$$\frac{a^{\frac{5}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(-\right.}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

```
[Out] a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hyper
((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b
*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

3.57 $\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=141

$$-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

```
[Out] 1/18*(-a*d+6*b*c)*x*(b*x^3+a)^(2/3)/b+1/6*d*x*(b*x^3+a)^(5/3)/b-1/18*a*(-a*
d+6*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)+1/27*a*(-a*d+6*b*c)*arctan(
1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 239}

$$-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] ((6*b*c - a*d)*x*(a + b*x^3)^(2/3))/(18*b) + (d*x*(a + b*x^3)^(5/3))/(6*b)
+ (a*(6*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(
9*Sqrt[3]*b^(4/3)) - (a*(6*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(18*b^(4/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx = \frac{dx\left(a+bx^3\right)^{5/3}}{6b} - \frac{(-6bc+ad)\int \left(a+bx^3\right)^{2/3}\,dx}{6b}$$

$$= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b} + \frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{(a(6bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b}$$

$$= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b} + \frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{a(6bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}} - \frac{a(6bc}{}$$

Mathematica [C] time = 0.10, size = 72, normalized size = 0.51

$$\frac{x\left(a+bx^3\right)^{2/3}\left(\frac{(6bc-ad)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}+d\left(a+bx^3\right)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d*(a + b*x^3) + ((6*b*c - a*d)*Hypergeometric2F1[-2/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(6*b)
```

fricas [A] time = 0.87, size = 424, normalized size = 3.01

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(6\,ab^2c-a^2bd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*
(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3
)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*a*b
*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c - a^
2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3*b^2*d*x
^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

maxima [B] time = 1.44, size = 322, normalized size = 2.28

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(bx^3+\cdots\right.}{\left(b-\frac{bx^3\cdots}{x^3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c + 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3), x)
```

sympy [C] time = 5.35, size = 82, normalized size = 0.58

$$\frac{a^{\frac{2}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{2}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

```
[Out] a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.58 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=111

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

```
[Out] 1/3*d*x*(b*x^3+a)^(2/3)/b-1/6*(-a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b
^(4/3)+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {388, 239}

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) - ((3*b*c - a*d)*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}}{3b} - \frac{(-3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}$$

$$= \frac{dx\left(a+bx^3\right)^{2/3}}{3b} + \frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

Mathematica [A] time = 0.15, size = 141, normalized size = 1.27

$$\frac{\frac{(3bc-ad)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}+dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3) + ((3*b*c - a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] +
Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/
(6*b^(1/3)))/(3*b)
```

fricas [A] time = 0.65, size = 362, normalized size = 3.26

$$\left[\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx - 3\sqrt{\frac{1}{3}}\left(3b^2c-abd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\right.\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)
*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(
2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))
/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c
- a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*d)*b^(2
/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 +
a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3 + c}{\left(b\,x^3 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.20, size = 244, normalized size = 2.20

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c+\frac{1}{18}\left(\frac{2\sqrt{3}\,a}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 4.44, size = 78, normalized size = 0.70

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.59 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=99

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

```
[Out] (-a*d+b*c)*x/a/b/(b*x^3+a)^(1/3)-1/2*d*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/
3)+1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2
)
```

Rubi [A] time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 239}

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(2*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b}$$

$$= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}} - \frac{d\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.62

$$\frac{dx^4\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)+4cx}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (4*c*x + d*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((b*
x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

fricas [B] time = 0.69, size = 488, normalized size = 4.93

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^2dx^3+a^2bd\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3
 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) +
```

```
2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*
sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*
sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2*(a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{d\, x^3 + c}{\left(b\, x^3 + a\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

maxima [A] time = 1.22, size = 134, normalized size = 1.35

$$-\frac{1}{6} d\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\, b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\, x}{\left(bx^3 + a\right)^{\frac{1}{3}} b} - \frac{\log\left(b^{\frac{2}{3}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}} b^{\frac{1}{3}}}{x} + \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{4}{3}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*
b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(
1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

sympy [C] time = 12.83, size = 71, normalized size = 0.72

$$\frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

```
[Out] c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamma
(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

3.60 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}} \, dx$

Optimal. Leaf size=47

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

```
[Out] 3/4*c*x/a^2/(b*x^3+a)^(1/3)+1/4*x*(d*x^3+c)/a/(b*x^3+a)^(4/3)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}} + \frac{(3c)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{4a}$$
$$= \frac{3cx}{4a^2\sqrt[3]{a+bx^3}} + \frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.79

$$\frac{x\left(4ac+adx^3+3bcx^3\right)}{4a^2\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

fricas [A] time = 0.83, size = 54, normalized size = 1.15

$$\frac{\left((3\,bc+ad)x^4+4\,acx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{4\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

maple [A] time = 0.05, size = 34, normalized size = 0.72

$$\frac{\left(ad\,x^{3}+3bc\,x^{3}+4ac\right)x}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(7/3),x)
```

```
[Out] 1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

maxima [A] time = 0.47, size = 51, normalized size = 1.09

$$-\frac{\left(b-\frac{4\left(bx^{3}+a\right)}{x^{3}}\right)cx^{4}}{4\left(bx^{3}+a\right)^{\frac{4}{3}}a^{2}}+\frac{dx^{4}}{4\left(bx^{3}+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b*
x^3 + a)^(4/3)*a)
```

mupad [B] time = 1.37, size = 33, normalized size = 0.70

$$\frac{4\,a\,c\,x+a\,d\,x^{4}+3\,b\,c\,x^{4}}{4\,a^{2}\left(b\,x^{3}+a\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(7/3),x)
```

```
[Out] (4*a*c*x + a*d*x^4 + 3*b*c*x^4)/(4*a^2*(a + b*x^3)^(4/3))
```

sympy [B] time = 82.05, size = 190, normalized size = 4.04

$$c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^{3}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^{4}\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^{3}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma\left(\frac{7}{3}\right)}\right)+\frac{}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^{3}}{a}}\,\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

```
[Out] c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

3.61 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=91

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

```
[Out] 1/7*(-a*d+b*c)*x/a/b/(b*x^3+a)^(7/3)+1/28*(a*d+6*b*c)*x/a^2/b/(b*x^3+a)^(4/
3)+3/28*(a*d+6*b*c)*x/a^3/b/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] ((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*x)/(28*a^2*b*(a
+ b*x^3)^(4/3)) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx = \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6bc+ad)\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{7ab}$$
$$= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}} + \frac{(3(6bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a^2b}$$
$$= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}} + \frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.65

$$\frac{7a^2\left(4cx+dx^4\right)+3abx^4\left(14c+dx^3\right)+18b^2cx^7}{28a^3\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (18*b^2*c*x^7 + 3*a*b*x^4*(14*c + d*x^3) + 7*a^2*(4*c*x + d*x^4))/(28*a^3*(
a + b*x^3)^(7/3))
```

fricas [A] time = 0.72, size = 87, normalized size = 0.96

$$\frac{\left(3\left(6\,b^2c+abd\right)x^7+7\left(6\,abc+a^2d\right)x^4+28\,a^2cx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{28\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x^
3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.04, size = 57, normalized size = 0.63

$$\frac{\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)x}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/28*x*(3*a*b*d*x^6+18*b^2*c*x^6+7*a^2*d*x^3+42*a*b*c*x^3+28*a^2*c)/(b*x^3+
a)^(7/3)/a^3
```

maxima [A] time = 0.61, size = 86, normalized size = 0.95

$$-\frac{\left(4\,b-\frac{7\left(bx^{3}+a\right)}{x^{3}}\right)dx^{7}}{28\left(bx^{3}+a\right)^{\frac{7}{3}}a^{2}}+\frac{\left(2\,b^{2}-\frac{7\left(bx^{3}+a\right)b}{x^{3}}+\frac{14\left(bx^{3}+a\right)^{2}}{x^{6}}\right)cx^{7}}{14\left(bx^{3}+a\right)^{\frac{7}{3}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a^3
)
```

mupad [B] time = 1.42, size = 87, normalized size = 0.96

$$\frac{3\,a\,d\,x\left(b\,x^{3}+a\right)^{2}-4\,a^{3}\,d\,x+18\,b\,c\,x\left(b\,x^{3}+a\right)^{2}+a^{2}\,d\,x\,\left(b\,x^{3}+a\right)+4\,a^{2}\,b\,c\,x+6\,a\,b\,c\,x\,\left(b\,x^{3}+a\right)}{28\,a^{3}\,b\left(b\,x^{3}+a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a +
b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^(7/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

3.62 $\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx$

Optimal. Leaf size=121

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

```
[Out] 1/10*(-a*d+b*c)*x/a/b/(b*x^3+a)^(10/3)+1/70*(a*d+9*b*c)*x/a^2/b/(b*x^3+a)^(
7/3)+3/140*(a*d+9*b*c)*x/a^3/b/(b*x^3+a)^(4/3)+9/140*(a*d+9*b*c)*x/a^4/b/(b
*x^3+a)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] ((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*x)/(70*a^2*b*(
a + b*x^3)^(7/3)) + (3*(9*b*c + a*d)*x)/(140*a^3*b*(a + b*x^3)^(4/3)) + (9*
(9*b*c + a*d)*x)/(140*a^4*b*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{10ab}$$

$$= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{(3(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{35a^2b}$$

$$= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{(9(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}}{140a^3b}$$

$$= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.03, size = 80, normalized size = 0.66

$$\frac{x\left(35a^3\left(4c+dx^3\right)+15a^2bx^3\left(21c+2dx^3\right)+9ab^2x^6\left(30c+dx^3\right)+81b^3cx^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*a
^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

fricas [A] time = 0.58, size = 121, normalized size = 1.00

$$\frac{\left(9\left(9\,b^3c+ab^2d\right)x^{10}+30\left(9\,ab^2c+a^2bd\right)x^7+140\,a^3cx+35\left(9\,a^2bc+a^3d\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3*
c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b
^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{13}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.04, size = 81, normalized size = 0.67

$$\frac{\left(9a\, b^2 d\, x^9 + 81 b^3 c\, x^9 + 30 a^2 b d\, x^6 + 270 a\, b^2 c\, x^6 + 35 a^3 d\, x^3 + 315 a^2 b c\, x^3 + 140 c\, a^3\right) x}{140 \left(b\, x^3 + a\right)^{\frac{10}{3}} a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(9*a*b^2*d*x^9+81*b^3*c*x^9+30*a^2*b*d*x^6+270*a*b^2*c*x^6+35*a^3*d
*x^3+315*a^2*b*c*x^3+140*a^3*c)/(b*x^3+a)^(10/3)/a^4
```

maxima [A] time = 0.50, size = 120, normalized size = 0.99

$$\frac{\left(14\, b^2 - \frac{40\left(bx^3+a\right)b}{x^3} + \frac{35\left(bx^3+a\right)^2}{x^6}\right) dx^{10}}{140\left(bx^3 + a\right)^{\frac{10}{3}} a^3} - \frac{\left(14\, b^3 - \frac{60\left(bx^3+a\right)b^2}{x^3} + \frac{105\left(bx^3+a\right)^2 b}{x^6} - \frac{140\left(bx^3+a\right)^3}{x^9}\right) cx^{10}}{140\left(bx^3 + a\right)^{\frac{10}{3}} a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^3
 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a
)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

mupad [B] time = 1.46, size = 105, normalized size = 0.87

$$\frac{x\left(\frac{c}{10\,a} - \frac{d}{10\,b}\right)}{\left(b\,x^3 + a\right)^{10/3}} + \frac{x\,\left(a\,d + 9\,b\,c\right)}{70\,a^2\,b\left(b\,x^3 + a\right)^{7/3}} + \frac{x\,\left(3\,a\,d + 27\,b\,c\right)}{140\,a^3\,b\left(b\,x^3 + a\right)^{4/3}} + \frac{x\,\left(9\,a\,d + 81\,b\,c\right)}{140\,a^4\,b\left(b\,x^3 + a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c/(10*a) - d/(10*b)))/(a + b*x^3)^(10/3) + (x*(a*d + 9*b*c))/(70*a^2*b*
(a + b*x^3)^(7/3)) + (x*(3*a*d + 27*b*c))/(140*a^3*b*(a + b*x^3)^(4/3)) + (
x*(9*a*d + 81*b*c))/(140*a^4*b*(a + b*x^3)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.63 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=151

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

```
[Out] 1/13*(-a*d+b*c)*x/a/b/(b*x^3+a)^(13/3)+1/130*(a*d+12*b*c)*x/a^2/b/(b*x^3+a)
^(10/3)+9/910*(a*d+12*b*c)*x/a^3/b/(b*x^3+a)^(7/3)+27/1820*(a*d+12*b*c)*x/a
^4/b/(b*x^3+a)^(4/3)+81/1820*(a*d+12*b*c)*x/a^5/b/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] ((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*x)/(130*a^2*b
*(a + b*x^3)^(10/3)) + (9*(12*b*c + a*d)*x)/(910*a^3*b*(a + b*x^3)^(7/3)) +
 (27*(12*b*c + a*d)*x)/(1820*a^4*b*(a + b*x^3)^(4/3)) + (81*(12*b*c + a*d)*
x)/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
```

```
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{(9(12bc+ad))\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a^2b}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{(27(12bc+ad))\int \frac{}{\left(a+b\right.}}{455a^3b}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 0.66

$$\frac{x\left(455a^4\left(4c+dx^3\right)+195a^3bx^3\left(28c+3dx^3\right)+351a^2b^2x^6\left(20c+dx^3\right)+81ab^3x^9\left(52c+dx^3\right)+972b^4cx^{12}\right)}{1820a^5\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3)
 + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a^5
*(a + b*x^3)^(13/3))
```

fricas [A] time = 0.67, size = 155, normalized size = 1.03

$$\frac{\left(81\left(12\,b^4c+ab^3d\right)x^{13}+351\left(12\,ab^3c+a^2b^2d\right)x^{10}+585\left(12\,a^2b^2c+a^3bd\right)x^7+1820\,a^4cx+455\left(12\,a^3bc+a^4d\right)\right.}{1820\left(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+10\,a^8b^2x^6+5\,a^9bx^3+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d)*
x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10
*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 105, normalized size = 0.70

$$\frac{\left(81a\,b^3d\,x^{12} + 972b^4c\,x^{12} + 351a^2b^2d\,x^9 + 4212a\,b^3c\,x^9 + 585a^3bd\,x^6 + 7020a^2b^2c\,x^6 + 455a^4d\,x^3 + 5460a^3bc\,x^3\right.}{1820\left(b\,x^3 + a\right)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/1820*x*(81*a*b^3*d*x^12+972*b^4*c*x^12+351*a^2*b^2*d*x^9+4212*a*b^3*c*x^9
+585*a^3*b*d*x^6+7020*a^2*b^2*c*x^6+455*a^4*d*x^3+5460*a^3*b*c*x^3+1820*a^4
*c)/(b*x^3+a)^(13/3)/a^5
```

maxima [A] time = 0.66, size = 154, normalized size = 1.02

$$-\frac{\left(140\,b^3 - \frac{546\left(bx^3+a\right)b^2}{x^3} + \frac{780\left(bx^3+a\right)^2b}{x^6} - \frac{455\left(bx^3+a\right)^3}{x^9}\right)dx^{13}}{1820\left(bx^3 + a\right)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4 - \frac{182\left(bx^3+a\right)b^3}{x^3} + \frac{390\left(bx^3+a\right)^2b^2}{x^6} - \frac{455\left(bx^3+a\right)^3b}{x^9} + \frac{455}{\quad}\right.}{455\left(bx^3 + a\right)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] -1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b
```

```
*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 4
55*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

mupad [B] time = 1.45, size = 132, normalized size = 0.87

$$\frac{x\left(\frac{c}{13\,a}-\frac{d}{13\,b}\right)}{\left(b\,x^3+a\right)^{13/3}}+\frac{x\,\left(a\,d+12\,b\,c\right)}{130\,a^2\,b\,\left(b\,x^3+a\right)^{10/3}}+\frac{x\,\left(9\,a\,d+108\,b\,c\right)}{910\,a^3\,b\,\left(b\,x^3+a\right)^{7/3}}+\frac{x\,\left(27\,a\,d+324\,b\,c\right)}{1820\,a^4\,b\,\left(b\,x^3+a\right)^{4/3}}+\frac{x\,\left(81\,a\,d+972\,b\,c\right)}{1820\,a^5\,b\,\left(b\,x^3+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c/(13*a) - d/(13*b)))/(a + b*x^3)^(13/3) + (x*(a*d + 12*b*c))/(130*a^2*
b*(a + b*x^3)^(10/3)) + (x*(9*a*d + 108*b*c))/(910*a^3*b*(a + b*x^3)^(7/3))
 + (x*(27*a*d + 324*b*c))/(1820*a^4*b*(a + b*x^3)^(4/3)) + (x*(81*a*d + 972
*b*c))/(1820*a^5*b*(a + b*x^3)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.64 $\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=85

$$\frac{a^2x\sqrt[3]{a+bx^3}\,(11bc-ad)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

```
[Out] 1/11*d*x*(b*x^3+a)^(10/3)/b+1/11*a^2*(-a*d+11*b*c)*x*(b*x^3+a)^(1/3)*hyperg
eom([-7/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{a^2x\sqrt[3]{a+bx^3}\,(11bc-ad)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*H
ypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/3)
)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\frac{dx\left(a+bx^3\right)^{10/3}}{11b}-\frac{(-11bc+ad)\int \left(a+bx^3\right)^{7/3}\,dx}{11b}$$
$$=\frac{dx\left(a+bx^3\right)^{10/3}}{11b}-\frac{\left(a^2(-11bc+ad)\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{7/3}\,dx}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$=\frac{dx\left(a+bx^3\right)^{10/3}}{11b}+\frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\;{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.91

$$\frac{x\sqrt[3]{a+bx^3}\left(d\left(a+bx^3\right)^3-\frac{a^2(ad-11bc)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^3 - (a^2*(-11*b*c + a*d)*Hypergeometric
2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(11*b)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2dx^9+\left(b^2c+2\,abd\right)x^6+\left(2\,abc+a^2d\right)x^3+a^2c\right)\left(bx^3+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c
)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{7/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(7/3)*(c + d*x^3), x)
```

sympy [C] time = 10.99, size = 265, normalized size = 3.12

$$\frac{a^{\frac{7}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{7}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c),x)
```

```
[Out] a**(7/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(7/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/3)*b*c*x**4*gamma(4/3)*hyp
er((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/
3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(10/3)) + a**(1/3)*b**2*c*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*d*x**10*gamm
a(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/
3))
```

3.65 $\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx$

Optimal. Leaf size=83

$$\frac{ax\sqrt[3]{a+bx^3}\,(8bc-ad)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

```
[Out] 1/8*d*x*(b*x^3+a)^(7/3)/b+1/8*a*(-a*d+8*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-
4/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{ax\sqrt[3]{a+bx^3}\,(8bc-ad)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperg
eometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)dx &= \frac{dx\left(a+bx^3\right)^{7/3}}{8b}-\frac{(-8bc+ad)\int \left(a+bx^3\right)^{4/3}dx}{8b}\\
&= \frac{dx\left(a+bx^3\right)^{7/3}}{8b}-\frac{\left(a(-8bc+ad)\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{4/3}dx}{8b\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{dx\left(a+bx^3\right)^{7/3}}{8b}+\frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 75, normalized size = 0.90

$$\frac{x\sqrt[3]{a+bx^3}\left(d\left(a+bx^3\right)^2-\frac{a(ad-8bc)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^2 - (a*(-8*b*c + a*d)*Hypergeometric2F1
[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(8*b)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bdx^6+(bc+ad)x^3+ac\right)\left(bx^3+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^6 + (b*c + a*d)*x^3 + a*c)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(bx^{3}+a\right)}^{\frac{4}{3}}{\left(dx^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3),x)
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3), x)
```

sympy [C] time = 7.40, size = 170, normalized size = 2.05

$$\frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{4}{3}}dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,bcx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,bdx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\right.}{3\Gamma\left(\frac{10}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c),x)
```

```
[Out] a**(4/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(4/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b*c*x**4*gamma(4/3)*hyper
((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b
*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

3.66 $\int \sqrt[3]{a+bx^3}\,\left(c+dx^3\right)\,dx$

Optimal. Leaf size=82

$$\frac{x\sqrt[3]{a+bx^3}\,(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

```
[Out] 1/5*d*x*(b*x^3+a)^(4/3)/b+1/5*(-a*d+5*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-1/
3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{x\sqrt[3]{a+bx^3}\,(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hypergeo
metric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)dx &= \frac{dx\left(a+bx^3\right)^{4/3}}{5b}-\frac{(-5bc+ad)\int \sqrt[3]{a+bx^3}\,dx}{5b}\\ &= \frac{dx\left(a+bx^3\right)^{4/3}}{5b}-\frac{\left((-5bc+ad)\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{5b\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{dx\left(a+bx^3\right)^{4/3}}{5b}+\frac{(5bc-ad)x\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 72, normalized size = 0.88

$$\frac{x\sqrt[3]{a+bx^3}\left(\frac{(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}+d\left(a+bx^3\right)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{3}+a\right)^{\frac{1}{3}} \left(dx^{3}+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x^3), x)
```

sympy [C] time = 4.97, size = 82, normalized size = 1.00

$$\frac{\sqrt[3]{a} c x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a} d x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.67 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=82

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

```
[Out] 1/2*d*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d+2*b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([1
/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hype
rgeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^3}{\left(a+bx^3\right)^{2/3}}\,dx &= \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{(-2bc+ad)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2b} \\
&= \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{\left((-2bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2b\left(a+bx^3\right)^{2/3}} \\
&= \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.89

$$
\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\left(a+bx^3\right)}{2b\left(a+bx^3\right)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3) + (2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\frac{dx^3+c}{\left(bx^3+a\right)^{\frac{2}{3}}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3 + c}{(b\,x^3 + a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3 + c}{(b\,x^3 + a)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x^3)/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 5.07, size = 78, normalized size = 0.95

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

3.68 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx$

Optimal. Leaf size=93

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/2*(-a*d+b*c)*x/a/b/(b*x^3+a)^(2/3)+1/2*(a*d+b*c)*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^(2
/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx = \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{(bc+ad)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}$$

$$= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{\left((bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2ab\left(a+bx^3\right)^{2/3}}$$

$$= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}}+\frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.71

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-adx}{ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

```
[Out] (-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3,
 4/3, -((b*x^3)/a)])/(a*b*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3 + c}{(b\,x^3 + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
[Out] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3 + c}{(b\,x^3 + a)^{5/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(5/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(5/3), x)
```

sympy [C] time = 16.36, size = 78, normalized size = 0.84

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

3.69 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx$

Optimal. Leaf size=94

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

```
[Out] 1/5*(-a*d+b*c)*x/a/b/(b*x^3+a)^(5/3)+1/5*(a*d+4*b*c)*x*(1+b*x^3/a)^(2/3)*hy
pergeom([1/3, 5/3],[4/3],-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

```
[Out] ((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x^3
)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx &= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{(4bc+ad)\int \frac{1}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}\\ &= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left((4bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{5a^2b\left(a+bx^3\right)^{2/3}}\\ &= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}}+\frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 0.80

$$\frac{x\left(\frac{\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a^2}-d\right)}{4b\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

```
[Out] (x*(-d + ((4*b*c + a*d)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 8/3, 4/3, -((b*x^3)/a)])/a^2))/(4*b*(a + b*x^3)^(5/3))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3
 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3 + c}{\left(b\,x^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3 + c}{\left(b\,x^3 + a\right)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(8/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(8/3), x)
```

sympy [C] time = 117.70, size = 78, normalized size = 0.83

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{8}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{8}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(8
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

3.70 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=262

$$\frac{x\left(a+bx^3\right)^{5/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{162b^2}+\frac{5ax\left(a+bx^3\right)^{2/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{486b^2}-\frac{5a^2\left(a^2d^2-6abcd+27b^2c^2\right)1}{486b^{7/3}}$$

```
[Out] 5/486*a*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+1/162*(a^2*d^2
-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(5/3)/b^2+1/108*d*(-4*a*d+15*b*c)*x*(b*x
^3+a)^(8/3)/b^2+1/12*d*x*(b*x^3+a)^(8/3)*(d*x^3+c)/b-5/486*a^2*(a^2*d^2-6*a
*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+5/729*a^2*(a^2*d^
2-6*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 195, 239}

$$\frac{x\left(a+bx^3\right)^{5/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{162b^2}+\frac{5ax\left(a+bx^3\right)^{2/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{486b^2}-\frac{5a^2\left(a^2d^2-6abcd+27b^2c^2\right)1}{486b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((
27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b
*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*
x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]*b^(7/3)) - (5*a^2*(27*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(
7/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}+\frac{\int \left(a+bx^3\right)^{5/3}\left(c(12bc-ad)+d(15bc-4ad)x^3\right)\,dx}{12b}$$

$$= \frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2}+\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}+\frac{\left(27b^2c^2-6abcd+a^2\right.}{27}$$

$$= \frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{162b^2}+\frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2}+\frac{dx\left(a+\right.}{}$$

$$= \frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2}+\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right.}{162b^2}$$

$$= \frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2}+\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right.}{162b^2}$$

Mathematica [A] time = 5.19, size = 238, normalized size = 0.91

$$\frac{10a^2\left(a^2d^2-6abcd+27b^2c^2\right)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)+3\sqrt[3]{}}{2916}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-20*a^3*d^2 + 15*a^2*b*d*(8*c + d*x^3) + 27
*b^3*x^3*(6*c^2 + 8*c*d*x^3 + 3*d^2*x^6) + 18*a*b^2*(24*c^2 + 22*c*d*x^3 +
7*d^2*x^6)) + 10*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(2*Sqrt[3]*ArcTan[(
1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a +
b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a +
b*x^3)^(1/3)]))/(2916*b^(7/3))
```

fricas [A] time = 0.85, size = 717, normalized size = 2.74

$$\left[\frac{30\sqrt{\frac{1}{3}}\left(27\,a^2b^3c^2-6\,a^3b^2cd+a^4bd^2\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2*c^2
- 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c
*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4
+ 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^
3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt(
-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(
```

```
-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2 - 6*a^3*b*c*
d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c*d + 7*a*b^3
*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4 + 4*(108*a*b
^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{5}{3}}\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

maxima [B] time = 1.29, size = 672, normalized size = 2.56

$$-\frac{1}{54}\left(\frac{10\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c^2 + 1/243*(10*sqrt(3)*a^3*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) +
 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3
)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x
^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*
b/x^9))*c*d - 1/2916*(20*sqrt(3)*a^4*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 10*a^4*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^
(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 20*a^4*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(7/3) + 3*(10*(b*x^3 + a)^(2/3)*a^4*b^3/x^2 - 36*(b*x^3 + a)^(
5/3)*a^4*b^2/x^5 - 75*(b*x^3 + a)^(8/3)*a^4*b/x^8 + 20*(b*x^3 + a)^(11/3)*a
^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4/x^6 - 4*(b*x^3
+ a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 13.13, size = 270, normalized size = 1.03

$$\frac{a^{\frac{5}{3}}c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{5}{3}}cdx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{5}{3}}d^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{2}{3}}bc^{2}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F}{3\Gamma\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar
```

```
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

3.71 $\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=219

$$\frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2}-\frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81b^{7/3}}+\frac{2a\left(2a^2d^2-9abcd\right.}{}$$

```
[Out] 1/81*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+2/27*d*(-a*d+3*
b*c)*x*(b*x^3+a)^(5/3)/b^2+1/9*d*x*(b*x^3+a)^(5/3)*(d*x^3+c)/b-1/81*a*(2*a^
2*d^2-9*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2/243*a*
(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 195, 239}

$$\frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2}-\frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81b^{7/3}}+\frac{2a\left(2a^2d^2-9abcd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*
(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d
*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2
- 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3)
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b} + \frac{\int \left(a+bx^3\right)^{2/3}\left(c(9bc-ad)+4d(3bc-ad)x^3\right)dx}{9b}$$

$$= \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2} + \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b} - \frac{(4ad(3bc-ad)-6bc(9$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2} + \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2} + \frac{dx\left(a+\right.}{}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2} + \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2} + \frac{dx\left(a+\right.}{}$$

Mathematica [A] time = 5.17, size = 203, normalized size = 0.93

$$\frac{3\sqrt[3]{b}\,x\left(a+bx^3\right)^{2/3}\left(-4a^2d^2+3abd\left(6c+dx^3\right)+9b^2\left(3c^2+3cdx^3+d^2x^6\right)\right)+a\left(2a^2d^2-9abcd+27b^2c^2\right)\left(\log\left(\right.\right.}{243b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-4*a^2*d^2 + 3*a*b*d*(6*c + d*x^3) + 9*b^2*
(3*c^2 + 3*c*d*x^3 + d^2*x^6)) + a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*(2*
Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (
b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(243*b^(7/3))
```

fricas [A] time = 0.63, size = 634, normalized size = 2.89

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(27\,ab^3c^2 - 9\,a^2b^2cd + 2\,a^3bd^2\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^3 - 3\left(bx^3 + a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - \left(bx^3 + a\right)^{\frac{1}{3}}b\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)^
(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-
b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x
)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9*a^2*
b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2
*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))
/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt
(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a^2*b*c*d + 2
*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2*d^2)*x^4
 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{2}{3}}\left(dx^3 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

maxima [B] time = 1.48, size = 552, normalized size = 2.52

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(bx^{3}\right.}{\left(b-\frac{bx}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c^2 + 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*c*d - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
```

```
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 7.27, size = 131, normalized size = 0.60

$$\frac{a^{\frac{2}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{2}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.72 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{7/3}}+\frac{(2a^2d^2-6abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{dx\left(a+bx^3\right)^{2/3}(}{18b^2}$$

```
[Out] 1/18*d*(-4*a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b^2+1/6*d*x*(b*x^3+a)^(2/3)*(d*x^3+
c)/b-1/18*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^
(7/3)+1/27*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {416, 388, 239}

$$-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{7/3}}+\frac{(2a^2d^2-6abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{dx\left(a+bx^3\right)^{2/3}(}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*(9*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(18*b^2) + (d*x*(a + b*x^3)^(2/3)*(
c + d*x^3))/(6*b) + ((9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(7/3)) - ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(7/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\int \frac{c(6bc-ad)+d(9bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$

$$= \frac{d(9bc-4ad)x\left(a+bx^3\right)^{2/3}}{18b^2} + \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\left(9b^2c^2-6abcd+2a^2d^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,d}{9b^2}$$

$$= \frac{d(9bc-4ad)x\left(a+bx^3\right)^{2/3}}{18b^2} + \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\left(9b^2c^2-6abcd+2a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2}{\sqrt[3]{}}}{\sqrt{}}\right)}{9\sqrt{3}\,b^{7/3}}$$

Mathematica [A] time = 5.15, size = 172, normalized size = 0.98

$$\frac{\left(2a^2d^2-6abcd+9b^2c^2\right)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)+3\sqrt[3]{b}\,dx\,(}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + (9*b^2*c^2
- 6*a*b*c*d + 2*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3
)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(54*b^(7/3))
```

fricas [A] time = 0.73, size = 554, normalized size = 3.17

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(9\,b^{3}c^{2}-6\,ab^{2}cd+2\,a^{2}bd^{2}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^{3}-3\left(bx^{3}+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^{2}-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^{3}-\left(bx^{3}+a\right)^{\frac{1}{3}}bx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)/
b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/
3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt
((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*s
qrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*s
qrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{2}}{\left(bx^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.28, size = 436, normalized size = 2.49

$$-\frac{1}{6}\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c^{2}+\frac{1}{9}\left(2\,\sqrt{3}\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 6.44, size = 126, normalized size = 0.72

$$\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.73 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=159

$$-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}}-\frac{dx\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3ab^2}+\frac{x\left(c+dx^3\right)}{ab\sqrt[3]{a-}}$$

```
[Out] -1/3*d*(-4*a*d+3*b*c)*x*(b*x^3+a)^(2/3)/a/b^2+(-a*d+b*c)*x*(d*x^3+c)/a/b/(b
*x^3+a)^(1/3)-1/3*d*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2
/9*d*(-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
7/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {413, 388, 239}

$$-\frac{dx\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3ab^2}-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}}+\frac{x\left(c+dx^3\right)}{ab\sqrt[3]{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] -(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(3*a*b^2) + ((b*c - a*d)*x*(c + d*
x^3))/(a*b*(a + b*x^3)^(1/3)) + (2*d*(3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)) - (d*(3*b*c - 2*a*d)*L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(7/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{\int \frac{acd-d(3bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= -\frac{d(3bc-4ad)x\left(a+bx^3\right)^{2/3}}{3ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{(2d(3bc-2ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b^2}$$

$$= -\frac{d(3bc-4ad)x\left(a+bx^3\right)^{2/3}}{3ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}} - \frac{d(\ldots}{\ldots}$$

Mathematica [A] time = 5.16, size = 168, normalized size = 1.06

$$\frac{d(3bc-2ad)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{9b^{7/3}} + \frac{x\left(a+bx^3\right)^{2/3}\left(\frac{3(bc-\ldots}{a(a+\ldots}\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d^2 + (3*(b*c - a*d)^2)/(a*(a + b*x^3))))/(3*b^2) + (
d*(3*b*c - 2*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/S
qrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/
(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(9*b^(7/3))
```

fricas [B] time = 0.58, size = 652, normalized size = 4.10

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(3a^2b^2cd-2a^3bd^2+\left(3ab^3cd-2a^2b^2d^2\right)x^3\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 -
3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*
d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/
3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(a*b^2*d
^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b
^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*
b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a^2*b*c*d
- 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (
b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(3*a^2*b
^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a*b^2*d^2*x^4 +
 (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*x^3 +
 a^2*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

maxima [B] time = 1.14, size = 301, normalized size = 1.89

$$\frac{1}{9}\,d^{2}\left(\frac{4\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\left(bx^{3}+a\right)a}{x^{3}}\right)}{\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}b^{3}}{x}-\frac{\left(bx^{3}+a\right)^{\frac{4}{3}}b^{2}}{x^{4}}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{b^{\frac{7}{3}}}+\frac{4\,a\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^2}{{\left(b\,x^3+a\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^3\right)^2}{\left(a + bx^3\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

3.74 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{7/3}} \, dx$

Optimal. Leaf size=152

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

```
[Out] 1/4*(-a*d+b*c)*(4*a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(1/3)+1/4*(-a*d+b*c)*x*(d*
x^3+c)/a/b/(b*x^3+a)^(4/3)-1/2*d^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+1
/3*d^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {413, 385, 239}

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((b*c - a*d)*(3*b*c + 4*a*d)*x)/(4*a^2*b^2*(a + b*x^3)^(1/3)) + ((b*c - a*d
)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) + (
a + b*x^3)^(1/3)])/(2*b^(7/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{\int \frac{c(3bc+ad)+4ad^2x^3}{\left(a+bx^3\right)^{4/3}}\,dx}{4ab}$$

$$= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b^2}$$

$$= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{d^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a}\right.}{2b^{7/3}}$$

Mathematica [A] time = 5.25, size = 180, normalized size = 1.18

$$\frac{x\left(\left(a+bx^3\right)\left(-5a^2d^2+2abcd+3b^2c^2\right)+a(bc-ad)^2\right)}{4a^2b^2\left(a+bx^3\right)^{4/3}} + \frac{d^2\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\right.}{6b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a*(b*c - a*d)^2 + (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*(a + b*x^3)))/(4*
a^2*b^2*(a + b*x^3)^(4/3)) + (d^2*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a +
```

```
b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1
+ (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(6*b^(
7/3))
```

fricas [B] time = 0.74, size = 719, normalized size = 4.73

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(a^2b^3d^2x^6+2\,a^3b^2d^2x^3+a^4bd^2\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt((
-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/
3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^
4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 + 2*a*b^3
*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2*b^3*d^2*
x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*
((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 +
a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*
b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

maxima [A] time = 1.19, size = 190, normalized size = 1.25

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)c^2x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a^2}+\frac{cdx^4}{2\left(bx^3+a\right)^{\frac{4}{3}}a}-\frac{1}{12}\left(\frac{3\left(b+\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{\left(bx^3+a\right)^{\frac{4}{3}}b^2}+\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c^2*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/2*c*d*x^4/
((b*x^3 + a)^(4/3)*a) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4
/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(7/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{2}}{\left(a + bx^{3}\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

3.75 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=78

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

```
[Out] 9/14*c^2*x/a^3/(b*x^3+a)^(1/3)+3/14*c*x*(d*x^3+c)/a^2/(b*x^3+a)^(4/3)+1/7*x
*(d*x^3+c)^2/a/(b*x^3+a)^(7/3)
```

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {378, 191}

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (9*c^2*x)/(14*a^3*(a + b*x^3)^(1/3)) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x
^3)^(4/3)) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx = \frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}} + \frac{(6c)\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{7a}$$
$$= \frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}} + \frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}} + \frac{\left(9c^2\right)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{14a^2}$$
$$= \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}} + \frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}} + \frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

Mathematica [A] time = 0.10, size = 126, normalized size = 1.62

$$\frac{x\sqrt[3]{\frac{bx^3}{a}+1}\left(a^2\left(14c^2+7cdx^3+2d^2x^6\right)+3abcx^3\left(7c+dx^3\right)+9b^2c^2x^6\right)}{14a^3\left(a+bx^3\right)^{7/3}\sqrt[3]{\frac{dx^3}{c}+1}\sqrt[3]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/3)*(9*b^2*c^2*x^6 + 3*a*b*c*x^3*(7*c + d*x^3) + a^2*(
14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)))/(14*a^3*(a + b*x^3)^(7/3)*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/3)*(1 + (d*x^3)/c)^(1/3))
```

fricas [A] time = 0.62, size = 103, normalized size = 1.32

$$\frac{\left(\left(9\,b^2c^2+3\,abcd+2\,a^2d^2\right)x^7+14\,a^2c^2x+7\left(3\,abc^2+a^2cd\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{14\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^2
 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x
^3 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{10}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.05, size = 76, normalized size = 0.97

$$\frac{\left(2a^2d^2x^6 + 3abcd\, x^6 + 9b^2c^2x^6 + 7a^2cd\, x^3 + 21ab\, c^2x^3 + 14a^2c^2\right) x}{14\left(b\, x^3 + a\right)^{\frac{7}{3}} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(10/3),x)
[Out] 1/14*x*(2*a^2*d^2*x^6+3*a*b*c*d*x^6+9*b^2*c^2*x^6+7*a^2*c*d*x^3+21*a*b*c^2*
x^3+14*a^2*c^2)/(b*x^3+a)^(7/3)/a^3
```

maxima [A] time = 0.51, size = 109, normalized size = 1.40

$$-\frac{\left(4\, b - \frac{7\left(bx^3+a\right)}{x^3}\right)cdx^7}{14\left(bx^3 + a\right)^{\frac{7}{3}} a^2} + \frac{d^2x^7}{7\left(bx^3 + a\right)^{\frac{7}{3}} a} + \frac{\left(2\, b^2 - \frac{7\left(bx^3+a\right)b}{x^3} + \frac{14\left(bx^3+a\right)^2}{x^6}\right)c^2x^7}{14\left(bx^3 + a\right)^{\frac{7}{3}} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
[Out] -1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*x
^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 +
a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

mupad [B] time = 1.43, size = 148, normalized size = 1.90

$$\frac{2\, a^4\, d^2\, x + 2\, a^2\, d^2\, x\left(b\, x^3 + a\right)^2 + 9\, b^2\, c^2\, x\left(b\, x^3 + a\right)^2 + 2\, a^2\, b^2\, c^2\, x - 4\, a^3\, d^2\, x\, \left(b\, x^3 + a\right) + 3\, a\, b^2\, c^2\, x\, \left(b\, x^3 + a\right) -}{14\, a^3\, b^2\left(b\, x^3 + a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(10/3),x)
```

```
[Out] (2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a^
2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3*b
*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^2*(
a + b*x^3)^(7/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

3.76 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{13/3}} \, dx$

Optimal. Leaf size=174

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{3cx\left(c+dx^3\right)(9bc-10ad)}{140a^3\left(a+bx^3\right)^{4/3}(bc-ad)}+\frac{x\left(c+dx^3\right)^2(9bc-10ad)}{70a^2\left(a+bx^3\right)^{7/3}(bc-ad)}+\frac{bx\left(c+dx^3\right)^3}{10a\left(a+bx^3\right)^{10/3}(bc-ad)}$$

```
[Out] 9/140*c^2*(-10*a*d+9*b*c)*x/a^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+3/140*c*(-10*a*d
+9*b*c)*x*(d*x^3+c)/a^3/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/70*(-10*a*d+9*b*c)*x*(
d*x^3+c)^2/a^2/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/10*b*x*(d*x^3+c)^3/a/(-a*d+b*c)
/(b*x^3+a)^(10/3)
```

Rubi [A] time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{x\left(c+dx^3\right)^2(9bc-10ad)}{70a^2\left(a+bx^3\right)^{7/3}(bc-ad)}+\frac{3cx\left(c+dx^3\right)(9bc-10ad)}{140a^3\left(a+bx^3\right)^{4/3}(bc-ad)}+\frac{bx\left(c+dx^3\right)^3}{10a\left(a+bx^3\right)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*c*(
9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((
9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^(7/3)) + (
b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{bx\left(c+dx^3\right)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}} + \frac{(9bc-10ad)\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx}{10a(bc-ad)}$$

$$= \frac{(9bc-10ad)x\left(c+dx^3\right)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{bx\left(c+dx^3\right)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}} + \frac{(3c(9bc-10ad))\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{35a^2(bc-ad)}$$

$$= \frac{3c(9bc-10ad)x\left(c+dx^3\right)}{140a^3(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{(9bc-10ad)x\left(c+dx^3\right)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{bx\left(c+dx^3\right)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}} + \cdots$$

$$= \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{3c(9bc-10ad)x\left(c+dx^3\right)}{140a^3(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{(9bc-10ad)x\left(c+dx^3\right)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}} + \cdots$$

Mathematica [A] time = 5.11, size = 106, normalized size = 0.61

$$\frac{x\left(10a^3\left(14c^2+7cdx^3+2d^2x^6\right)+3a^2bx^3\left(105c^2+20cdx^3+2d^2x^6\right)+18ab^2cx^6\left(15c+dx^3\right)+81b^3c^2x^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*d
*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140*a
^4*(a + b*x^3)^(10/3))
```

fricas [A] time = 0.73, size = 152, normalized size = 0.87

$$\frac{\left(3\left(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2\right)x^{10}+10\left(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2\right)x^7+140\,a^3c^2x+35\left(9\,a^2bc^2+2\,a^3cd\right)x^4\right)(}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2 +
 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c*d
)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*
a^7*b*x^3 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.05, size = 115, normalized size = 0.66

$$\frac{\left(6a^2b\,d^2x^9+18a\,b^2cd\,x^9+81b^3c^2x^9+20a^3d^2x^6+60a^2bcd\,x^6+270a\,b^2c^2x^6+70a^3cd\,x^3+315a^2b\,c^2x^3+140c^2a}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(6*a^2*b*d^2*x^9+18*a*b^2*c*d*x^9+81*b^3*c^2*x^9+20*a^3*d^2*x^6+60*
a^2*b*c*d*x^6+270*a*b^2*c^2*x^6+70*a^3*c*d*x^3+315*a^2*b*c^2*x^3+140*a^3*c^
2)/(b*x^3+a)^(10/3)/a^4
```

maxima [A] time = 0.71, size = 159, normalized size = 0.91

$$-\frac{\left(7\,b-\frac{10\left(bx^3+a\right)}{x^3}\right)d^2x^{10}}{70\left(bx^3+a\right)^{\frac{10}{3}}a^2}+\frac{\left(14\,b^2-\frac{40\left(bx^3+a\right)b}{x^3}+\frac{35\left(bx^3+a\right)^2}{x^6}\right)cdx^{10}}{70\left(bx^3+a\right)^{\frac{10}{3}}a^3}-\frac{\left(14\,b^3-\frac{60\left(bx^3+a\right)b^2}{x^3}+\frac{105\left(bx^3+a\right)^2b}{x^6}-\frac{140\left(bx^3+a\right)^3}{x^9}\right)c}{140\left(bx^3+a\right)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 + a)
^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b
/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

mupad [B] time = 1.45, size = 176, normalized size = 1.01

$$\frac{x\left(\frac{c^2}{10\,a}+\frac{a\left(\frac{d^2}{10\,b}-\frac{c\,d}{5\,a}\right)}{b}\right)}{\left(b\,x^3+a\right)^{10/3}}-\frac{x\left(\frac{d^2}{7\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+9\,b^2\,c^2}{70\,a^2\,b^2}\right)}{\left(b\,x^3+a\right)^{7/3}}+\frac{x\left(2\,a^2\,d^2+6\,a\,b\,c\,d+27\,b^2\,c^2\right)}{140\,a^3\,b^2\left(b\,x^3+a\right)^{4/3}}+\frac{x\left(6\,a^2\,d^2+18\,a\,b\,c\,d+8\right.}{140\,a^4\,b^2\left(b\,x^3+a\right)^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c^2/(10*a) + (a*(d^2/(10*b) - (c*d)/(5*a)))/b))/(a + b*x^3)^(10/3) - (x
*(d^2/(7*b^2) - (9*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(70*a^2*b^2)))/(a + b*x^3
)^(7/3) + (x*(2*a^2*d^2 + 27*b^2*c^2 + 6*a*b*c*d))/(140*a^3*b^2*(a + b*x^3)
^(4/3)) + (x*(6*a^2*d^2 + 81*b^2*c^2 + 18*a*b*c*d))/(140*a^4*b^2*(a + b*x^3
)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.77 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$

Optimal. Leaf size=211

$$\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2\left(a+bx^3\right)^{10/3}}+\frac{9x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^4b^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(2a^2d^2+9abcd+54b^2c^2\right)}{455a^3b^2\left(a+bx^3\right)^{7/3}}$$

```
[Out] 2/65*(-a*d+b*c)*(a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^(10/3)+1/455*(2*a^2*d^2+9*a
*b*c*d+54*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(7/3)+3/910*(2*a^2*d^2+9*a*b*c*d+54*
b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(4/3)+9/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x
/a^5/b^2/(b*x^3+a)^(1/3)+1/13*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(13/3)
```

Rubi [A] time = 0.13, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 192, 191}

$$\frac{9x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^4b^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(2a^2d^2+9abcd+54b^2c^2\right)}{455a^3b^2\left(a+bx^3\right)^{7/3}}+\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^(10/3)) + ((54*b^2*
c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^(7/3)) + (3*(54*b^
2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^(4/3)) + (9*(54*
b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^(1/3)) + ((b*c
 - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\int \frac{c(12bc+ad)+d(9bc+4ad)x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{65a^2b^2}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(6\left(54b^2c^2\right.\right.}{}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{(}{}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{9}{}$$

Mathematica [A] time = 5.18, size = 138, normalized size = 0.65

$$\frac{x\left(65a^4\left(14c^2+7cdx^3+2d^2x^6\right)+39a^3bx^3\left(70c^2+15cdx^3+2d^2x^6\right)+9a^2b^2x^6\left(390c^2+39cdx^3+2d^2x^6\right)+81ab\right.}{910a^5\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*c
*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*a^
2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/3))
```

fricas [A] time = 0.64, size = 200, normalized size = 0.95

$$\frac{\left(9\left(54\,b^{4}c^{2}+9\,ab^{3}cd+2\,a^{2}b^{2}d^{2}\right)x^{13}+39\left(54\,ab^{3}c^{2}+9\,a^{2}b^{2}cd+2\,a^{3}bd^{2}\right)x^{10}+65\left(54\,a^{2}b^{2}c^{2}+9\,a^{3}bcd+2\,a^{4}d^{2}\right)\cdots\right.}{910\left(a^{5}b^{5}x^{15}+5\,a^{6}b^{4}x^{12}+10\,a^{7}b^{3}x^{9}+10\,a^{8}b^{2}x^{6}+5\,a^{9}bx^{3}+a^{1}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^2
 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d + 2
*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3 + a
)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^6 +
5*a^9*b*x^3 + a^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{2}}{\left(bx^{3}+a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 156, normalized size = 0.74

$$\frac{\left(18a^{2}b^{2}d^{2}x^{12}+81a\,b^{3}cd\,x^{12}+486b^{4}c^{2}x^{12}+78a^{3}b\,d^{2}x^{9}+351a^{2}b^{2}cd\,x^{9}+2106a\,b^{3}c^{2}x^{9}+130a^{4}d^{2}x^{6}+585a^{3}bcd\cdots\right.}{910\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/910*x*(18*a^2*b^2*d^2*x^12+81*a*b^3*c*d*x^12+486*b^4*c^2*x^12+78*a^3*b*d^
2*x^9+351*a^2*b^2*c*d*x^9+2106*a*b^3*c^2*x^9+130*a^4*d^2*x^6+585*a^3*b*c*d*
```

```
x^6+3510*a^2*b^2*c^2*x^6+455*a^4*c*d*x^3+2730*a^3*b*c^2*x^3+910*a^4*c^2)/(b
*x^3+a)^(13/3)/a^5
```

maxima [A] time = 0.50, size = 210, normalized size = 1.00

$$\frac{\left(35\,b^2-\frac{91\,(bx^3+a)b}{x^3}+\frac{65\,(bx^3+a)^2}{x^6}\right)d^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}-\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)cdx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\,(bx^3}{x^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*x
^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4) +
1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(
b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13/3)*a
^5)
```

mupad [B] time = 1.43, size = 217, normalized size = 1.03

$$\frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{\left(b\,x^3+a\right)^{13/3}}-\frac{x\left(\frac{d^2}{10\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{130\,a^2\,b^2}\right)}{\left(b\,x^3+a\right)^{10/3}}+\frac{x\left(2\,a^2\,d^2+9\,a\,b\,c\,d+54\,b^2\,c^2\right)}{455\,a^3\,b^2\left(b\,x^3+a\right)^{7/3}}+\frac{x\left(6\,a^2\,d^2+27\,a\,b\,c\,d+\right.}{910\,a^4\,b^2\left(b\,x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^3)^(13/3) -
 (x*(d^2/(10*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(130*a^2*b^2)))/(a +
 b*x^3)^(10/3) + (x*(2*a^2*d^2 + 54*b^2*c^2 + 9*a*b*c*d))/(455*a^3*b^2*(a +
 b*x^3)^(7/3)) + (x*(6*a^2*d^2 + 162*b^2*c^2 + 27*a*b*c*d))/(910*a^4*b^2*(a
 + b*x^3)^(4/3)) + (x*(18*a^2*d^2 + 486*b^2*c^2 + 81*a*b*c*d))/(910*a^5*b^2
*(a + b*x^3)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.78 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} \, dx$

Optimal. Leaf size=253

$$\frac{x(bc-ad)(4ad+15bc)}{208a^2b^2(a+bx^3)^{13/3}}+\frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}}+\frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}}$$

```
[Out] 1/208*(-a*d+b*c)*(4*a*d+15*b*c)*x/a^2/b^2/(b*x^3+a)^(13/3)+1/520*(a^2*d^2+6
*a*b*c*d+45*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^(10/3)+9/3640*(a^2*d^2+6*a*b*c*d+4
5*b^2*c^2)*x/a^4/b^2/(b*x^3+a)^(7/3)+27/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)
*x/a^5/b^2/(b*x^3+a)^(4/3)+81/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^6/b^2
/(b*x^3+a)^(1/3)+1/16*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^(16/3)
```

Rubi [A] time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 192, 191}

$$\frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}}+\frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}}+\frac{x(a^2d^2+6abcd+45b^2c^2)}{520a^3b^2(a+bx^3)^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] ((b*c - a*d)*(15*b*c + 4*a*d)*x)/(208*a^2*b^2*(a + b*x^3)^(13/3)) + ((45*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(520*a^3*b^2*(a + b*x^3)^(10/3)) + (9*(45*b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(3640*a^4*b^2*(a + b*x^3)^(7/3)) + (27*(45
*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^5*b^2*(a + b*x^3)^(4/3)) + (81*(
45*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^6*b^2*(a + b*x^3)^(1/3)) + ((b
*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
```

```
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{19/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{16ab\left(a+bx^3\right)^{16/3}} + \frac{\int \frac{c(15bc+ad)+4d(3bc+ad)x^3}{\left(a+bx^3\right)^{16/3}}\,dx}{16ab}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{16ab\left(a+bx^3\right)^{16/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx}{52a^2b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{16ab\left(a+bx^3\right)^{16/3}} + \frac{\left(9\left(45b^2c^2+\right.\right.}{}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{9\left(45b^2c^2+6abcd+a^2d^2\right)x}{3640a^4b^2\left(a+bx^3\right)^{7/3}} + \frac{(bc}{16a}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{9\left(45b^2c^2+6abcd+a^2d^2\right)x}{3640a^4b^2\left(a+bx^3\right)^{7/3}} + \frac{27\left(\right.}{}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{9\left(45b^2c^2+6abcd+a^2d^2\right)x}{3640a^4b^2\left(a+bx^3\right)^{7/3}} + \frac{27\left(\right.}{}$$

Mathematica [A] time = 5.15, size = 169, normalized size = 0.67

$$\frac{x\left(520a^5\left(14c^2+7cdx^3+2d^2x^6\right)+156a^4bx^3\left(175c^2+40cdx^3+6d^2x^6\right)+144a^3b^2x^6\left(325c^2+39cdx^3+3d^2x^6\right)+\right.}{7280a^6\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(5
20*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6) +
 144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175*c^
2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

fricas [A] time = 0.83, size = 246, normalized size = 0.97

$$\frac{\left(81\left(45\,b^5c^2+6\,ab^4cd+a^2b^3d^2\right)x^{16}+432\left(45\,ab^4c^2+6\,a^2b^3cd+a^3b^2d^2\right)x^{13}+936\left(45\,a^2b^3c^2+6\,a^3b^2cd+a^4bd^2\right.\right.}{7280\left(a^6b^6x^{18}+6\,a^7b^5x^{15}+15\,a^8b^4x^{12}+20\,a^9b^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c^
2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c*d
 + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d +
a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^6*
b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10*b^2*
x^6 + 6*a^11*b*x^3 + a^12)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{19}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

maple [A] time = 0.05, size = 197, normalized size = 0.78

$$\frac{\left(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(19/3),x)
```

```
[Out] 1/7280*x*(81*a^2*b^3*d^2*x^15+486*a*b^4*c*d*x^15+3645*b^5*c^2*x^15+432*a^3*
b^2*d^2*x^12+2592*a^2*b^3*c*d*x^12+19440*a*b^4*c^2*x^12+936*a^4*b*d^2*x^9+5
616*a^3*b^2*c*d*x^9+42120*a^2*b^3*c^2*x^9+1040*a^5*d^2*x^6+6240*a^4*b*c*d*x
^6+46800*a^3*b^2*c^2*x^6+3640*a^5*c*d*x^3+27300*a^4*b*c^2*x^3+7280*a^5*c^2)
/(b*x^3+a)^(16/3)/a^6
```

maxima [A] time = 0.64, size = 261, normalized size = 1.03

$$-\frac{\left(455\,b^3-\frac{1680\left(bx^3+a\right)b^2}{x^3}+\frac{2184\left(bx^3+a\right)^2b}{x^6}-\frac{1040\left(bx^3+a\right)^3}{x^9}\right)d^2x^{16}}{7280\left(bx^3+a\right)^{\frac{16}{3}}a^4}+\frac{\left(455\,b^4-\frac{2240\left(bx^3+a\right)b^3}{x^3}+\frac{4368\left(bx^3+a\right)^2b^2}{x^6}-\frac{4160\left(bx^3+a\right.}{x^9}\right.}{3640\left(bx^3+a\right)^{\frac{16}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1/145
6*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*
x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*c
^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

mupad [B] time = 1.48, size = 257, normalized size = 1.02

$$\frac{x\left(\frac{c^2}{16\,a}+\frac{a\left(\frac{d^2}{16\,b}-\frac{c\,d}{8\,a}\right)}{b}\right)}{\left(b\,x^3+a\right)^{16/3}}-\frac{x\left(\frac{d^2}{13\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+15\,b^2\,c^2}{208\,a^2\,b^2}\right)}{\left(b\,x^3+a\right)^{13/3}}+\frac{x\left(a^2\,d^2+6\,a\,b\,c\,d+45\,b^2\,c^2\right)}{520\,a^3\,b^2\left(b\,x^3+a\right)^{10/3}}+\frac{x\left(9\,a^2\,d^2+54\,a\,b\,c\,d+405\cdots\right.}{3640\,a^4\,b^2\left(b\,x^3+a\right)^{7\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (x
*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a + b*
x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b*x^
3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(a +
b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^5*b^2
*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(7280*a
^6*b^2*(a + b*x^3)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

3.79 $\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=135

$$\frac{a^2x\sqrt[3]{a+bx^3}\left(2a^2d^2-14abcd+77b^2c^2\right){}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c\right.}{14b}$$

```
[Out] 1/154*d*(-4*a*d+17*b*c)*x*(b*x^3+a)^(10/3)/b^2+1/14*d*x*(b*x^3+a)^(10/3)*(d
*x^3+c)/b+1/77*a^2*(2*a^2*d^2-14*a*b*c*d+77*b^2*c^2)*x*(b*x^3+a)^(1/3)*hype
rgeom([-7/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 246, 245}

$$\frac{a^2x\sqrt[3]{a+bx^3}\left(2a^2d^2-14abcd+77b^2c^2\right){}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c\right.}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(17*b*c - 4*a*d)*x*(a + b*x^3)^(10/3))/(154*b^2) + (d*x*(a + b*x^3)^(10/
3)*(c + d*x^3))/(14*b) + (a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a +
b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(77*b^2*(1 +
(b*x^3)/a)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b}+\frac{\int \left(a+bx^3\right)^{7/3}\left(c(14bc-ad)+d(17bc-4ad)x^3\right)dx}{14b}$$

$$= \frac{d(17bc-4ad)x\left(a+bx^3\right)^{10/3}}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b}-\frac{(ad(17bc-4ad)-11}{}$$

$$= \frac{d(17bc-4ad)x\left(a+bx^3\right)^{10/3}}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b}-\frac{\left(a^2(ad(17bc-4ad)-\right.}{}$$

$$= \frac{d(17bc-4ad)x\left(a+bx^3\right)^{10/3}}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b}+\frac{a^2\left(77b^2c^2-14abcd\right.}{}$$

Mathematica [A] time = 3.59, size = 177, normalized size = 1.31

$$\frac{ax\sqrt[3]{a+bx^3}\left(-9bx^3\Gamma\left(-\frac{4}{3}\right)\left(c+dx^3\right)^2 {}_3F_2\left(-\frac{4}{3},\frac{4}{3},2;1,\frac{13}{3};-\frac{bx^3}{a}\right)-3bx^3\Gamma\left(-\frac{4}{3}\right)\left(11c^2+16cdx^3+5d^2x^6\right){}_2F_1\left(-\frac{4}{3},\frac{4}{3}\right.\right.}{280\Gamma\left(-\frac{7}{3}\right)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-7/3]*H
ypergeometric2F1[-7/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[-4/3]*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b*x^3)/a
)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-4/3]*HypergeometricPFQ[{-4/3, 4/3, 2}, {1
, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-7/3])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2d^2x^{12} + 2\left(b^2cd + abd^2\right)x^9 + \left(b^2c^2 + 4\,abcd + a^2d^2\right)x^6 + a^2c^2 + 2\left(abc^2 + a^2cd\right)x^3\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^12 + 2*(b^2*c*d + a*b*d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^3)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{7}{3}}\left(dx^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(b\,x^3 + a\right)^{\frac{7}{3}} \left(d\,x^3 + c\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{7}{3}}\left(dx^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(7/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 12.60, size = 418, normalized size = 3.10

$$\frac{a^{\frac{7}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{7}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{7}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2a^{\frac{4}{3}}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2}{3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(7/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(7/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(7/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
2*a**(4/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3)) + 4*a**(4/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(4/3)*b*d**
2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(13/3)) + a**(1/3)*b**2*c**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10
/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(1/3)*b**2*c*d*x**10
*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(13/3)) + a**(1/3)*b**2*d**2*x**13*gamma(13/3)*hyper((-1/3, 13/3), (16/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(16/3))
```

3.80 $\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=133

$$\frac{ax\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right){}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+d\right.}{11b}$$

```
[Out] 1/44*d*(-2*a*d+7*b*c)*x*(b*x^3+a)^(7/3)/b^2+1/11*d*x*(b*x^3+a)^(7/3)*(d*x^3
+c)/b+1/44*a*(2*a^2*d^2-11*a*b*c*d+44*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom(
[-4/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 246, 245}

$$\frac{ax\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right){}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+d\right.}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(7*b*c - 2*a*d)*x*(a + b*x^3)^(7/3))/(44*b^2) + (d*x*(a + b*x^3)^(7/3)*(
c + d*x^3))/(11*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(44*b^2*(1 + (b*x^3
)/a)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}+\frac{\int \left(a+bx^3\right)^{4/3}\left(c(11bc-ad)+2d(7bc-2ad)x^3\right)\,dx}{11b}$$

$$= \frac{d(7bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}-\frac{(2ad(7bc-2ad)-8bc(1}{8}$$

$$= \frac{d(7bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}-\frac{\left(a(2ad(7bc-2ad)-8bc\right.}{}$$

$$= \frac{d(7bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}+\frac{a\left(44b^2c^2-11abcd+2a\right.}{4}$$

Mathematica [A] time = 5.19, size = 171, normalized size = 1.29

$$\frac{x\left(2a^2\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2-11abcd+44b^2c^2\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(4a^3d^2-2a^2bd\left(11c+dx^3\right)-3ab^2\left(4\right.\right.\right.}{220b^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(-((a + b*x^3)*(4*a^3*d^2 - 2*a^2*b*d*(11*c + d*x^3) - 3*a*b^2*(44*c^2 +
 33*c*d*x^3 + 10*d^2*x^6) - b^3*x^3*(44*c^2 + 55*c*d*x^3 + 20*d^2*x^6))) +
2*a^2*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*(1 + (b*x^3)/a)^(2/3)*Hypergeom
etric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(220*b^2*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bd^2x^9 + \left(2\,bcd + ad^2\right)x^6 + \left(bc^2 + 2\,acd\right)x^3 + ac^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^9 + (2*b*c*d + a*d^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2
)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{4}{3}}\left(dx^3 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(b\,x^3 + a\right)^{\frac{4}{3}}\left(d\,x^3 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{4}{3}}\left(dx^3 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 7.06, size = 270, normalized size = 2.03

$$\frac{a^{\frac{4}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{4}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{4}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}\,bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2}{3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

3.81 $\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=131

$$\frac{x\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right){}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

```
[Out] 1/40*d*(-4*a*d+11*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d*x*(b*x^3+a)^(4/3)*(d*x^3
+c)/b+1/10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-1/3
, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 246, 245}

$$\frac{x\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right){}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(11*b*c - 4*a*d)*x*(a + b*x^3)^(4/3))/(40*b^2) + (d*x*(a + b*x^3)^(4/3)*
(c + d*x^3))/(8*b) + ((10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1/3
)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(10*b^2*(1 + (b*x^3)/a)^
(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2 dx = \frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b} + \frac{\int \sqrt[3]{a+bx^3}\left(c(8bc-ad)+d(11bc-4ad)x^3\right) dx}{8b}$$

$$= \frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2} + \frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b} + \frac{\left(10b^2c^2-4abcd+a^2d^2\right)}{10b^2}$$

$$= \frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2} + \frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b} + \frac{\left(\left(10b^2c^2-4abcd+a^2d^2\right)\right.}{10b^2\sqrt[3]{}}$$

$$= \frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2} + \frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b} + \frac{\left(10b^2c^2-4abcd+a^2d^2\right)}{10b^2}$$

Mathematica [A] time = 4.00, size = 179, normalized size = 1.37

$$\frac{x\sqrt[3]{a+bx^3}\left(-9bx^3\Gamma\left(\frac{2}{3}\right)\left(c+dx^3\right)^2 {}_3F_2\left(\frac{2}{3},\frac{4}{3},2;1,\frac{13}{3};-\frac{bx^3}{a}\right)-3bx^3\Gamma\left(\frac{2}{3}\right)\left(11c^2+16cdx^3+5d^2x^6\right) {}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{13}{3};-\right.\right.}{280a\Gamma\left(-\frac{1}{3}\right)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hyp
ergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)] -
 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1, 13/3
}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(d^{2}x^{6}+2\,cdx^{3}+c^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx^{3}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx^{3}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 3.94, size = 131, normalized size = 1.00

$$\frac{\sqrt[3]{a}\,c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}\,cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.82 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=132

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2-5abcd+5b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\left(a+bx^3\right)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}\,(2bc-ad)}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}{5b}$$

```
[Out] 2/5*d*(-a*d+2*b*c)*x*(b*x^3+a)^(1/3)/b^2+1/5*d*x*(b*x^3+a)^(1/3)*(d*x^3+c)/
b+1/5*(2*a^2*d^2-5*a*b*c*d+5*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2
/3],[4/3],-b*x^3/a)/b^2/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2-5abcd+5b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\left(a+bx^3\right)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}\,(2bc-ad)}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*(a + b*x^3)^(1/3))/(5*b^2) + (d*x*(a + b*x^3)^(1/3)*(c
 + d*x^3))/(5*b) + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(
2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*b^2*(a + b*x^3)^(2/
3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{dx\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}{5b} + \frac{\int \frac{c(5bc-ad)+4d(2bc-ad)x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{5b}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}{5b} - \frac{(4ad(2bc-ad)-2bc(5bc-ad))\int \frac{1}{\left(a+bx^3\right.}}{10b^2}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}{5b} - \frac{\left((4ad(2bc-ad)-2bc(5bc-ad))\left(1+\frac{b}{}\right.\right.}{10b^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}{5b} + \frac{\left(5b^2c^2-5abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}}{5b^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 5.14, size = 104, normalized size = 0.79

$$\frac{x\left(\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2-5abcd+5b^2c^2\right)\,_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-d\left(a+bx^3\right)\left(2ad-b\left(5c+dx^3\right)\right)\right)}{5b^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(-(d*(a + b*x^3)*(2*a*d - b*(5*c + d*x^3))) + (5*b^2*c^2 - 5*a*b*c*d + 2
*a^2*d^2)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/
a)]))/(5*b^2*(a + b*x^3)^(2/3))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2x^6 + 2\,cdx^3 + c^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)/(b*x^3 + a)^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3 + c\right)^2}{\left(b\,x^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(2/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(2/3), x)
```

sympy [C] time = 4.27, size = 126, normalized size = 0.95

$$\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(2/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((2
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

3.83 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=146

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-2a^2d^2+2abcd+b^2c^2\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}\,(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

```
[Out] -1/2*d*(-2*a*d+b*c)*x*(b*x^3+a)^(1/3)/a/b^2+1/2*(-a*d+b*c)*x*(d*x^3+c)/a/b/
(b*x^3+a)^(2/3)+1/2*(-2*a^2*d^2+2*a*b*c*d+b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/a/b^2/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-2a^2d^2+2abcd+b^2c^2\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}\,(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] -(d*(b*c - 2*a*d)*x*(a + b*x^3)^(1/3))/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^
3))/(2*a*b*(a + b*x^3)^(2/3)) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 + (
b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b^2*(a
 + b*x^3)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{5/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{\int \frac{c(bc+ad)-2d(bc-2ad)x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}}+-\frac{(-2ad(bc-2ad)-2bc(bc+ad))\int \frac{1}{\left(a+bx^3\right)^{2/3}}}{4ab^2}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}}+-\frac{\left((-2ad(bc-2ad)-2bc(bc+ad))\left(1+\frac{b}{}\right.\right.}{4ab^2\left(a+bx^3\right)^{2/3}}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2}+\frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{\left(b^2c^2+2abcd-2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1}{2ab^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 3.76, size = 171, normalized size = 1.17

$$\frac{x\Gamma\left(\frac{2}{3}\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-3bx^3\left(c+dx^3\right)^2\,{}_3F_2\left(\frac{4}{3},2,\frac{8}{3};1,\frac{13}{3};-\frac{bx^3}{a}\right)-bx^3\left(11c^2+16cdx^3+5d^2x^6\right)\,{}_2F_1\left(\frac{4}{3},\frac{8}{3};\frac{13}{3};-\frac{bx^3}{a}\right)\right.}{84a^2\Gamma\left(\frac{5}{3}\right)\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(4*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 5/3, 10/3, -((b*x^3)/a)] - b*x^3*(11*c^2 + 16*c*d*x^3
 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 8/3, 13/3, -((b*x^3)/a)] - 3*b*x^3*(c
+ d*x^3)^2*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b*x^3)/a)]))/(84*
a^2*(a + b*x^3)^(2/3)*Gamma[5/3])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^3+c^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(5/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(5/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(5/3), x)
```

3.84 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}} \, dx$

Optimal. Leaf size=147

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2+abcd+2b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}+\frac{2x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

```
[Out] 2/5*(c^2/a^2-d^2/b^2)*x/(b*x^3+a)^(2/3)+1/5*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x
^3+a)^(5/3)+1/5*(2*a^2*d^2+a*b*c*d+2*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom
([1/3, 2/3],[4/3],-b*x^3/a)/a^2/b^2/(b*x^3+a)^(2/3)
```

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2+abcd+2b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}+\frac{2x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*(c^2/a^2 - d^2/b^2)*x)/(5*(a + b*x^3)^(2/3)) + ((b*c - a*d)*x*(c + d*x^3
))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x*(1 + (b
*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*a^2*b^2*(
a + b*x^3)^(2/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{8/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\int \frac{c(4bc+ad)+d(bc+4ad)x^3}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\left(2b^2c^2+abcd+2a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{5a^2b^2}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\left(\left(2b^2c^2+abcd+2a^2d^2\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5a^2b^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\left(2b^2c^2+abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\right.}{5a^2b^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 5.16, size = 128, normalized size = 0.87

$$\frac{x\left(\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2+abcd+2b^2c^2\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+\left(a+bx^3\right)\left(-3a^2d^2+abcd+2b^2c^2\right)+a(bc-\right.}{5a^2b^2\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (x*(a*(b*c - a*d)^2 + (2*b^2*c^2 + a*b*c*d - 3*a^2*d^2)*(a + b*x^3) + (2*b^
2*c^2 + a*b*c*d + 2*a^2*d^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometr
ic2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(5*a^2*b^2*(a + b*x^3)^(5/3))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(d^2x^6 + 2\,cdx^3 + c^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3 + c\right)^2}{\left(b\,x^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3 + c\right)^2}{\left(b\,x^3 + a\right)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(8/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(8/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^3\right)^2}{\left(a + bx^3\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(8/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(8/3), x)
```

3.85 $\int \frac{\left(a+bx^3\right)^3}{\left(c+dx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=109

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

```
[Out] 1/10*x*(b*x^3+a)^3/c/(d*x^3+c)^(10/3)+9/70*a*x*(b*x^3+a)^2/c^2/(d*x^3+c)^(7
/3)+27/140*a^2*x*(b*x^3+a)/c^3/(d*x^3+c)^(4/3)+81/140*a^3*x/c^4/(d*x^3+c)^(
1/3)
```

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {378, 191}

$$\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^2)/(70*c^2
*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^(4/3)) +
(81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^3}{\left(c+dx^3\right)^{13/3}}\,dx = \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{(9a)\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^{10/3}}\,dx}{10c}$$

$$= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}} + \frac{\left(27a^2\right)\int \frac{a+bx^3}{\left(c+dx^3\right)^{7/3}}\,dx}{35c^2}$$

$$= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}} + \frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}} + \frac{\left(81a^3\right)\int \frac{1}{\left(c+dx^3\right)^{4/3}}\,dx}{140c^3}$$

$$= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}} + \frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}$$

Mathematica [A] time = 0.04, size = 120, normalized size = 1.10

$$\frac{x\left(a^3\left(140c^3+315c^2dx^3+270cd^2x^6+81d^3x^9\right)+3a^2bcx^3\left(35c^2+30cdx^3+9d^2x^6\right)+6ab^2c^2x^6\left(10c+3dx^3\right)+14\right.}{140c^4\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*c
^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x^6
 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

fricas [A] time = 0.67, size = 166, normalized size = 1.52

$$\frac{\left(\left(14\,b^3c^3+18\,ab^2c^2d+27\,a^2bcd^2+81\,a^3d^3\right)x^{10}+30\left(2\,ab^2c^3+3\,a^2bc^2d+9\,a^3cd^2\right)x^7+140\,a^3c^3x+105\left(a^2bc^3+\right.\right.}{140\left(c^4d^4x^{12}+4\,c^5d^3x^9+6\,c^6d^2x^6+4\,c^7dx^3+c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 + 3
0*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(a^
```

```
2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*x^9
 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^3}{(dx^3 + c)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

maple [A] time = 0.05, size = 134, normalized size = 1.23

$$\frac{\left(81a^3d^3x^9 + 27a^2bc\,d^2x^9 + 18a\,b^2c^2d\,x^9 + 14b^3c^3x^9 + 270a^3c\,d^2x^6 + 90a^2b\,c^2d\,x^6 + 60a\,b^2c^3x^6 + 315a^3c^2d\,x^3 + \right.}{140\left(d\,x^3 + c\right)^{\frac{10}{3}} c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/(d*x^3+c)^(13/3),x)
```

```
[Out] 1/140*x*(81*a^3*d^3*x^9+27*a^2*b*c*d^2*x^9+18*a*b^2*c^2*d*x^9+14*b^3*c^3*x^
9+270*a^3*c*d^2*x^6+90*a^2*b*c^2*d*x^6+60*a*b^2*c^3*x^6+315*a^3*c^2*d*x^3+1
05*a^2*b*c^3*x^3+140*a^3*c^3)/(d*x^3+c)^(10/3)/c^4
```

maxima [A] time = 0.63, size = 182, normalized size = 1.67

$$\frac{b^3x^{10}}{10\left(dx^3+c\right)^{\frac{10}{3}}c} - \frac{3\,ab^2\left(7\,d - \frac{10\left(dx^3+c\right)}{x^3}\right)x^{10}}{70\left(dx^3+c\right)^{\frac{10}{3}}c^2} + \frac{3\left(14\,d^2 - \frac{40\left(dx^3+c\right)d}{x^3} + \frac{35\left(dx^3+c\right)^2}{x^6}\right)a^2bx^{10}}{140\left(dx^3+c\right)^{\frac{10}{3}}c^3} - \frac{\left(14\,d^3 - \frac{60\left(dx^3+c\right)d^2}{x^3} + \frac{105}{\cdots}\right.}{140\left(dx^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^3
)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 + 35
*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3 - 6
0*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9)*a^
3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

mupad [B] time = 1.56, size = 271, normalized size = 2.49

$$\frac{x\left(\frac{a^3}{10\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{10\,d}-\frac{3\,a\,b^2}{10\,c}\right)}{d}+\frac{3\,a^2\,b}{10\,c}\right)}{d}\right)}{\left(d\,x^3+c\right)^{10/3}}-\frac{x\left(\frac{b^3}{4\,d^3}-\frac{27\,a^3\,d^3+9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-7\,b^3\,c^3}{140\,c^3\,d^3}\right)}{\left(d\,x^3+c\right)^{4/3}}+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{9\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2}{70\,c^2\,d^3}\right.}{\left(d\,x^3+c\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/(c + d*x^3)^(13/3),x)
```

```
[Out] (x*(a^3/(10*c) - (c*((c*(b^3/(10*d) - (3*a*b^2)/(10*c)))/d + (3*a^2*b)/(10*
c)))/d))/(c + d*x^3)^(10/3) - (x*(b^3/(4*d^3) - (27*a^3*d^3 - 7*b^3*c^3 + 6
*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(140*c^3*d^3)))/(c + d*x^3)^(4/3) + (x*((c*(b
^3/(7*d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (9*a^3*d^3 + b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2)/(70*c^2*d^3)))/(c + d*x^3)^(7/3) + (x*(81*a^3*d
^3 + 14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2))/(140*c^4*d^3*(c + d*x^3
)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{b^{2/3}\left(20a^2d^2-24abcd+9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18d^3}+\frac{b^{2/3}\left(20a^2d^2-24abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,d^3}-\frac{(bc}{}$$

```
[Out] -1/18*b*(-11*a*d+6*b*c)*x*(b*x^3+a)^(2/3)/d^2+1/6*b*x*(b*x^3+a)^(5/3)/d-1/6
*(-a*d+b*c)^(8/3)*ln(d*x^3+c)/c^(2/3)/d^3+1/2*(-a*d+b*c)^(8/3)*ln((-a*d+b*c
)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^3-1/18*b^(2/3)*(20*a^2*d^2-24*
a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/27*b^(2/3)*(20*a^2*
d^2-24*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2
))/d^3*3^(1/2)-1/3*(-a*d+b*c)^(8/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d^3*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{8/3}}{c+dx^3}\,dx = \frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^2x\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [**C**] time = 1.23, size = 655, normalized size = 1.98

$$3bx^4\sqrt[3]{\frac{bx^3}{a}+1}\,\sqrt[3]{bc-ad}\,\left(20a^2d^2-24abcd+9b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2\sqrt[3]{c}\left(9a^3d^2\sqrt[3]{a+bx^3}\,\log\left(\frac{\sqrt[3]{c}\,x\,\sqrt[3]{bc}}{\sqrt[3]{ax^3}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*x^4*(1 + (b*x^
3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1
/3)*(-18*a*b^2*c^(5/3)*(b*c - a*d)^(1/3)*x + 42*a^2*b*c^(2/3)*d*(b*c - a*d)
^(1/3)*x - 18*b^3*c^(5/3)*(b*c - a*d)^(1/3)*x^4 + 51*a*b^2*c^(2/3)*d*(b*c -
 a*d)^(1/3)*x^4 + 9*b^3*c^(2/3)*d*(b*c - a*d)^(1/3)*x^7 + 2*Sqrt[3]*a*(3*b^
2*c^2 - 7*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)
^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*a*(3*b^2*c^2 - 7*a*b*c*
d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a
*x^3)^(1/3)] + 3*a*b^2*c^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/
3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)
] - 7*a^2*b*c*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b
+ a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + 9*a^3*d
^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3
) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*c*d^2*(b*c - a*
d)^(1/3)*(a + b*x^3)^(1/3))
```

fricas [B] time = 15.64, size = 643, normalized size = 1.94

$$-\frac{18\,\sqrt{3}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\,\sqrt{3}\left(bx^{3}+a\right)^{\frac{1}{3}}c\left(\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{c^{2}}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\,\sqrt{3}\left(9\,b^{2}c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 +
a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) +
2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/3*(s
qrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*
x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*
d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(
-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (
b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^2*d^2*x^4 - 2*
(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{8}{3}}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{8/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx$

Optimal. Leaf size=273

$$\frac{b^{2/3}(3bc-5ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6d^2}-\frac{b^{2/3}(3bc-5ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^2}+\frac{(bc-ad)^{5/3}\log\left(c+dx^3\right)}{6c^{2/3}d^2}-\frac{(bc-ad)}{}$$

```
[Out] 1/3*b*x*(b*x^3+a)^(2/3)/d+1/6*(-a*d+b*c)^(5/3)*ln(d*x^3+c)/c^(2/3)/d^2-1/2*
(-a*d+b*c)^(5/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^2
+1/6*b^(2/3)*(-5*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^2-1/9*b^(2/3)*
(-5*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1
/2)+1/3*(-a*d+b*c)^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+
a)^(1/3))*3^(1/2))/c^(2/3)/d^2*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 0.22, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/3}}{c+dx^3}\,dx=\frac{\left(a\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$=\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.71, size = 443, normalized size = 1.62

$$2\sqrt[3]{c}\left(3a^2d\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6b^2c^{2/3}x^4\sqrt[3]{bc-ad}-abc\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(-3*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[
4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1/3)*(6*a*b*c^(2/3)*(b
*c - a*d)^(1/3)*x + 6*b^2*c^(2/3)*(b*c - a*d)^(1/3)*x^4 + 2*Sqrt[3]*a*(-(b*
c) + 3*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*(b*c - 3*a*d)*(a + b*x^3)^(1/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*b*c*(a + b*x^3)^(1/3)*Lo
g[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d
)^(1/3)*x)/(b + a*x^3)^(1/3)] + 3*a^2*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b +
 a*x^3)^(1/3)]))/(36*c*d*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

fricas [B] time = 1.75, size = 535, normalized size = 1.96

$$6\left(bx^3+a\right)^{\frac{2}{3}}bdx+6\sqrt{3}\,(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\,\left(-b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*
x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c - 5*a*
d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3*b*c - 5
*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3
+ a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*
x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a
)^(2/3)*(b*c - a*d))/x^2))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{5/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{5}{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

3.88 $\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=233

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a\right.}}{2c^{2/3}d}$$

```
[Out] -1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(
1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
```

```
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.22, size = 161, normalized size = 0.69

$$\frac{4acx\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [B] time = 1.02, size = 469, normalized size = 2.01

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\,\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\,\sqrt{3}\,\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,bx-2\,\sqrt{3}\left(bx^3 \cdots\right.}{3\,bx \cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
```

```
a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.89 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=148

$$\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/
3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)
*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c
- a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*
x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{}{\sqrt[3]{a}}\right.}{3c^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{c}} + \text{Subs}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{c^{2/3}\sqrt[3]{bc}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] time = 0.15, size = 168, normalized size = 1.14

$$\frac{\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))
/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[
c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^
(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.90 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=179

$$-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)
+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4
/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3
*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a +
 b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3
)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{2}{c^{2/3}+\sqrt[3]{c}\right.}{}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}}\right.}{6c^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a}}{\sqrt[3]{a+bx^3}}\right.}{6c^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}\right.}{}$$

Mathematica [C] time = 0.80, size = 256, normalized size = 1.43

$$-\frac{28c^3\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-28c^3\left(a+bx^3\right)^2+21c^2dx^3\left(a+bx^3\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-21c^2dx^3\left(a\right.}{7c^3x^2\left(a+bx^3\right)^{7/3}(ad-b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/7*(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^
3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21
*c^2*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2
```

```
, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(-(b*c) + a*d)*x^2*(a
 + b*x^3)^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.91 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=226

$$\frac{bx(3bc-7ad)}{4a^2\sqrt[3]{a+bx^3}\,(bc-ad)^2}+\frac{d^2\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{7/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{7/3}}+\frac{bx}{4a\left(a+bx^3\right)^{\cdots}}$$

[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/4*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^
2/(b*x^3+a)^(1/3)+1/6*d^2*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(7/3)-1/2*d^2*ln((
-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(7/3)+1/3*d^2
arctan(1/3(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/
3)/(-a*d+b*c)^(7/3)*3^(1/2)

Rubi [C] time = 2.58, antiderivative size = 621, normalized size of antiderivative = 2.75, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{-9c^2x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-9d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-18cdx^{12}(bc-a\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] -(70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 + 105*c^3*d*(b*c - a*d)*x^6*(a + b*x
^3)^2 + 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 + 280*c^5*(a + b*x^3)^3 +
420*c^4*d*x^3*(a + b*x^3)^3 + 180*c^3*d^2*x^6*(a + b*x^3)^3 - 280*c^5*(a +
b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 33*c^2*(b*c - a*d)^3*x^9*Hype
rgeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 60*c*d*(b
*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] - 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[
{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c*d*(b*c -
 a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3},
```

```
{1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*(b*c - a*d)^2*x^5*(a
 + b*x^3)^(10/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{70c^4(bc-ad)x^3\left(a+bx^3\right)^2+105c^3d(bc-ad)x^6\left(a+bx^3\right)^2+45c^2d^2(bc-ad)x^9\left(\right.}{}$$

Mathematica [C] time = 2.67, size = 621, normalized size = 2.75

$$\frac{9c^2x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+9d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+18cdx^{12}(bc-ad)^3\,{}_3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] (-70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 - 105*c^3*d*(b*c - a*d)*x^6*(a + b*x
^3)^2 - 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 - 280*c^5*(a + b*x^3)^3 -
420*c^4*d*x^3*(a + b*x^3)^3 - 180*c^3*d^2*x^6*(a + b*x^3)^3 + 280*c^5*(a +
b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
```

```
+ 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 33*c^2*(b*c - a*d)^3*x^9*Hype
rgeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 60*c*d*(b
*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[
{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 18*c*d*(b*c -
 a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3},
{1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*(b*c - a*d)^2*x^5*(a
 + b*x^3)^(10/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{7}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

3.92 $\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=280

$$\frac{bx(6bc-13ad)}{28a^2\left(a+bx^3\right)^{4/3}(bc-ad)^2}+\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{28a^3\sqrt[3]{a+bx^3}\,(bc-ad)^3}-\frac{d^3\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{10/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}$$

```
[Out] 1/7*b*x/a/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/28*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^3+a)^(4/3)+1/28*b*(67*a^2*d^2-57*a*b*c*d+18*b^2*c^2)*x/a^3/(-a*d+b
*c)^3/(b*x^3+a)^(1/3)-1/6*d^3*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(10/3)+1/2*d^3
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(10/3)-1
/3*d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))
/c^(2/3)/(-a*d+b*c)^(10/3)*3^(1/2)
```

Rubi [C] time = 6.64, antiderivative size = 1172, normalized size of antiderivative = 4.19, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{-4158d^3(bc-ad)^4\,{}_2F_1\left(2,\frac{13}{3};\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}-2268d^3(bc-ad)^4\,{}_3F_2\left(2,2,\frac{13}{3};1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}-378d^3(bc-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] -(7280*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 16380*c^4*d*(b*c - a*d)^2*x^9*
(a + b*x^3)^2 + 14040*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 4212*c^2*d
^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 12740*c^6*(b*c - a*d)*x^3*(a + b*x^3)
^3 + 28665*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 24570*c^4*d^2*(b*c - a*d)*
x^9*(a + b*x^3)^3 + 7371*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 + 50960*c^7
*(a + b*x^3)^4 + 114660*c^6*d*x^3*(a + b*x^3)^4 + 98280*c^5*d^2*x^6*(a + b*
x^3)^4 + 29484*c^4*d^3*x^9*(a + b*x^3)^4 - 50960*c^7*(a + b*x^3)^4*Hypergeo
metric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 114660*c^6*d*x^
3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 98280*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] - 29484*c^4*d^3*x^9*(a + b*x^3)^4*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5796*c^3*(b*c -
a*d)^4*x^12*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 15246*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric2F1[2, 13/3, 16/3, ((b*
```

```
c - a*d)*x^3)/(c*(a + b*x^3))] - 13608*c*d^2*(b*c - a*d)^4*x^18*Hypergeomet
ric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4158*d^3*(b*c -
a*d)^4*x^21*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 2646*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 7560*c^2*d*(b*c - a*d)^4*x^15*Hyper
geometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
7182*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2268*d^3*(b*c - a*d)^4*x^21*Hypergeometr
icPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*c^3
*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 1134*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric
PFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 113
4*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3},
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*d^3*(b*c - a*d)^4*x^21*Hypergeome
tricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/
(5096*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(13/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{10/3}\left(c+dx^3\right)}\,dx}{a^3\sqrt[3]{a+bx^3}}$$

$$= -\frac{7280c^5(bc-ad)^2x^6\left(a+bx^3\right)^2+16380c^4d(bc-ad)^2x^9\left(a+bx^3\right)^2+14040c^3d^2(}{}$$

Mathematica [A] time = 5.75, size = 277, normalized size = 0.99

$$\frac{bx\left(\left(a+bx^3\right)^2\left(67a^2d^2-57abcd+18b^2c^2\right)+4a^2(bc-ad)^2+a\left(a+bx^3\right)(ad-bc)(13ad-6bc)\right)}{28a^3\left(a+bx^3\right)^{7/3}(bc-ad)^3}-\frac{d^3\left(\log\left(\frac{\sqrt[3]{c}\,x\,\sqrt[3]{\cdots}}{\sqrt[3]{ax\cdots}}\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] (b*x*(4*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-6*b*c + 13*a*d)*(a + b*x^3)
+ (18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2)*(a + b*x^3)^2))/(28*a^3*(b*c - a*d
)^3*(a + b*x^3)^(7/3)) - (d^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3
)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)
^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(6*c^(2/3)*(b*c
 - a*d)^(10/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{10}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{10}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{10}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{10/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{10}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(10/3)*(c + d*x**3)), x)
```

3.93 $\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3} \, dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a
)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.61, size = 346, normalized size = 5.77

$$\frac{x\left(\frac{4\left(bx^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(2a^2d+abdx^3+b^2x^3\left(c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+ \cdots\right)}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^{3}+a\right)^{4/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{4}{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.94 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^
(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.17, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(bcF_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.95 $\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^(
2/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.04, size = 161, normalized size = 2.73

$$-\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(dx^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{2}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.96 $\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a)
^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}\left(c+dx^3\right)}\,dx}{a\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.28, size = 332, normalized size = 5.35

$$\frac{x\Bigg(\frac{4\left(bx^3\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4ac\left(2ad-b\left(2c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}-\frac{bdx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\Bigg(}{}}{8a\left(a+bx^3\right)^{2/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-((b*d*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c) + (4*(4*a*c*(2*a*d - b*(2*c + d*x^3))*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)

[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{5}{3}}\left(dx^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)),x)

[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{5}{3}}\left(c+dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c),x)

[Out] Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)), x)

3.97 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,1,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(b*x^3+
a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}\left(c+dx^3\right)}\,dx}{a^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.87, size = 429, normalized size = 6.92

$$-\frac{x\left(\frac{4\left(bx^3\left(c+dx^3\right)\left(11a^2d+ab\left(9dx^3-6c\right)-4b^2cx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4ac\left(10a^3d^2-a^2bd\left(20c+dx^3\right)+ab^2\left(10c^2-\right.\right.}{\left(a+bx^3\right)\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\right.\right.}\right.}{40a^2\left(a+bx^3\right)^{2/3}(bc-ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*(x*((b*d*(-4*b*c + 9*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(4*a*c*(10*a^3*d^2 + 4*b^3*c*
x^3*(2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3 - 9
*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(
c + d*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3))*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((a + b*x^3)*(c + d*x^3)*(-4*a*c*Appel
lF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3
, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{8}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{8/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{8}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)), x)
```

3.98 $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2} \, dx$

Optimal. Leaf size=351

$$\frac{b^{5/3}(3bc-4ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3d^3}-\frac{2b^{5/3}(3bc-4ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^3}+\frac{(bc-ad)^{5/3}(ad+3bc)\log\left(c+dx^3\right)}{9c^{5/3}d^3}$$

```
[Out] 1/3*b*(-a*d+2*b*c)*x*(b*x^3+a)^(2/3)/c/d^2-1/3*(-a*d+b*c)*x*(b*x^3+a)^(5/3)
/c/d/(d*x^3+c)+1/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^3-1/3
*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3)
)/c^(5/3)/d^3+1/3*b^(5/3)*(-4*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3
-2/9*b^(5/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/d^3*3^(1/2)+2/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c
)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^3*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.18, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2 x\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int\frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx=\frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 1.08, size = 698, normalized size = 1.99

$$\frac{1}{18}\left(\frac{2a^3\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a^2b\left(\log\left(\cdots\right.\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3))))/d^2 - (9*b^3
*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/3))
+ (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3)) - (2*a*b
^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
```

```
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)))/18
```

fricas [B] time = 12.13, size = 819, normalized size = 2.33

$$\frac{2\sqrt{3}\left(3\,b^2c^3-2\,abc^2d-a^2cd^2+\left(3\,b^2c^2d-2\,abcd^2-a^2d^3\right)x^3\right)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\,\sqrt{3}\left(bx^3+a\right)}{3\,(bc-a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*
c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1
/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4*a*b
*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)
*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^3 - 2*
a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 - 4*a*b*c^
2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c
*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)
^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) +
 (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 + (2*b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{8}{3}}}{\left(dx^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{8/3}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
[Out] Timed out
```

3.99 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=301

$$-\frac{b^{5/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}-\frac{(bc-ad)^{2/3}(2ad+3bc)\log\left(c+dx^3\right)}{18c^{5/3}d^2}+\frac{(bc-ad)^{2/3}(2ad+}{}$$

```
[Out] -1/3*(-a*d+b*c)*x*(b*x^3+a)^(2/3)/c/d/(d*x^3+c)-1/18*(-a*d+b*c)^(2/3)*(2*a*
d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^2+1/6*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*ln((-a*
d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/d^2-1/2*b^(5/3)*ln(-b^(1/3)
*x+(b*x^3+a)^(1/3))/d^2+1/3*b^(5/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))/d^2*3^(1/2)-1/9*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*arctan(1/3*(1+2
*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^2*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 0.20, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {430, 429}

$$\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
```

```
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^2}\,dx = \frac{\left(a\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{\left(c+dx^3\right)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{ax\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.64, size = 450, normalized size = 1.50

$$\frac{\frac{4a^2\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{9b^2c^{2/3}x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d\sqrt[3]{a+bx^3}}-\frac{12c^{2/}}{}}{36c^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-12*c^(2/3)*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(d*(c + d*x^3)) + (9*b^2*c^(
2/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/(d*(a + b*x^3)^(1/3)) + (4*a^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b
*c - a*d)^(1/3) + (2*a*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2
/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/
3)))/(36*c^(5/3))
```

fricas [B] time = 1.34, size = 631, normalized size = 2.10

$$-\frac{2\sqrt{3}\left(\left(3\,bcd+2\,ad^2\right)x^3+3\,bc^2+2\,acd\right)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\,\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c
- a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3
)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x^3 + b*c
^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(b*c*d*
x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^
2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2
 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x
^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^
2))/(c*d^3*x^3 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.100 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=182

$$\frac{a\log\left(c+dx^3\right)}{9c^{5/3}\sqrt[3]{bc-ad}}-\frac{a\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/c/(d*x^3+c)+1/9*a*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3
)-1/3*a*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(
1/3)+2/9*a*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1
/2))/c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 241, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2a\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}+\frac{a\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*ArcTan[(c^(1/3) + (2*(b*c -
a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*
c - a*d)^(1/3)) - (2*a*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3
)])/(9*c^(5/3)*(b*c - a*d)^(1/3)) + (a*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*
c^(5/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx = \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{(2a)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{3c}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+}\right.}{9c^{5/3}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+b}}\right.}{3c^{4/3}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{(2a)\,\mathrm{Su}}{}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}\right.}{9c^{5/3}\sqrt[3]{bc-}}$$

Mathematica [C] time = 0.04, size = 78, normalized size = 0.43

$$\frac{x\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}\sqrt[3]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{2}{3}}}{\left(d\,x^3 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\right)^{2/3}}{\left(d\,x^3 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

3.101 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=217

$$\frac{(3bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}-\frac{dx\left(a+b\right.}{3c\left(c+dx^3\right)}$$

```
[Out] -1/3*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)+1/18*(-2*a*d+3*b*c)*ln(d*x^
3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/6*(-2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1
/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/9*(-2*a*d+3*b*c)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*
c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{18c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\ldots}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] -(d*x*(a + b*x^3)^(2/3))/(3*c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)*A
rcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3
))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) - ((3*b*c - 2*a*d)*Log[c^(1/3) -
 ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*c^(5/3)*(b*c - a*d)^(4/3)) +
((3*b*c - 2*a*d)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)]])/(18*c^(5/3)*(b*c - a*d)^(
4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}\,dx = -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)}+\frac{(3bc-}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log\left(c^{2/3}+\right.}{18c^{5/3}(}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\right.}{9c^{5/3}(bc-ad)^{4/}}$$

Mathematica [C] time = 0.14, size = 99, normalized size = 0.46

$$\frac{x\left(\left(c+dx^3\right)(3bc-2ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-cd\left(a+bx^3\right)\right)}{3c^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]

[Out] (x*(-(c*d*(a + b*x^3)) + (3*b*c - 2*a*d)*(c + d*x^3)*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(3*c^2*(b*c - a*d)*(a + b*x^3
)^(1/3)*(c + d*x^3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{1/3}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

3.102 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{d(3bc-ad)\log\left(c+dx^3\right)}{9c^{5/3}(bc-ad)^{7/3}}+\frac{d(3bc-ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}-\frac{2d(3bc-ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{7/3}}+\frac{bx}{3ac\sqrt[3]{a}}$$

```
[Out] 1/3*b*(a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/(
b*x^3+a)^(1/3)/(d*x^3+c)-1/9*d*(-a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^
(7/3)+1/3*d*(-a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(
5/3)/(-a*d+b*c)^(7/3)-2/9*d*(-a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [C] time = 1.93, antiderivative size = 625, normalized size of antiderivative = 2.39, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{c\left(a+bx^3\right)^{2/3}\left(-\frac{54d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^5\left(a+bx^3\right)^3}-\frac{108dx^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^4\left(a+bx^3\right)^3}-\frac{54x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\right.}{c^3\left(a+bx^3\right)^3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] -(c*(a + b*x^3)^(2/3)*(6860 + (13720*d*x^3)/c + (6300*d^2*x^6)/c^2 - (525*(
b*c - a*d)*x^3)/(c*(a + b*x^3)) - (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)
) - (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) - 6860*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (13720*d*x^3*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c - (6300*d^2*x^6*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 + (2240*
(b*c - a*d)*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(c*(a + b*x^3)) + (5320*d*(b*c - a*d)*x^6*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^2*(a + b*x^3)) + (2520*d^2*(b*
c - a*d)*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))])/(c^3*(a + b*x^3)) - (54*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2, 2, 7/3
}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(a + b*x^3)^3) - (10
8*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))])/(c^4*(a + b*x^3)^3) - (54*d^2*(b*c - a*d)^3*x^15*
```

```
HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
])/(c^5*(a + b*x^3)^3)))/(420*(b*c - a*d)^2*x^5*(c + d*x^3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^2}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}\left(6860+\frac{13720dx^3}{c}+\frac{6300d^2x^6}{c^2}-\frac{525(bc-ad)x^3}{c\left(a+bx^3\right)}-\frac{1890d(bc-ad)x^6}{c^2\left(a+bx^3\right)}-\frac{945d^2(bc-}{c^3\left(a+b\right.}\right.}{}$$

Mathematica [C] time = 2.03, size = 625, normalized size = 2.39

$$\frac{c\left(a+bx^3\right)^{2/3}\left(\frac{54d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^5\left(a+bx^3\right)^3}+\frac{108dx^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^4\left(a+bx^3\right)^3}+\frac{54x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)}{c\left(bx^3+\right.}\right)}{c^3\left(a+bx^3\right)^3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] (c*(a + b*x^3)^(2/3)*(-6860 - (13720*d*x^3)/c - (6300*d^2*x^6)/c^2 + (525*(
b*c - a*d)*x^3)/(c*(a + b*x^3)) + (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)
```

```
) + (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) + 6860*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + (13720*d*x^3*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c + (6300*d^2*x^6*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 - (2240*
(b*c - a*d)*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(c*(a + b*x^3)) + (5320*d*(-(b*c) + a*d)*x^6*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^2*(a + b*x^3)) + (2520*d^2*
(-(b*c) + a*d)*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))])/(c^3*(a + b*x^3)) + (54*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2,
2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(a + b*x^3)^3)
 + (108*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*(a + b*x^3)^3) + (54*d^2*(b*c - a*d)^3
*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))])/(c^5*(a + b*x^3)^3)))/(420*(b*c - a*d)^2*x^5*(c + d*x^3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3} + a\right)^{\frac{4}{3}} \left(dx^{3} + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^{3} + a\right)^{4/3}\, \left(d\,x^{3} + c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{3}\right)^{\frac{4}{3}} \left(c + dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

3.103 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=324

$$\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}\,(bc-ad)^3}+\frac{d^2(9bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{10/3}}-\frac{d^2(9bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}+\frac{d^2(9bc}{3}$$

```
[Out] 1/12*b*(4*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)+1/12*b*(-4*a^2*d^2-
33*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+
b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)+1/18*d^2*(-2*a*d+9*b*c)*ln(d*x^3+c)/c^(5/3)/
(-a*d+b*c)^(10/3)-1/6*d^2*(-2*a*d+9*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x
^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(10/3)+1/9*d^2*(-2*a*d+9*b*c)*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(-a*d+b*c)^
(10/3)*3^(1/2)
```

Rubi [C] time = 5.69, antiderivative size = 1214, normalized size of antiderivative = 3.75, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{6804d^3(bc-ad)^4\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}+1134d^3(bc-ad)^4\,{}_4F_3\left(2,2,2,\frac{10}{3};1,1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}+2154}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (26130*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 89505*c^4*d*(b*c - a*d)^2*x^9*
(a + b*x^3)^2 + 84240*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 26325*c^2*
d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 748020*c^6*(b*c - a*d)*x^3*(a + b*x^
3)^3 + 2113020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 1916460*c^4*d^2*(b*c -
 a*d)*x^9*(a + b*x^3)^3 + 589680*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 - 2
002000*c^7*(a + b*x^3)^4 - 5460000*c^6*d*x^3*(a + b*x^3)^4 - 4914000*c^5*d^
2*x^6*(a + b*x^3)^4 - 1506960*c^4*d^3*x^9*(a + b*x^3)^4 - 1248520*c^6*(b*c
- a*d)*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 3478020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3*Hypergeometri
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3144960*c^4*d^2*(b*c
 - a*d)*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 966420*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2002000*c^7*(a +
```

```
b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
+ 5460000*c^6*d*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] + 4914000*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1506960*c^4*d^3*x^9*(
a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 7938*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 22680*c^2*d*(b*c - a*d)^4*x^15*Hyper
geometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
21546*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6804*d^3*(b*c - a*d)^4*x^21*Hypergeomet
ricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c
^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 3402*c^2*d*(b*c - a*d)^4*x^15*Hypergeometr
icPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3
402*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*d^3*(b*c - a*d)^4*x^21*Hyperge
ometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
])/(21840*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(10/3)*(c + d*x^3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)^2}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= \frac{26130c^5(bc-ad)^2x^6\left(a+bx^3\right)^2 + 89505c^4d(bc-ad)^2x^9\left(a+bx^3\right)^2 + 84240c^3d^2}{}$$

Mathematica [A] time = 5.94, size = 288, normalized size = 0.89

$$\frac{1}{36}\left(3x\left(a+bx^3\right)^{2/3}\left(\frac{3b^2(11ad-3bc)}{a^2\left(a+bx^3\right)(ad-bc)^3} + \frac{3b^2}{a\left(a+bx^3\right)^2(bc-ad)^2} - \frac{4d^3}{c\left(c+dx^3\right)(bc-ad)^3}\right) + \frac{2d^2(9bc-2ad}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (3*x*(a + b*x^3)^(2/3)*((3*b^2)/(a*(b*c - a*d)^2*(a + b*x^3)^2) + (3*b^2*(-
3*b*c + 11*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)) - (4*d^3)/(c*(b*c - a*d
)^3*(c + d*x^3))) + (2*d^2*(9*b*c - 2*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c
^(5/3)*(b*c - a*d)^(10/3)))/36
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{7}{3}}\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**2), x)
```

3.104 $\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3
/a)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^2}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{\left(c+dx^3\right)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.32, size = 341, normalized size = 5.68

$$\frac{x\left(\frac{4c\left(x^3\left(a+bx^3\right)(ad-bc)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(3a^2d+abdx^3-b^2cx^3\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+bx^3\left(\frac{bx}{a}\right.}{12c^2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(b*(2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d - b^2*c*x^3 + a*b*d*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + (-(b*c) + a*d)*x^3
*(a + b*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x
^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(12*c^2*d*(a + b*x^3)^
(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3}\right)^{\frac{4}{3}}}{\left(c + dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3)**2, x)
```

3.105 $\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^2} \, dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/a
)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{\left(c+dx^3\right)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.21, size = 232, normalized size = 3.93

$$\frac{x\left(\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}\right)}{c+dx^3}+\frac{bx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}\right)}{12\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

maple [F]   time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(dx^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^{3}+a\right)^{1/3}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^{3}}}{\left(c+dx^{3}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

3.106 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a)
^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)^2}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.27, size = 393, normalized size = 6.66

$$\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(bdx^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(c+dx^3\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+4c\left(3ad-3bc+bdx^3\right)\right)-}{12c^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)(bc-ad)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(4*c*(-3*b*
c + 3*a*d + b*d*x^3) + b*d*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4
/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - d*x^4*(4*c*(a + b*x^3) + b*
x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1,
 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(dx^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{2}{3}}\left(c+dx^{3}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**2), x)
```

3.107 $\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,2,4/3,-b*x^3/a,-d*x^3/c)/a/c^2/(b*x^3+
a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}\left(c+dx^3\right)^2}\,dx}{a\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.57, size = 386, normalized size = 6.23

$$x\left(\frac{c\left(16ac\left(6a^2d^2+2abd\left(dx^3-6c\right)+3b^2c\left(2c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3\left(2a^2d^2+2abd^2x^3+3b^2c\left(c+dx^3\right)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\right.\right.\right.}{\left(c+dx^3\right)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\right.$$
$$\overline{\qquad 24ac^2\left(a+bx^3\right)^{2/3}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(b*d*(3*b*c + 2*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + (c*(16*a*c*(6*a^2*d^2 + 2*a*b*d*(-6*c + d*x
^3) + 3*b^2*c*(2*c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)] - 4*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3
, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{5}{3}} \left(dx^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{5/3}\,\left(d\,x^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{5}{3}} \left(c+dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)**2), x)
```

3.108 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^2/(b*x^
3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}\left(c+dx^3\right)^2}\,dx}{a^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.99, size = 550, normalized size = 8.87

$$\frac{\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5a^2d^2+21abcd-6b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(ad-bc)^3} + \frac{4c\left(x^4\left(5a^4d^3+10a^3bd^3x^3+a^2b^2d\left(24c^2+24cdx^3+5d^2x^6\right)+3ab^3c\left(-3c^2+4cdx^3+7d^2\right.\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] ((b*d*(-6*b^2*c^2 + 21*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 + (4*c*
(-4*a*c*x*(15*a^4*d^3 - 6*b^4*c^2*x^3*(2*c + d*x^3) + 5*a^3*b*d^2*(-9*c + 4
*d*x^3) + a^2*b^2*d*(45*c^2 - 21*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-5*c^2 +
 11*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^4*(5*a^4*d^3 + 10*a^3*b*d^3*x^3 - 6*b^4*c^2*x^3*(c + d*x^3) + a^2
*b^2*d*(24*c^2 + 24*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-3*c^2 + 4*c*d*x^3 +
7*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*
d)^3*(a + b*x^3)*(c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(60*a^2*c^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{8}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)^{8/3}\,\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{8}{3}}\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)**2), x)
```

3.109 $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=541

$$-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{18c^2d^4}+\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{18c^2d^3}-\frac{(bc-ad)^{8/}}{}$$

```
[Out] -1/18*b*(-a*d+2*b*c)*(-5*a^2*d^2-18*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c
^2/d^4+1/18*b*(-5*a^2*d^2-10*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(5/3)/c^2/d^3-
1/6*(-a*d+b*c)*x*(b*x^3+a)^(11/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+12
*b*c)*x*(b*x^3+a)^(8/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(8/3)*(5*a^2*d^2+
18*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^5+1/18*(-a*d+b*c)^(8/3)*(5*a^2
*d^2+18*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/
c^(8/3)/d^5-1/18*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/d^5+1/27*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^5*3^(1/2)-1/27*(-a*d+b*c)^
(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^5*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^4x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^4*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -14/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx=\frac{\left(a^4\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=\frac{a^4x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 2.34, size = 1171, normalized size = 2.16

$$\frac{1}{108}\left(\frac{10\left(2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{ax^3+b}}\right)+\log\left(\frac{(bc-ad)^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)\right)a^5}{c^{8/3}\sqrt[3]{bc-ad}}+\frac{6b\left(2\sqrt{3}\cdots\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*x^
3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*(1 +
 (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(
d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/
```

```
3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (1
0*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(
1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]
+ Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) + (36*a*b^4
*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3)) - (72*a^2*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/
3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (30*a^3*b
^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (6*a^4*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{14}{3}}}{\left(dx^3 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{14}{3}}}{\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{14}{3}}}{\left(dx^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{14/3}}{\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(14/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(14/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

3.110 $\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} dx$

Optimal. Leaf size=458

$$\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{18c^2d^3}+\frac{(bc-ad)^{5/3}\left(5a^2d^2+12abcd+27b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^4}-\frac{(bc-ad)^{5/3}\left(5\right.}{}$$

```
[Out] 1/18*b*(-5*a^2*d^2-7*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c^2/d^3-1/6*(-a*
d+b*c)*x*(b*x^3+a)^(8/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+9*b*c)*x*(b
*x^3+a)^(5/3)/c^2/d^2/(d*x^3+c)+1/54*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d
+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^4-1/18*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*
b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/d^
4+1/6*b^(8/3)*(-11*a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^4-1/9*b^(8/3
)*(-11*a*d+9*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^4*3
^(1/2)+1/27*(-a*d+b*c)^(5/3)*(5*a^2*d^2+12*a*b*c*d+27*b^2*c^2)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^4*3^(1/2
)
```

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^3x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^3*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -11/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx = \frac{\left(a^3\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^3x\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 1.84, size = 908, normalized size = 1.98

$$\frac{1}{108}\left(\frac{10\left(2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{ax^3+b}}\right)+\log\left(\frac{(bc-ad)^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)\right)a^4}{c^{8/3}\sqrt[3]{bc-ad}}+\frac{4b\left(2\sqrt{3}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*(
b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b*x
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^3*(
a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a^4*(2
*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/S
qrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^
(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1
/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^3*c^(1/3)
```

```
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a^2*b^2*(2*Sqr
t[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[
3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3
) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (4*a^3*b*(2*Sqrt[
3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]
] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3)
+ ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)
/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

fricas [B] time = 79.23, size = 1246, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*
d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 +
 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)
*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*
d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2
)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
)/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3
 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*
(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b^3*c^5 - 11*a
*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a
*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)
/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^
6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)
*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (27
*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d
^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d -
15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2
)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(6*b^3*c^2*d^3*x^7 + (27*
b^3*c^3*d^2 - 25*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 + 5*a^3*d^5)*x^4 + 2*(9*b^3*
```

```
c^4*d - 8*a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3 + 4*a^3*c*d^4)*x)*(b*x^3 + a)^(2/
3))/(c^2*d^6*x^6 + 2*c^3*d^5*x^3 + c^4*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{11}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{11}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{11}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{11/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(11/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(11/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.111 $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^3}+\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\right.}{18c^{8/3}d^3}$$

```
[Out] -1/6*(-a*d+b*c)*x*(b*x^3+a)^(5/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+6*
b*c)*x*(b*x^3+a)^(2/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(2/3)*(5*a^2*d^2+6
*a*b*c*d+9*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^3+1/18*(-a*d+b*c)^(2/3)*(5*a^2*d^
2+6*a*b*c*d+9*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/
3)/d^3-1/2*b^(8/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3+1/3*b^(8/3)*arctan(1/
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)-1/27*(-a*d+b*c)^(2/3
)*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^3*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.16, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx = \frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 1.16, size = 651, normalized size = 1.66

$$\frac{\frac{10a^3\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{2a^2bc\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{\ldots}\right.\right.}{d\sqrt[3]{bc-a\ldots}}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*d
*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a + b
*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
 + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (6
*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
```

```
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108*c^(8/3))
```

fricas [B] time = 8.89, size = 954, normalized size = 2.44

$$-\frac{2\sqrt{3}\left(\left(9\,b^{2}c^{2}d^{2}+6\,abcd^{3}+5\,a^{2}d^{4}\right)x^{6}+9\,b^{2}c^{4}+6\,abc^{3}d+5\,a^{2}c^{2}d^{2}+2\left(9\,b^{2}c^{3}d+6\,abc^{2}d^{2}+5\,a^{2}cd^{3}\right)x^{3}\right)\left(\frac{b^{2}c^{2}}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4
 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d
^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(
b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d
*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 +
a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4
)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^
2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((
c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c -
a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*lo
g(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x^6 + 2*b^2*c
^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/
3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d^2 + 6*a*b*c*d
^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^
3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)
+ (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^
3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a*b*c*d^3 - 5*a^2*d^
4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)*x)*(b*x^3 + a)^(2/3))/
(c^2*d^5*x^6 + 2*c^3*d^4*x^3 + c^4*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{8}{3}}}{\left(dx^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}{\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{8}{3}}}{\left(dx^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{8/3}}{\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

3.112 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=217

$$\frac{5a^2\log\left(c+dx^3\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}$$

```
[Out] 1/6*x*(b*x^3+a)^(5/3)/c/(d*x^3+c)^2+5/18*a*x*(b*x^3+a)^(2/3)/c^2/(d*x^3+c)+
5/54*a^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-5/18*a^2*ln((-a*d+b*c)^(1/3)*
x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+5/27*a^2*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(
1/3)*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{54c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*x*(a + b*x^3)^(2/3))/(18*c
^2*(c + d*x^3)) + (5*a^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x
^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (5*a
^2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*c^(8/3)*(b*c
 - a*d)^(1/3)) + (5*a^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c -
 a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}\,dx = \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{(5a)\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx}{6c}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{9c^2}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^2}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}} + \frac{\left(5a^2\right)\text{Sul}}{}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)} - \frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}} + \frac{\left(5a^2\right)\text{Subst}\left(\int \frac{}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-}}\right.}{18}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)} - \frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\right.}{54c^{8/3}\sqrt[3]{bc-ad}}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2} + \frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)} + \frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}} - \frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}} + \frac{5}{}$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.36

$$\frac{ax\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}\sqrt[3]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*Hypergeometric2F1[-5/3, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(1/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

maple [**F**] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{5}{3}}}{\left(d\,x^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{5/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=267

$$\frac{a(6bc-5ad)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{4/3}}-\frac{a(6bc-5ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}+\frac{x\left(a+bx\right.}{18c^2\left(c\right.}$$

```
[Out] -1/6*d*x*(b*x^3+a)^(5/3)/c/(-a*d+b*c)/(d*x^3+c)^2+1/18*(-5*a*d+6*b*c)*x*(b*
x^3+a)^(2/3)/c^2/(-a*d+b*c)/(d*x^3+c)+1/54*a*(-5*a*d+6*b*c)*ln(d*x^3+c)/c^(
8/3)/(-a*d+b*c)^(4/3)-1/18*a*(-5*a*d+6*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(
b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)+1/27*a*(-5*a*d+6*b*c)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)
^(4/3)*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {382, 378, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a+bx^3\right)^{2/3}(6bc-5ad)}{18c^2\left(c+dx^3\right)(bc-ad)}-\frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{54c^{8/3}(bc-ad)^{4/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] -(d*x*(a + b*x^3)^(5/3))/(6*c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a*d)
*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)*(c + d*x^3)) + (a*(6*b*c - 5*a*d)
*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1
/3))])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(4/3)) - (a*(6*b*c - 5*a*d)*Log[c^(1/
3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*c^(8/3)*(b*c - a*d)^(4/3
)) + (a*(6*b*c - 5*a*d)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c -
 a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^3}\,dx = -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx}{6c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)}+\frac{(a(6bc-5ad))\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{9c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)}+\frac{(a(6bc-5ad))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,d\right.}{9c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)}+\frac{(a(6bc-5ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\right.}{27c^{8/3}(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)}-\frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}}+$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)}-\frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}}+$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2}+\frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)}+\frac{a(6bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}-$$

Mathematica [C] time = 0.22, size = 153, normalized size = 0.57

$$\frac{x\left(c\left(-a^2d\left(8c+5dx^3\right)+ab\left(6c^2-5cdx^3-5d^2x^6\right)+3b^2cx^3\left(2c+dx^3\right)\right)-2a\left(c+dx^3\right)^2(5ad-6bc)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\right.\right.}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(c*(3*b^2*c*x^3*(2*c + d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(6*c^2 - 5*c
*d*x^3 - 5*d^2*x^6)) - 2*a*(-6*b*c + 5*a*d)*(c + d*x^3)^2*Hypergeometric2F1
```

```
[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(18*c^3*(b*c - a*d)*(a +
 b*x^3)^(1/3)*(c + d*x^3)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{2}{3}}}{\left(d\,x^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

3.114 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=307

$$\frac{\left(5a^2d^2-12abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{7/3}}-\frac{\left(5a^2d^2-12abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}+\frac{\left(5a^2d^2-12abcd\cdots\right.}{9\sqrt{3}\cdots}$$

```
[Out] -1/6*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)^2-1/18*d*(-5*a*d+9*b*c)*x*(
b*x^3+a)^(2/3)/c^2/(-a*d+b*c)^2/(d*x^3+c)+1/54*(5*a^2*d^2-12*a*b*c*d+9*b^2*
c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(7/3)-1/18*(5*a^2*d^2-12*a*b*c*d+9*b^2*
c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(7/3
)+1/27*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(7/3)*3^(1/2)
```

Rubi [C] time = 0.31, antiderivative size = 167, normalized size of antiderivative = 0.54, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{x\left(cd\left(-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)+3b^2cx^3\left(4c+3dx^3\right)\right)-2\left(c+dx^3\right)^2\left(5a^2d^2-12abcd+9b^2\cdots\right.\right.}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] -(x*(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^2
 + c*d*x^3 - 5*d^2*x^6)) - 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*x^
3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(1
8*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)^3}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= -\frac{x\left(cd\left(3b^2cx^3\left(4c+3dx^3\right)-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)\right)-2\left(9b\right.\right.}{18c^3(bc-ad)^2\sqrt[3]{a+bx^3}\,\left(c+\right.}$$

Mathematica [C] time = 0.38, size = 168, normalized size = 0.55

$$\frac{x\left(2\left(c+dx^3\right)^2\left(5a^2d^2-12abcd+9b^2c^2\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-cd\left(-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x\right.\right.\right.}{18c^3\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(-(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^
2 + c*d*x^3 - 5*d^2*x^6))) + 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*
x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/
(18*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**3), x)
```

3.115 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=377

$$\frac{dx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2+15abcd+18b^2c^2\right)}{18ac^2\left(c+dx^3\right)(bc-ad)^3}-\frac{d\left(5a^2d^2-18abcd+27b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{10/3}}+\frac{d\left(5a^2d^2-18abcd+27\right.}{18c^{8/}}$$

```
[Out] -1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(1/3)/(d*x^3+c)^2+1/6*b*(a*d+6*b*c)*x/a/c/(
-a*d+b*c)^2/(b*x^3+a)^(1/3)/(d*x^3+c)+1/18*d*(-5*a^2*d^2+15*a*b*c*d+18*b^2*
c^2)*x*(b*x^3+a)^(2/3)/a/c^2/(-a*d+b*c)^3/(d*x^3+c)-1/54*d*(5*a^2*d^2-18*a*
b*c*d+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(10/3)+1/18*d*(5*a^2*d^2-1
8*a*b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3
)/(-a*d+b*c)^(10/3)-1/27*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+
2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(
10/3)*3^(1/2)
```

Rubi [C] time = 2.74, antiderivative size = 428, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{65c^2\left(a+bx^3\right)^2\left(-28\left(c+dx^3\right)^2\left(a^2\left(843c^2dx^3+500c^3+375cd^2x^6+27d^3x^9\right)+9abcx^3\left(73c^2+104cdx^3+33d^2\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -(65*c^2*(a + b*x^3)^2*(14000*a^2*c^5 + 21896*a*b*c^5*x^3 + 48104*a^2*c^4*d
*x^3 + 8391*b^2*c^5*x^6 + 70802*a*b*c^4*d*x^6 + 60807*a^2*c^3*d^2*x^6 + 244
17*b^2*c^4*d*x^9 + 81534*a*b*c^3*d^2*x^9 + 33657*a^2*c^2*d^3*x^9 + 23409*b^
2*c^3*d^2*x^12 + 38652*a*b*c^2*d^3*x^12 + 7155*a^2*c*d^4*x^12 + 7425*b^2*c^
2*d^3*x^15 + 5940*a*b*c*d^4*x^15 + 243*a^2*d^5*x^15 - 28*(c + d*x^3)^2*(27*
b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 + 33*d^2*x^
6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) - 486*(b*c - a*d)^
4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3}, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))])/(16380*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(7/3
)*(c + d*x^3)^2)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^3}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{65c^2\left(a+bx^3\right)^2\left(14000a^2c^5+21896abc^5x^3+48104a^2c^4dx^3+8391b^2c^5x^6+7\right.}{}$$

Mathematica [C] time = 3.08, size = 428, normalized size = 1.14

$$-\frac{486x^{12}\left(c+dx^3\right)^3(bc-ad)^4\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+65c^2\left(a+bx^3\right)^2\left(28\left(c+dx^3\right)^2\left(a^2\left(500c^3+843\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/16380*(65*c^2*(a + b*x^3)^2*(-14000*a^2*c^5 - 21896*a*b*c^5*x^3 - 48104*
a^2*c^4*d*x^3 - 8391*b^2*c^5*x^6 - 70802*a*b*c^4*d*x^6 - 60807*a^2*c^3*d^2*
x^6 - 24417*b^2*c^4*d*x^9 - 81534*a*b*c^3*d^2*x^9 - 33657*a^2*c^2*d^3*x^9 -
 23409*b^2*c^3*d^2*x^12 - 38652*a*b*c^2*d^3*x^12 - 7155*a^2*c*d^4*x^12 - 74
25*b^2*c^2*d^3*x^15 - 5940*a*b*c*d^4*x^15 - 243*a^2*d^5*x^15 + 28*(c + d*x^
3)^2*(27*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 +
```

```
33*d^2*x^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 486*(b*
c - a*d)^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)^3*x^8*(a + b*x^3
)^(7/3)*(c + d*x^3)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.116 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=463

$$\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)(bc-ad)^3}+\frac{d^2\left(5a^2d^2-24abcd+54b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{13/3}}-\frac{d^2\left(5a^2d^2-24abcd+54b^2c^2\right)}{18c^{8/3}(bc-}$$

```
[Out] -1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)^2+1/12*b*(2*a*d+3*b*c)*x/a/
c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)/(d*x^3+c)+1/12*b*(-2*a^2*d^2-42*a*b*c*d+9*b^
2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)/(d*x^3+c)+1/36*d*(10*a^3*d^3-42
*a^2*b*c*d^2-135*a*b^2*c^2*d+27*b^3*c^3)*x*(b*x^3+a)^(2/3)/a^2/c^2/(-a*d+b*
c)^4/(d*x^3+c)+1/54*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/
3)/(-a*d+b*c)^(13/3)-1/18*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(13/3)+1/27*d^2*(5*a
^2*d^2-24*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b
*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(13/3)*3^(1/2)
```

Rubi [C] time = 8.66, antiderivative size = 1990, normalized size of antiderivative = 4.30, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] -(522756*c^6*(b*c - a*d)^3*x^9*(a + b*x^3)^2 + 1516320*c^5*d*(b*c - a*d)^3*
x^12*(a + b*x^3)^2 + 2198664*c^4*d^2*(b*c - a*d)^3*x^15*(a + b*x^3)^2 + 141
5232*c^3*d^3*(b*c - a*d)^3*x^18*(a + b*x^3)^2 + 341172*c^2*d^4*(b*c - a*d)^
3*x^21*(a + b*x^3)^2 + 28042560*c^7*(b*c - a*d)^2*x^6*(a + b*x^3)^3 + 10760
2560*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3 + 157697280*c^5*d^2*(b*c - a*d)^
2*x^12*(a + b*x^3)^3 + 101088000*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3 +
 24261120*c^3*d^4*(b*c - a*d)^2*x^18*(a + b*x^3)^3 - 265470660*c^8*(b*c - a
*d)*x^3*(a + b*x^3)^4 - 1019636800*c^7*d*(b*c - a*d)*x^6*(a + b*x^3)^4 - 14
66086440*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4 - 930252960*c^5*d^3*(b*c - a
*d)*x^12*(a + b*x^3)^4 - 221899860*c^4*d^4*(b*c - a*d)*x^15*(a + b*x^3)^4 +
 335877360*c^9*(a + b*x^3)^5 + 1279532800*c^8*d*x^3*(a + b*x^3)^5 + 1823334
240*c^7*d^2*x^6*(a + b*x^3)^5 + 1151579520*c^6*d^3*x^9*(a + b*x^3)^5 + 2739
39120*c^5*d^4*x^12*(a + b*x^3)^5 - 67420080*c^7*(b*c - a*d)^2*x^6*(a + b*x^
```

```
3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 25
9692160*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 377700960*c^5*d^2*(b*c - a*d)^2*x^12
*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 241113600*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3*Hypergeometric2F
1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57723120*c^3*d^4*(b*c -
 a*d)^2*x^18*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 349440000*c^8*(b*c - a*d)*x^3*(a + b*x^3)^4*Hypergeomet
ric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1339520000*c^7*d*(
b*c - a*d)*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 1921920000*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1218147840*
c^5*d^3*(b*c - a*d)*x^12*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 290384640*c^4*d^4*(b*c - a*d)*x^15*(a + b*x
^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3
35877360*c^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] - 1279532800*c^8*d*x^3*(a + b*x^3)^5*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1823334240*c^7*d^2*x^6*(a +
 b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 1151579520*c^6*d^3*x^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 273939120*c^5*d^4*x^12*(a + b*x^3)^5*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57834*c^4*(b
*c - a*d)^5*x^15*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 224532*c^3*d*(b*c - a*d)^5*x^18*HypergeometricP
FQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3265
92*c^2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 210924*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 51030*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 10/3}, {1
, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5103*c^4*(b*c - a*d)^5*x^1
5*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 20412*c^3*d*(b*c - a*d)^5*x^18*HypergeometricPFQ[{2, 2,
2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 30618*c^
2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19
/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 20412*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))] - 5103*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 2, 1
0/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(524160*c^6*(b*c
 - a*d)^4*x^11*(a + b*x^3)^(10/3)*(c + d*x^3)^2)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)^3}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{522756c^6(bc-ad)^3x^9\left(a+bx^3\right)^2+1516320c^5d(bc-ad)^3x^{12}\left(a+bx^3\right)^2+21986}{}$$

Mathematica [A] time = 5.88, size = 337, normalized size = 0.73

$$\frac{1}{36}x\left(a+bx^3\right)^{2/3}\left(\frac{27b^3(bc-5ad)}{a^2\left(a+bx^3\right)(bc-ad)^4}-\frac{9b^3}{a\left(a+bx^3\right)^2(ad-bc)^3}+\frac{2d^3(5ad-21bc)}{c^2\left(c+dx^3\right)(bc-ad)^4}-\frac{6d^3}{c\left(c+dx^3\right)^2(bc-a}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*((-9*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^3)^2) + (27*b^3
*(b*c - 5*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^3)) - (6*d^3)/(c*(b*c - a*d)^3*
(c + d*x^3)^2) + (2*d^3*(-21*b*c + 5*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^3)))
)/36 + (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3)
 - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(54*c^(8/3)*(b*c - a*d)^(13/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)

[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{7/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

3.117 $\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^3} \, dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3
/a)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{\left(c+dx^3\right)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.52, size = 316, normalized size = 5.27

$$\frac{x\left(\frac{4c\left(\frac{4a^2c\left(c+dx^3\right)(10ad+bc)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+8a^2cd+5a^2d^2x^3-abc^2+10abcdx^3+5abd^2x^6-b^2c^2x^3+2b^2cdx^6 \cdots}{\left(c+dx^3\right)^2} \cdots\right)}{72c^3d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(b*(2*b*c + 5*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-(a*b*c^2) + 8*a^2*c*d - b^2*c^2*x^3 +
10*a*b*c*d*x^3 + 5*a^2*d^2*x^3 + 2*b^2*c*d*x^6 + 5*a*b*d^2*x^6 + (4*a^2*c*(
b*c + 10*a*d)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*
(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Appel
lF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(72*c
^3*d*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.118 $\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3/a
)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^3}\,dx=\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{\left(c+dx^3\right)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$=\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] time = 0.62, size = 431, normalized size = 7.31

$$\frac{\frac{c\left(16acx\left(3a^2d\left(6c+5dx^3\right)+ab\left(-18c^2-7cdx^3+5d^2x^6\right)-b^2cx^3\left(7c+4dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^4\left(a^2d\left(8c+5dx^3\right)+ab\left(-7c^2+4cdx^3+5d^2x^6\right)-b^2cx^3\left(7c+\right.\right.\right.}{\left(c+dx^3\right)^2\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx}{a}\right.\right.}}{72c^3\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (-(b*(-4*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]) + (c*(16*a*c*x*(-(b^2*c*x^3*(7*c + 4*d*x^3)) +
 3*a^2*d*(6*c + 5*d*x^3) + a*b*(-18*c^2 - 7*c*d*x^3 + 5*d^2*x^6))*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^4*(-(b^2*c*x^3*(7*c + 4
*d*x^3)) + a^2*d*(8*c + 5*d*x^3) + a*b*(-7*c^2 + 4*c*d*x^3 + 5*d^2*x^6))*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*A
ppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1
[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1
, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*c^3*(b*c - a*d)*(a + b*x^3)^(2/3
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a)
^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)^3}\,dx}{\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 0.77, size = 442, normalized size = 7.49

$$\frac{x\left(\frac{4c\left(dx^3\left(a^2d\left(8c+5dx^3\right)+ab\left(-13c^2-2cdx^3+5d^2x^6\right)-b^2cx^3\left(13c+10dx^3\right)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(3a^2d^2\left(6c+5\right.\right.\right.}{\left(c+dx^3\right)^2\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\right.\right.}\right.}{72c^3\left(a+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(5*b*d*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d^2*(6*c + 5*d*x^3) +
b^2*c*(18*c^2 + 5*c*d*x^3 - 10*d^2*x^6) + a*b*d*(-36*c^2 - 25*c*d*x^3 + 5*d
^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + d*x^3*(a^
2*d*(8*c + 5*d*x^3) - b^2*c*x^3*(13*c + 10*d*x^3) + a*b*(-13*c^2 - 2*c*d*x^
3 + 5*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c +
d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(72*c^3*(b*c - a*
d)^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,{\left(d\,x^3+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**3), x)
```

3.120 $\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,3,4/3,-b*x^3/a,-d*x^3/c)/a/c^3/(b*x^3+
a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}\left(c+dx^3\right)^3}\,dx}{a\left(a+bx^3\right)^{2/3}}$$

$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 1.10, size = 531, normalized size = 8.56

$$\frac{\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5a^2d^2-16abcd-9b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(ad-bc)^3}-\frac{4c\left(4acx\left(3a^3d^3\left(6c+5dx^3\right)+a^2bd^2\left(-54c^2-43cdx^3+5d^2x^6\right)+ab^2cd\left(54c^2+35cdx^3-16d\right.\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] ((b*d*(-9*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 - (4*c*
(4*a*c*x*(3*a^3*d^3*(6*c + 5*d*x^3) + a*b^2*c*d*(54*c^2 + 35*c*d*x^3 - 16*d
^2*x^6) - 9*b^3*c^2*(2*c^2 + 3*c*d*x^3 + d^2*x^6) + a^2*b*d^2*(-54*c^2 - 43
*c*d*x^3 + 5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)] + x^4*(9*b^3*c^2*(c + d*x^3)^2 - a^3*d^3*(8*c + 5*d*x^3) + a*b^2*c*d^2*x
^3*(19*c + 16*d*x^3) + a^2*b*d^2*(19*c^2 + 8*c*d*x^3 - 5*d^2*x^6))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)^3*(c + d*x^3)^2*(
4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*a*c^3*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{5}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{5/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.121 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,3,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^3/(b*x^
3+a)^(2/3)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}\left(c+dx^3\right)^3}\,dx}{a^2\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 1.62, size = 515, normalized size = 8.31

$$x\left(bdx^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(25a^3d^3-110a^2bcd^2-171ab^2c^2d+36b^3c^3\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(\frac{4ac\left(c+dx^3\right)\left(50a^4d^4-235a^3\cdots\right.}{4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\cdots}\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(b*d*(36*b^3*c^3 - 171*a*b^2*c^2*d - 110*a^2*b*c*d^2 + 25*a^3*d^3)*x^3*(
1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 + (4*c*((36*b^5*c^3*x^3*(c + d*x^3)^2 + 9*a*b^4*c^2*(6*c - 19*d*x^3)*(c +
d*x^3)^2 + 5*a^5*d^4*(8*c + 5*d*x^3) + 5*a^3*b^2*d^3*x^3*(-50*c^2 - 36*c*d*
x^3 + 5*d^2*x^6) + 5*a^4*b*d^3*(-25*c^2 - 6*c*d*x^3 + 10*d^2*x^6) - a^2*b^3
*c*d*(189*c^3 + 378*c^2*d*x^3 + 314*c*d^2*x^6 + 110*d^3*x^9))/(a + b*x^3) +
 (4*a*c*(36*b^4*c^4 - 171*a*b^3*c^3*d + 540*a^2*b^2*c^2*d^2 - 235*a^3*b*c*d
^3 + 50*a^4*d^4)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(3
60*a^2*c^3*(b*c - a*d)^4*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{8}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{8/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.122 $\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx$

Optimal. Leaf size=155

$$\frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}+\frac{84ax\left(a+bx^3\right)^{3/4}}{325c^2\left(c+dx^3\right)^{13/12}}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

```
[Out] 4/25*x*(b*x^3+a)^(7/4)/c/(d*x^3+c)^(25/12)+84/325*a*x*(b*x^3+a)^(3/4)/c^2/(
d*x^3+c)^(13/12)+189/325*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1
/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(1/4)/(d*x^3+c)^(
1/12)
```

Rubi [A] time = 0.06, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}+\frac{84ax\left(a+bx^3\right)^{3/4}}{325c^2\left(c+dx^3\right)^{13/12}}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (84*a*x*(a + b*x^3)^(3
/4))/(325*c^2*(c + d*x^3)^(13/12)) + (189*a^2*x*((c*(a + b*x^3))/(a*(c + d*
x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d
*x^3)))])/(325*c^3*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
```

```
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^{7/4}}{\left(c+dx^3\right)^{37/12}}\,dx &= \frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}+\frac{(21a)\int \frac{\left(a+bx^3\right)^{3/4}}{\left(c+dx^3\right)^{25/12}}\,dx}{25c}\\
&= \frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}+\frac{84ax\left(a+bx^3\right)^{3/4}}{325c^2\left(c+dx^3\right)^{13/12}}+\frac{\left(189a^2\right)\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}\,dx}{325c^2}\\
&= \frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}+\frac{84ax\left(a+bx^3\right)^{3/4}}{325c^2\left(c+dx^3\right)^{13/12}}+\frac{189a^2x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{325c^3\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 0.58

$$\frac{ax\left(a+bx^3\right)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-7/4, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*
x^3)/c)^(1/4))
```

fricas [F] time = 77.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{7}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{d^4x^{12}+4\,cd^3x^9+6\,c^2d^2x^6+4\,c^3dx^3+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(7/4)*(d*x^3 + c)^(11/12)/(d^4*x^12 + 4*c*d^3*x^9 + 6*
c^2*d^2*x^6 + 4*c^3*d*x^3 + c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{7}{4}}}{\left(dx^3 + c\right)^{\frac{37}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{7}{4}}}{\left(d\,x^3 + c\right)^{\frac{37}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

```
[Out] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{7}{4}}}{\left(dx^3 + c\right)^{\frac{37}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\right)^{7/4}}{\left(d\,x^3 + c\right)^{37/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x)
```

```
[Out] int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/4)/(d*x**3+c)**(37/12),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} \, dx$

Optimal. Leaf size=155

$$\frac{45a^2x\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{133c^3\left(a+bx^3\right)^{3/4}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

```
[Out] 4/19*x*(b*x^3+a)^(5/4)/c/(d*x^3+c)^(19/12)+60/133*a*x*(b*x^3+a)^(1/4)/c^2/(
d*x^3+c)^(7/12)+45/133*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/1
2)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(3
/4)
```

Rubi [A] time = 0.06, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{45a^2x\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{133c^3\left(a+bx^3\right)^{3/4}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (60*a*x*(a + b*x^3)^(1
/4))/(133*c^2*(c + d*x^3)^(7/12)) + (45*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(133*c^3*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
```

```
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/4}}{\left(c+dx^3\right)^{31/12}}\,dx = \frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}} + \frac{(15a)\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}}\,dx}{19c}$$

$$= \frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}} + \frac{\left(45a^2\right)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{133c^2}$$

$$= \frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}} + \frac{45a^2x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\ {}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\cdots\right.}{133c^3\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 0.58

$$\frac{ax\sqrt[4]{a+bx^3}\sqrt[4]{\frac{dx^3}{c}+1}\ {}_2F_1\left(-\frac{5}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^2\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-5/4, 1/3, 4
/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(1/4)*(c +
d*x^3)^(7/12))
```

fricas [F] time = 3.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{5}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(5/4)*(d*x^3 + c)^(5/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^
2*d*x^3 + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{5}{4}}}{\left(dx^3 + c\right)^{\frac{31}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{5}{4}}}{\left(d\,x^3 + c\right)^{\frac{31}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

```
[Out] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{5}{4}}}{\left(dx^3 + c\right)^{\frac{31}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^3+a\right)}^{5/4}}{{\left(d\,x^3+c\right)}^{31/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x)
```

```
[Out] int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)
```

```
[Out] Timed out
```

3.124 $\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx$

Optimal. Leaf size=122

$$\frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}+\frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}}$$

```
[Out] 4/13*x*(b*x^3+a)^(3/4)/c/(d*x^3+c)^(13/12)+9/13*a*x*(c*(b*x^3+a)/a/(d*x^3+c
))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c^2/(b*x^3
+a)^(1/4)/(d*x^3+c)^(1/12)
```

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}+\frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^
3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
```

```
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/4}}{\left(c+dx^3\right)^{25/12}}\,dx = \frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}} + \frac{(9a)\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}\,dx}{13c}$$

$$= \frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}} + \frac{9ax\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{13c^2\sqrt[4]{a+bx^3}\ \sqrt[12]{c+dx^3}}$$

Mathematica [A] time = 0.02, size = 89, normalized size = 0.73

$$\frac{x\left(a+bx^3\right)^{3/4}\ {}_2F_1\left(-\frac{3}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{3/4}\ \sqrt[12]{c+dx^3}\ \sqrt[4]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*x^
3)/c)^(1/4))
```

fricas [F] time = 47.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c
^2*d*x^3 + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{4}}}{\left(dx^3+c\right)^{\frac{25}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{3}{4}}}{\left(d\,x^3+c\right)^{\frac{25}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
[Out] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{4}}}{\left(dx^3+c\right)^{\frac{25}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{3/4}}{\left(d\,x^3+c\right)^{25/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x)
```

```
[Out] int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/4)/(d*x**3+c)**(25/12),x)
```

```
[Out] Timed out
```

3.125 $\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}} \, dx$

Optimal. Leaf size=122

$$\frac{3ax\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

```
[Out] 4/7*x*(b*x^3+a)^(1/4)/c/(d*x^3+c)^(7/12)+3/7*a*x*(c*(b*x^3+a)/a/(d*x^3+c))^
(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+
c))/c^2/(b*x^3+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{3ax\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/
(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}}\,dx = \frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}+\frac{(3a)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{7c}$$

$$= \frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}+\frac{3ax\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.02, size = 89, normalized size = 0.73

$$\frac{x\sqrt[4]{a+bx^3}\sqrt[4]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-1/4, 1/3, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(1 + (b*x^3)/a)^(1/4)*(c + d*x^
3)^(7/12))
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{d^2x^6+2\,cdx^3+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(d^2*x^6 + 2*c*d*x^3 + c^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{4}}}{\left(dx^3+c\right)^{\frac{19}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{4}}}{\left(d\,x^3+c\right)^{\frac{19}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
[Out] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{4}}}{\left(dx^3+c\right)^{\frac{19}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/4}}{\left(d\,x^3+c\right)^{19/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x)
```

```
[Out] int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/4)/(d*x**3+c)**(19/12),x)
```

```
[Out] Timed out
```

3.126 $\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}} dx$

Optimal. Leaf size=87

$$\frac{x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\sqrt[4]{a+bx^3}\ \sqrt[12]{c+dx^3}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x^
3/a/(d*x^3+c))/c/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {380}

$$\frac{x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\sqrt[4]{a+bx^3}\ \sqrt[12]{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3,
 -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^(1
/12))
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}} dx = \frac{x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{c\sqrt[4]{a+bx^3}\ \sqrt[12]{c+dx^3}}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.99

$$\frac{x\sqrt[4]{\frac{bx^3}{a}+1}\left(\frac{dx^3}{c}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(1 + (d*x^3)/c)^(3/4)*Hypergeometric2F1[1/4, 1/3,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(1/4)*(c + d*x^3)^
(13/12))
```

fricas [F] time = 51.16, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{bd^2x^9+\left(2\,bcd+ad^2\right)x^6+\left(bc^2+2\,acd\right)x^3+ac^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b*d^2*x^9 + (2*b*c*d + a*d^
2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{4}}\left(d\,x^3+c\right)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

```
[Out] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{1/4}\,\left(d\,x^3+c\right)^{13/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x)
```

```
[Out] int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,\left(c+dx^3\right)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/4)*(c + d*x**3)**(13/12)), x)
```

3.127 $\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$

Optimal. Leaf size=87

$$\frac{x\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\left(a+bx^3\right)^{3/4}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/
3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(3/4)
```

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {380}

$$\frac{x\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\left(a+bx^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometri
c2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(
3/4))
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx=\frac{x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\left(c+dx^3\right)^{5/12}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.02, size = 86, normalized size = 0.99

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[4]{\frac{dx^3}{c}+1}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[1/3, 3/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(3/4)*(c + d*x^3)^
(7/12))
```

fricas [F] time = 4.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{bdx^6+(bc+ad)x^3+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{3}{4}}\left(d\,x^3+c\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

```
[Out] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{3/4}\,\left(d\,x^3+c\right)^{7/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x)
```

```
[Out] int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{3}{4}}\left(c+dx^3\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

3.128 $\int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}} \, dx$

Optimal. Leaf size=87

$$\frac{x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4} {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\left(a+bx^3\right)^{5/4}}$$

```
[Out] x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4
/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(5/4)
```

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {380}

$$\frac{x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4} {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\left(a+bx^3\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometr
ic2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^
(5/4))
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{5/4} \sqrt[12]{c+dx^3}} \, dx = \frac{x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\left(c+dx^3\right)^{11/12} {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{c\left(a+bx^3\right)^{5/4}}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 1.02

$$\frac{x\sqrt[4]{\frac{bx^3}{a}+1}\,_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{a\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}\,\sqrt[4]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*Hypergeometric2F1[1/3, 5/4, 4/3, ((-(b*c) + a*d)*x
^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3)/
c)^(1/4))
```

fricas [F] time = 64.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{b^2dx^9+\left(b^2c+2\,abd\right)x^6+\left(2\,abc+a^2d\right)x^3+a^2c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^2*d*x^9 + (b^2*c + 2*a*b*
d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{4}}\left(dx^3+c\right)^{\frac{1}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{5}{4}}\left(d\,x^3+c\right)^{\frac{1}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

```
[Out] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{4}}\left(dx^3+c\right)^{\frac{1}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{5/4}\,\left(d\,x^3+c\right)^{1/12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x)
```

```
[Out] int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{5}{4}}\sqrt[12]{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(5/4)*(c + d*x**3)**(1/12)), x)
```

3.129 $\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx$

Optimal. Leaf size=121

$$\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}$$

```
[Out] 4/9*x*(d*x^3+c)^(5/12)/a/(b*x^3+a)^(3/4)+5/9*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3
/4)*(d*x^3+c)^(5/12)*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c)
)/a/(b*x^3+a)^(3/4)
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a
*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(
((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx = \frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}} + \frac{(5c)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{9a}$$

$$= \frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}} + \frac{5x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.05, size = 89, normalized size = 0.74

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{7}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a\left(a+bx^3\right)^{3/4}\left(\frac{dx^3}{c}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 7/4, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(3/4)*(1 + (d*x^3)/
c)^(3/4))
```

fricas [F] time = 2.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{\frac{5}{12}}}{\left(bx^{3}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{\frac{5}{12}}}{\left(b\,x^{3}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
[Out] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{\frac{5}{12}}}{\left(bx^{3}+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^{5/12}}{{\left(b\,x^3+a\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x)
```

```
[Out] int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^3\right)^{\frac{5}{12}}}{\left(a + bx^3\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(5/12)/(b*x**3+a)**(7/4),x)
[Out] Integral((c + d*x**3)**(5/12)/(a + b*x**3)**(7/4), x)
```

3.130 $\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}\,dx$

Optimal. Leaf size=121

$$\frac{11x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}$$

```
[Out] 4/15*x*(d*x^3+c)^(11/12)/a/(b*x^3+a)^(5/4)+11/15*x*(c*(b*x^3+a)/a/(d*x^3+c)
)^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x
^3+c))/a/(b*x^3+a)^(5/4)
```

Rubi [A] time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{11x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}\,dx = \frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}} + \frac{(11c)\int \frac{1}{\left(a+bx^3\right)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{15a}$$

$$= \frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}} + \frac{11x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\left(c+dx^3\right)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.74

$$\frac{x\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{9}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a^2\sqrt[4]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 9/4, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(1/4)*(1 + (d*x^
3)/c)^(5/4))
```

fricas [F] time = 64.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{11}{12}}}{\left(bx^3+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^{\frac{11}{12}}}{\left(b\,x^3+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
[Out] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{11}{12}}}{\left(bx^3+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^{11/12}}{\left(b\,x^3+a\right)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x)
```

```
[Out] int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(11/12)/(b*x**3+a)**(9/4),x)
```

```
[Out] Timed out
```

3.131 $\int \frac{\left(c+dx^3\right)^{17/12}}{\left(a+bx^3\right)^{11/4}} \, dx$

Optimal. Leaf size=153

$$\frac{85cx\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

```
[Out] 68/189*c*x*(d*x^3+c)^(5/12)/a^2/(b*x^3+a)^(3/4)+4/21*x*(d*x^3+c)^(17/12)/a/
(b*x^3+a)^(7/4)+85/189*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)
*hypergeom([1/3, 3/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(3/4
)
```

Rubi [A] time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{85cx\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

```
[Out] (68*c*x*(c + d*x^3)^(5/12))/(189*a^2*(a + b*x^3)^(3/4)) + (4*x*(c + d*x^3)^
(17/12))/(21*a*(a + b*x^3)^(7/4)) + (85*c*x*((c*(a + b*x^3))/(a*(c + d*x^3)
))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(189*a^2*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
```

```
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{17/12}}{\left(a+bx^3\right)^{11/4}}\,dx = \frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}} + \frac{(17c)\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx}{21a}$$

$$= \frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}} + \frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}} + \frac{\left(85c^2\right)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{189a^2}$$

$$= \frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}} + \frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}} + \frac{85cx\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc}{a(}\right.}{189a^2\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.59

$$\frac{cx\left(\frac{bx^3}{a}+1\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{11}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a^2\left(a+bx^3\right)^{3/4}\left(\frac{dx^3}{c}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

```
[Out] (c*x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 11/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(3/4)*(1 + (d*
x^3)/c)^(3/4))
```

fricas [F] time = 2.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{17}{12}}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(17/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{17}{12}}}{\left(bx^3+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^{\frac{17}{12}}}{\left(b\,x^3+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

```
[Out] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{17}{12}}}{\left(bx^3+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^{17/12}}{{\left(b\,x^3+a\right)}^{11/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x)
```

```
[Out] int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{\left(c+dx^3\right)^{23/12}}{\left(a+bx^3\right)^{13/4}}\,dx$

Optimal. Leaf size=153

$$\frac{253cx\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

```
[Out] 92/405*c*x*(d*x^3+c)^(11/12)/a^2/(b*x^3+a)^(5/4)+4/27*x*(d*x^3+c)^(23/12)/a
/(b*x^3+a)^(9/4)+253/405*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/
12)*hypergeom([1/3, 5/4],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(
5/4)
```

Rubi [A] time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{253cx\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

```
[Out] (92*c*x*(c + d*x^3)^(11/12))/(405*a^2*(a + b*x^3)^(5/4)) + (4*x*(c + d*x^3)
^(23/12))/(27*a*(a + b*x^3)^(9/4)) + (253*c*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c - a
*d)*x^3)/(a*(c + d*x^3)))])/(405*a^2*(a + b*x^3)^(5/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
```

```
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{23/12}}{\left(a+bx^3\right)^{13/4}}\,dx = \frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}+\frac{(23c)\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}\,dx}{27a}$$

$$= \frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}+\frac{\left(253c^2\right)\int \frac{1}{\left(a+bx^3\right)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{405a^2}$$

$$= \frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}+\frac{253cx\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\left(c+dx^3\right)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\right.}{405a^2\left(a+bx^3\right)^{5/4}}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 0.59

$$\frac{cx\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{13}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a^3\sqrt[4]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

```
[Out] (c*x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 13/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^3*(a + b*x^3)^(1/4)*(1 + (d
*x^3)/c)^(5/4))
```

fricas [F] time = 66.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{23}{12}}}{b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(23/12)/(b^4*x^12 + 4*a*b^3*x^9 + 6*
a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{23}{12}}}{\left(bx^3+a\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^{\frac{23}{12}}}{\left(b\,x^3+a\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

```
[Out] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{23}{12}}}{\left(bx^3+a\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^{23/12}}{{\left(b\,x^3+a\right)}^{13/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x)
```

```
[Out] int((c + d*x^3)^(23/12)/(a + b*x^3)^(13/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(23/12)/(b*x**3+a)**(13/4),x)
```

```
[Out] Timed out
```

3.133 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx$

Optimal. Leaf size=79

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

```
[Out] x*(b*x^3+a)^m*(d*x^3+c)^p*AppellF1(1/3,-m,-p,4/3,-b*x^3/a,-d*x^3/c)/((1+b*x
^3/a)^m)/((1+d*x^3/c)^p)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((
d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p\,dx = \left(\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m \left(c+dx^3\right)^p\,dx$$
$$= \left(\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\left(c+dx^3\right)^p\left(1+\frac{dx^3}{c}\right)^{-p}\right)\int \left(1+\frac{bx^3}{a}\right)^m \left(1+\frac{dx^3}{c}\right)^p\,dx$$
$$= x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\left(c+dx^3\right)^p\left(1+\frac{dx^3}{c}\right)^{-p} F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

Mathematica [B] time = 0.43, size = 172, normalized size = 2.18

$$\frac{4acx\left(a+bx^3\right)^m\left(c+dx^3\right)^p F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3x^3\left(bcmF_1\left(\frac{4}{3};1-m,-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adpF_1\left(\frac{4}{3};-m,1-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a
), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)
/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^m\left(dx^3+c\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^m\left(dx^3+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^p,x)
[Out] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(b x^{3} + a\right)}^{m} {\left(d x^{3} + c\right)}^{p}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^m\,{\left(d\,x^3+c\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3)^p,x)
[Out] int((a + b*x^3)^m*(c + d*x^3)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
[Out] Timed out
```

3.134 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx$

Optimal. Leaf size=167

$$\frac{x\left(c+dx^3\right)^{q+1}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right){}_2F_1\left(1,q+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{c}\right)}{cd^2(3q+4)(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}(4bc-a}{d^2(3q+4)(3q+}$$

```
[Out] -b*(4*b*c-a*d*(10+3*q))*x*(d*x^3+c)^(1+q)/d^2/(9*q^2+33*q+28)+b*x*(b*x^3+a)
*(d*x^3+c)^(1+q)/d/(7+3*q)+(4*b^2*c^2-2*a*b*c*d*(7+3*q)+a^2*d^2*(9*q^2+33*q
+28))*x*(d*x^3+c)^(1+q)*hypergeom([1, 4/3+q],[4/3],-d*x^3/c)/c/d^2/(9*q^2+3
3*q+28)
```

Rubi [A] time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {416, 388, 246, 245}

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right){}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)}{d^2(3q+4)(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}(}{d^2(3q+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*q)*(7 + 3*
q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) + ((4*b^2*c^2 -
2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^q*Hypergeo
metric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(d^2*(4 + 3*q)*(7 + 3*q)*(1 + (d*x^3
)/c)^q)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^q\,dx = \frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\int \left(c+dx^3\right)^q\left(-a(bc-ad(7+3q))-b(4bc-ad(10+3\ldots}{d(7+3q)}$$

$$= -\frac{b(4bc-ad(10+3q))x\left(c+dx^3\right)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\left(4b^2c^2-2abcd\ldots}{}$$

$$= -\frac{b(4bc-ad(10+3q))x\left(c+dx^3\right)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\left(\left(4b^2c^2-2abc\ldots}{}$$

$$= -\frac{b(4bc-ad(10+3q))x\left(c+dx^3\right)^{1+q}}{d^2(4+3q)(7+3q)} + \frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)} + \frac{\left(4b^2c^2-2abcd\ldots}{}$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.63

$$\frac{1}{14}x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(14a^2\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+bx^3\left(7a\,{}_2F_1\left(\frac{4}{3},-q;\frac{7}{3};-\frac{dx^3}{c}\right)+2bx^3\,{}_2F_1\left(\frac{7}{3},-q;\frac{10}{3};-\frac{dx\ldots}{c\ldots}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(14*a^2*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)] + b*
x^3*(7*a*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)] + 2*b*x^3*Hypergeome
tric2F1[7/3, -q, 10/3, -((d*x^3)/c)])))/(14*(1 + (d*x^3)/c)^q)
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)\left(dx^3 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x^3 + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^2 \left(dx^3 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \left(b\,x^3 + a\right)^2 \left(d\,x^3 + c\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^q,x)
[Out] int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^2 \left(dx^3 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3 + a\right)^2 \left(d\,x^3 + c\right)^q \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^q,x)
```

```
[Out] int((a + b*x^3)^2*(c + d*x^3)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**q,x)
```

```
[Out] Timed out
```

3.135 $\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx$

Optimal. Leaf size=84

$$\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}-\frac{x\left(c+dx^3\right)^{q+1}(bc-ad(3q+4))\,{}_2F_1\left(1,q+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{c}\right)}{cd(3q+4)}$$

```
[Out] b*x*(d*x^3+c)^(1+q)/d/(4+3*q)-(b*c-a*d*(4+3*q))*x*(d*x^3+c)^(1+q)*hypergeom
([1, 4/3+q],[4/3],-d*x^3/c)/c/d/(4+3*q)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right){}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) + ((a - (b*c)/(4*d + 3*d*q))*x*(c +
 d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(1 + (d*x^3)/c)^q
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx = \frac{bx\left(c+dx^3\right)^{1+q}}{d(4+3q)} - \left(-a+\frac{bc}{4d+3dq}\right)\int \left(c+dx^3\right)^q\,dx$$
$$= \frac{bx\left(c+dx^3\right)^{1+q}}{d(4+3q)} - \left(\left(-a+\frac{bc}{4d+3dq}\right)\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \left(1+\frac{dx^3}{c}\right)^q\,dx$$
$$= \frac{bx\left(c+dx^3\right)^{1+q}}{d(4+3q)} + \left(a-\frac{bc}{4d+3dq}\right)x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)$$

Mathematica [A] time = 0.04, size = 90, normalized size = 1.07

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left((ad(3q+4)-bc)\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+b\left(c+dx^3\right)\left(\frac{dx^3}{c}+1\right)^q\right)}{d(3q+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(b*(c + d*x^3)*(1 + (d*x^3)/c)^q + (-(b*c) + a*d*(4 + 3*q)
)*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)]))/(d*(4 + 3*q)*(1 + (d*x^3)
/c)^q)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)\left(dx^3+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)*(d*x^3 + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right) \left(d\,x^{3}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^q,x)
```

```
[Out] int((b*x^3+a)*(d*x^3+c)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{3}+a\right)\left(dx^{3}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^{3}+a\right)\,\left(d\,x^{3}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^q,x)
```

```
[Out] int((a + b*x^3)*(c + d*x^3)^q, x)
```

sympy [C] time = 82.86, size = 75, normalized size = 0.89

$$\frac{ac^{q}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},-q\\\frac{4}{3}\end{matrix}\right|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{bc^{q}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{4}{3},-q\\\frac{7}{3}\end{matrix}\right|\frac{dx^{3}e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**q,x)
```

```
[Out] a*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(3*g
amma(4/3)) + b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*exp_pol
ar(I*pi)/c)/(3*gamma(7/3))
```

3.136 $\int \frac{(c+dx^3)^q}{a+bx^3}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

```
[Out] x*(d*x^3+c)^q*AppellF1(1/3,1,-q,4/3,-b*x^3/a,-d*x^3/c)/a/((1+d*x^3/c)^q)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*
(1 + (d*x^3)/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^q}{a+bx^3}\,dx = \left(\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^q}{a+bx^3}\,dx$$
$$= \frac{x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

Mathematica [B] time = 0.20, size = 162, normalized size = 2.84

$$\frac{4acx\left(c+dx^3\right)^q F_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(adqF_1\left(\frac{4}{3};1-q,1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bcF_1\left(\frac{4}{3};-q,2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+4acF_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
])/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)] -
 b*c*AppellF1[4/3, -q, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^3+c\right)^q}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^q}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{q}}{b\,x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a),x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{q}}{bx^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^{3}+c\right)^{q}}{b\,x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^q/(a + b*x^3),x)
```

```
[Out] int((c + d*x^3)^q/(a + b*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.137 $\int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c}+1\right)^{-q} F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

```
[Out] x*(d*x^3+c)^q*AppellF1(1/3,2,-q,4/3,-b*x^3/a,-d*x^3/c)/a^2/((1+d*x^3/c)^q)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c}+1\right)^{-q} F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^
2*(1 + (d*x^3)/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^q}{\left(a+bx^3\right)^2}\,dx = \left(\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^q}{\left(a+bx^3\right)^2}\,dx$$

$$= \frac{x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Mathematica [B] time = 0.26, size = 162, normalized size = 2.84

$$\frac{4acx\left(c+dx^3\right)^q F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^2 \Big(3x^3 \Big(adqF_1\left(\frac{4}{3};2,1-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bcF_1\left(\frac{4}{3};3,-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\Big)+4acF_1\Big(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 - 2*b*c*AppellF1[4/3, 3, -q, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^3+c\right)^q}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^q}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{3}+c\right)^{q}}{\left(b\,x^{3}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a)^2,x)
[Out] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{q}}{\left(bx^{3}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^{3}+c\right)^{q}}{\left(b\,x^{3}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^q/(a + b*x^3)^2,x)
[Out] int((c + d*x^3)^q/(a + b*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
[Out] Timed out
```

3.138 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3\,dx$

Optimal. Leaf size=298

$$\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b^3(3m+4)(3m+7)(3m+10)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-12a^2\right.}{}$$

```
[Out] d*(28*a^2*d^2-a*b*c*d*(92+15*m)+b^2*c^2*(9*m^2+60*m+118))*x*(b*x^3+a)^(1+m)
/b^3/(10+3*m)/(9*m^2+33*m+28)-d*(7*a*d-b*c*(16+3*m))*x*(b*x^3+a)^(1+m)*(d*x
^3+c)/b^2/(9*m^2+51*m+70)+d*x*(b*x^3+a)^(1+m)*(d*x^3+c)^2/b/(10+3*m)-(28*a^
3*d^3-12*a^2*b*c*d^2*(10+3*m)+3*a*b^2*c^2*d*(9*m^2+51*m+70)-b^3*c^3*(27*m^3
+189*m^2+414*m+280))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^3/
(10+3*m)/(9*m^2+33*m+28)/((1+b*x^3/a)^m)
```

Rubi [A] time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {416, 528, 388, 246, 245}

$$-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(-12a^2bcd^2(3m+10)+28a^3d^3+3ab^2c^2d\left(9m^2+51m+70\right)-b^3c^3\left(27m^3+189m^2+\right.\right.}{b^3(3m+4)(3m+7)(3m+10)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a +
 b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16
 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d
*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^
2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 4
14*m + 189*m^2 + 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -
((b*x^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
```

```
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)^2}{b(10+3m)} + \frac{\int \left(a+bx^3\right)^m \left(c+dx^3\right)\left(-c(ad-bc(10+3m))-d(7\right.}{b(10+3m)}$$

$$= -\frac{d(7ad-bc(16+3m))x\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b^2(7+3m)(10+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)^2}{b(10+3m)} + \int$$

$$= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x\left(a+bx^3\right)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-}{}$$

$$= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x\left(a+bx^3\right)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-}{}$$

$$= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x\left(a+bx^3\right)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-}{}$$

Mathematica [A] time = 5.08, size = 137, normalized size = 0.46

$$\frac{1}{140}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(140c^3\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(105c^2\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\left(30c\,{}_2F_1\left(\frac{7}{3}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d
*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*(30*c
*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)] + 7*d*x^3*Hypergeometric2F1
[10/3, -m, 13/3, -((b*x^3)/a)]))))/(140*(1 + (b*x^3)/a)^m)
```

fricas [F] time = 1.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3\right)\left(bx^3+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3+c\right)^3\left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \left(d\,x^{3}+c\right)^{3} \left(b\,x^{3}+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(dx^{3}+c\right)}^{3} {\left(bx^{3}+a\right)}^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^3+a\right)}^m\,{\left(d\,x^3+c\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^m*(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
[Out] Timed out
```

3.139 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=176

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right){}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)}-\frac{dx\left(a+bx^3\right)^{m}}{b^2(3m}$$

[Out] -d*(4*a*d-b*c*(10+3*m))*x*(b*x^3+a)^(1+m)/b^2/(9*m^2+33*m+28)+d*x*(b*x^3+a)^(1+m)*(d*x^3+c)/b/(7+3*m)+(4*a^2*d^2-2*a*b*c*d*(7+3*m)+b^2*c^2*(9*m^2+33*m+28))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^2/(9*m^2+33*m+28)/((1+b*x^3/a)^m)

Rubi [A] time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {416, 388, 246, 245}

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right){}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)}-\frac{dx\left(a+bx^3\right)^{m}}{b^2(3m}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)^m*(c + d*x^3)^2,x]

[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*m)*(7 + 3*m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) + ((4*a^2*d^2 - 2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3)/a)^m)

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\int \left(a+bx^3\right)^m \left(-c(ad-bc(7+3m))-d(4ad-bc(10\right.}{b(7+3m)}$$

$$= -\frac{d(4ad-bc(10+3m))x\left(a+bx^3\right)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(4a^2d^2-2\right.}{}$$

$$= -\frac{d(4ad-bc(10+3m))x\left(a+bx^3\right)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(\left(4a^2d^2-\right.\right.}{}$$

$$= -\frac{d(4ad-bc(10+3m))x\left(a+bx^3\right)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(4a^2d^2-2\right.}{}$$

Mathematica [A] time = 5.04, size = 106, normalized size = 0.60

$$\frac{1}{14}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(14c^2\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(7c\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d*
x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*Hypergeome
tric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(1 + (b*x^3)/a)^m)
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^6 + 2\,cdx^3 + c^2\right)\left(bx^3 + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="fricas")
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3 + c\right)^2 \left(bx^3 + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(d\,x^3 + c\right)^2 \left(b\,x^3 + a\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3 + c\right)^2 \left(bx^3 + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3 + a\right)^m \left(d\,x^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^m*(c + d*x^3)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.140 $\int \left(a+bx^3\right)^m \left(c+dx^3\right) dx$

Optimal. Leaf size=93

$$\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}(ad-bc(3m+4))\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b(3m+4)}$$

```
[Out] d*x*(b*x^3+a)^(1+m)/b/(4+3*m)-(a*d-b*c*(4+3*m))*x*(b*x^3+a)^m*hypergeom([1/
3, -m],[4/3],-b*x^3/a)/b/(4+3*m)/((1+b*x^3/a)^m)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right){}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*m))*x*(a +
 b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^m
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right) dx = \frac{dx\left(a+bx^3\right)^{1+m}}{b(4+3m)} - \left(-c+\frac{ad}{4b+3bm}\right)\int \left(a+bx^3\right)^m dx$$
$$= \frac{dx\left(a+bx^3\right)^{1+m}}{b(4+3m)} - \left(\left(-c+\frac{ad}{4b+3bm}\right)\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m dx$$
$$= \frac{dx\left(a+bx^3\right)^{1+m}}{b(4+3m)} + \left(c-\frac{ad}{4b+3bm}\right)x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.97

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left((bc(3m+4)-ad)\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^m\right)}{b(3m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*(d*(a + b*x^3)*(1 + (b*x^3)/a)^m + (-(a*d) + b*c*(4 + 3*m)
)*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*m)*(1 + (b*x^3)
/a)^m)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3+c\right)\left(bx^3+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^3+c\right)\left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="giac")
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \left(d\,x^{3}+c\right) \left(b\,x^{3}+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c),x)
[Out] int((b*x^3+a)^m*(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^{3}+c\right)\left(bx^{3}+a\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^{3}+a\right)^{m}\,\left(d\,x^{3}+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m*(c + d*x^3),x)
[Out] int((a + b*x^3)^m*(c + d*x^3), x)
```

sympy [C] time = 100.31, size = 75, normalized size = 0.81

$$\frac{a^{m}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},-m\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{m}dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{4}{3},-m\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c),x)
[Out] a**m*c*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(4/3)) + a**m*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3))
```

3.141 $\int \left(a+bx^3\right)^m \, dx$

Optimal. Leaf size=44

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

```
[Out] x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/((1+b*x^3/a)^m)
```

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m,x]
```

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3
)/a)^m
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^m \, dx &= \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m \, dx\\ &= x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [C] time = 0.20, size = 196, normalized size = 4.45

$$\frac{2^{-m}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-m}\left(\frac{i\left(\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-m}\left(a+bx^3\right)^m F_1\left(m+1;-m,-m;m+2;-\frac{i\left(\sqrt[3]{b}\,x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},\right.}{\sqrt[3]{b}\,(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^m*AppellF1[1 + m, -m, -m, 2 +
 m, ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3]
 - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^m*b^(1/3)*(1 + m)*((a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^m*((I*(1 + (b^(1/3)
*x)/a^(1/3)))/(3*I + Sqrt[3]))^m)
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^3+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m,x)
```

```
[Out] int((b*x^3+a)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m, x)
```

mupad [B] time = 1.35, size = 41, normalized size = 0.93

$$\frac{x\left(b\,x^3+a\right)^m\,{}_2\mathrm{F}_1\left(\frac{1}{3},-m;\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m,x)
[Out] (x*(a + b*x^3)^m*hypergeom([1/3, -m], 4/3, -(b*x^3)/a))/((b*x^3)/a + 1)^m
```

sympy [C] time = 15.56, size = 34, normalized size = 0.77

$$\frac{a^m x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m,x)
[Out] a**m*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

3.142 $\int \frac{(a+bx^3)^m}{c+dx^3}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,1,4/3,-b*x^3/a,-d*x^3/c)/c/((1+b*x^3/a)^m)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*
(1 + (b*x^3)/a)^m)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^m}{c+dx^3}\,dx = \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^m}{c+dx^3}\,dx$$
$$= \frac{x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

Mathematica [B] time = 0.25, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(3x^3\left(adF_1\left(\frac{4}{3};-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-bcmF_1\left(\frac{4}{3};1-m,1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^m}{dx^3+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^m}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{m}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c),x)
[Out] int((b*x^3+a)^m/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{m}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^{3}+a\right)^{m}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3),x)
[Out] int((a + b*x^3)^m/(c + d*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c),x)
[Out] Timed out
```

3.143 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/((1+b*x^3/a)^m)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
2*(1 + (b*x^3)/a)^m)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^m}{\left(c+dx^3\right)^2}\,dx=\left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^m}{\left(c+dx^3\right)^2}\,dx$$
$$=\frac{x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

Mathematica [B] time = 0.26, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)^2\left(-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2adF_1\left(\frac{4}{3};-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^m}{d^2x^6+2\,cdx^3+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

maple [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{m}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{m}}{\left(dx^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^{3}+a\right)^{m}}{\left(d\,x^{3}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^m/(c + d*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
[Out] Timed out
```

3.144 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

```
[Out] x*(b*x^3+a)^m*AppellF1(1/3,-m,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/((1+b*x^3/a)^m)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
3*(1 + (b*x^3)/a)^m)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^m}{\left(c+dx^3\right)^3}\,dx = \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^m}{\left(c+dx^3\right)^3}\,dx$$
$$= \frac{x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

Mathematica [B] time = 0.41, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)^3\left(-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-m,4;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{b}{}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^m}{d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^m}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^m}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^m/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.145 $\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

```
[Out] x*(d*x^3+c)^(a*d/(-3*a*d+3*b*c))/a/c/((b*x^3+a)^(b*c/(-3*a*d+3*b*c)))
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {381}

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*c
 - 3*a*d)),x]
```

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3*
a*d)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.98

$$\frac{x\left(a+bx^3\right)^{\frac{bc}{3ad-3bc}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/
(3*b*c - 3*a*d)),x]
```

```
[Out] (x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d))
)/(a*c)
```

fricas [A] time = 1.32, size = 91, normalized size = 1.72

$$\frac{bdx^7 + (bc + ad)x^4 + acx}{\left(bx^3 + a\right)^{\frac{4\,bc-3\,ad}{3\,(bc-ad)}}\left(dx^3 + c\right)^{\frac{3\,bc-4\,ad}{3\,(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c
- a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3 + c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

maple [A] time = 0.04, size = 71, normalized size = 1.34

$$\frac{x\left(b\,x^3 + a\right)^{1-\frac{3ad-4bc}{3(ad-bc)}}\left(d\,x^3 + c\right)^{1-\frac{4ad-3bc}{3(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x)
```

```
[Out] (b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*b*c)/(a
*d-b*c))/a/c*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

mupad [B] time = 1.90, size = 131, normalized size = 2.47

$$\frac{x\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}+\frac{x^4\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}\,(a\,d+b\,c)}{a\,c}+\frac{b\,d\,x^7\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}}{a\,c}}{\left(d\,x^3+c\right)^{\frac{a\,d}{3\,a\,d-3\,b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1)/(c + d*x^3)^((a*d)/(3*a*d - 3*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1) + (x^4*(a + b*x^3)^((b*c)/(3*a*d
 - 3*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^7*(a + b*x^3)^((b*c)/(3*a*d - 3*
b*c) - 1))/(a*c))/(c + d*x^3)^((a*d)/(3*a*d - 3*b*c) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+3
*b*c)),x)
```

```
[Out] Timed out
```

3.146 $\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx$

Optimal. Leaf size=94

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

```
[Out] a*c^4*x+1/5*c^3*(4*a*d+b*c)*x^5+2/9*c^2*d*(3*a*d+2*b*c)*x^9+2/13*c*d^2*(2*a
*d+3*b*c)*x^13+1/17*d^3*(a*d+4*b*c)*x^17+1/21*b*d^4*x^21
```

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{5}c^3x^5(4ad+bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx &= \int \left(ac^4+c^3(bc+4ad)x^4+2c^2d(2bc+3ad)x^8+2cd^2(3bc+2ad)x^{12}+d^3(4bc+ad)x\right.\\ &= ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+\end{aligned}$$

Mathematica [A] time = 0.02, size = 94, normalized size = 1.00

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

fricas [A] time = 0.90, size = 98, normalized size = 1.04

$$\frac{1}{21}x^{21}d^4b+\frac{4}{17}x^{17}d^3cb+\frac{1}{17}x^{17}d^4a+\frac{6}{13}x^{13}d^2c^2b+\frac{4}{13}x^{13}d^3ca+\frac{4}{9}x^9dc^3b+\frac{2}{3}x^9d^2c^2a+\frac{1}{5}x^5c^4b+\frac{4}{5}x^5dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^4*b + 4/17*x^17*d^3*c*b + 1/17*x^17*d^4*a + 6/13*x^13*d^2*c^2*b
 + 4/13*x^13*d^3*c*a + 4/9*x^9*d*c^3*b + 2/3*x^9*d^2*c^2*a + 1/5*x^5*c^4*b
+ 4/5*x^5*d*c^3*a + x*c^4*a
```

giac [A] time = 0.16, size = 98, normalized size = 1.04

$$\frac{1}{21}\,bd^4x^{21}+\frac{4}{17}\,bcd^3x^{17}+\frac{1}{17}\,ad^4x^{17}+\frac{6}{13}\,bc^2d^2x^{13}+\frac{4}{13}\,acd^3x^{13}+\frac{4}{9}\,bc^3dx^9+\frac{2}{3}\,ac^2d^2x^9+\frac{1}{5}\,bc^4x^5+\frac{4}{5}\,ac^3dx^5+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^13
 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5
+ 4/5*a*c^3*d*x^5 + a*c^4*x
```

maple [A] time = 0.04, size = 97, normalized size = 1.03

$$\frac{b\,d^4x^{21}}{21}+\frac{\left(a\,d^4+4bc\,d^3\right)x^{17}}{17}+\frac{\left(4ac\,d^3+6c^2d^2b\right)x^{13}}{13}+\frac{\left(6a\,c^2d^2+4c^3db\right)x^9}{9}+a\,c^4x+\frac{\left(4a\,c^3d+b\,c^4\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^4,x)
```

```
[Out] 1/21*b*d^4*x^21+1/17*(a*d^4+4*b*c*d^3)*x^17+1/13*(4*a*c*d^3+6*b*c^2*d^2)*x^
13+1/9*(6*a*c^2*d^2+4*b*c^3*d)*x^9+1/5*(4*a*c^3*d+b*c^4)*x^5+a*c^4*x
```

maxima [A] time = 0.65, size = 96, normalized size = 1.02

$$\frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\left(4\,bcd^3+ad^4\right)x^{17}+\frac{2}{13}\left(3\,bc^2d^2+2\,acd^3\right)x^{13}+\frac{2}{9}\left(2\,bc^3d+3\,ac^2d^2\right)x^9+ac^4x+\frac{1}{5}\left(bc^4+4\,ac^3d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

mupad [B] time = 1.30, size = 88, normalized size = 0.94

$$x^5\left(\frac{b\,c^4}{5}+\frac{4\,a\,d\,c^3}{5}\right)+x^{17}\left(\frac{a\,d^4}{17}+\frac{4\,b\,c\,d^3}{17}\right)+\frac{b\,d^4\,x^{21}}{21}+a\,c^4\,x+\frac{2\,c^2\,d\,x^9\,\left(3\,a\,d+2\,b\,c\right)}{9}+\frac{2\,c\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^4,x)
```

```
[Out] x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^17*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*d
^4*x^21)/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^13*(2*
a*d + 3*b*c))/13
```

sympy [A] time = 0.09, size = 107, normalized size = 1.14

$$ac^4x+\frac{bd^4x^{21}}{21}+x^{17}\left(\frac{ad^4}{17}+\frac{4bcd^3}{17}\right)+x^{13}\left(\frac{4acd^3}{13}+\frac{6bc^2d^2}{13}\right)+x^9\left(\frac{2ac^2d^2}{3}+\frac{4bc^3d}{9}\right)+x^5\left(\frac{4ac^3d}{5}+\frac{bc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*a
*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) + x*
*5*(4*a*c**3*d/5 + b*c**4/5)
```

3.147 $\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

```
[Out] a*c^3*x+1/5*c^2*(3*a*d+b*c)*x^5+1/3*c*d*(a*d+b*c)*x^9+1/13*d^2*(a*d+3*b*c)*
x^13+1/17*b*d^3*x^17
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^4+3cd(bc+ad)x^8+d^2(3bc+ad)x^{12}+bd^3x^{16}\right)dx\\ &= ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

fricas [A] time = 0.96, size = 74, normalized size = 1.06

$$\frac{1}{17}x^{17}d^3b + \frac{3}{13}x^{13}d^2cb + \frac{1}{13}x^{13}d^3a + \frac{1}{3}x^9dc^2b + \frac{1}{3}x^9d^2ca + \frac{1}{5}x^5c^3b + \frac{3}{5}x^5dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^3*b + 3/13*x^13*d^2*c*b + 1/13*x^13*d^3*a + 1/3*x^9*d*c^2*b + 1
/3*x^9*d^2*c*a + 1/5*x^5*c^3*b + 3/5*x^5*d*c^2*a + x*c^3*a
```

giac [A] time = 0.15, size = 74, normalized size = 1.06

$$\frac{1}{17}\,bd^3x^{17} + \frac{3}{13}\,bcd^2x^{13} + \frac{1}{13}\,ad^3x^{13} + \frac{1}{3}\,bc^2dx^9 + \frac{1}{3}\,acd^2x^9 + \frac{1}{5}\,bc^3x^5 + \frac{3}{5}\,ac^2dx^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 + 1
/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

maple [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{b\,d^3x^{17}}{17} + \frac{\left(a\,d^3 + 3bc\,d^2\right)x^{13}}{13} + \frac{\left(3ac\,d^2 + 3b\,c^2d\right)x^9}{9} + a\,c^3x + \frac{\left(3a\,c^2d + b\,c^3\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^3,x)
```

```
[Out] 1/17*b*d^3*x^17+1/13*(a*d^3+3*b*c*d^2)*x^13+1/9*(3*a*c*d^2+3*b*c^2*d)*x^9+1
/5*(3*a*c^2*d+b*c^3)*x^5+a*c^3*x
```

maxima [A] time = 0.65, size = 70, normalized size = 1.00

$$\frac{1}{17}\,bd^3x^{17} + \frac{1}{13}\left(3\,bcd^2 + ad^3\right)x^{13} + \frac{1}{3}\left(bc^2d + acd^2\right)x^9 + \frac{1}{5}\left(bc^3 + 3\,ac^2d\right)x^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x
^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

mupad [B] time = 1.24, size = 66, normalized size = 0.94

$$x^5\left(\frac{b\,c^3}{5}+\frac{3\,a\,d\,c^2}{5}\right)+x^{13}\left(\frac{a\,d^3}{13}+\frac{3\,b\,c\,d^2}{13}\right)+\frac{b\,d^3\,x^{17}}{17}+a\,c^3\,x+\frac{c\,d\,x^9\,\left(a\,d+b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^3,x)
```

```
[Out] x^5*((b*c^3)/5 + (3*a*c^2*d)/5) + x^13*((a*d^3)/13 + (3*b*c*d^2)/13) + (b*d
^3*x^17)/17 + a*c^3*x + (c*d*x^9*(a*d + b*c))/3
```

sympy [A] time = 0.08, size = 76, normalized size = 1.09

$$ac^3x+\frac{bd^3x^{17}}{17}+x^{13}\left(\frac{ad^3}{13}+\frac{3bcd^2}{13}\right)+x^9\left(\frac{acd^2}{3}+\frac{bc^2d}{3}\right)+x^5\left(\frac{3ac^2d}{5}+\frac{bc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c*
d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

3.148 $\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

```
[Out] a*c^2*x+1/5*c*(2*a*d+b*c)*x^5+1/9*d*(a*d+2*b*c)*x^9+1/13*b*d^2*x^13
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^4+d(2bc+ad)x^8+bd^2x^{12}\right)dx\\ &= ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/13

fricas [A] time = 0.77, size = 50, normalized size = 1.00

$$\frac{1}{13}x^{13}d^2b + \frac{2}{9}x^9dcb + \frac{1}{9}x^9d^2a + \frac{1}{5}x^5c^2b + \frac{2}{5}x^5dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b + 2/9*x^9*d*c*b + 1/9*x^9*d^2*a + 1/5*x^5*c^2*b + 2/5*x^5*d*c*a + x*c^2*a
```

giac [A] time = 0.16, size = 50, normalized size = 1.00

$$\frac{1}{13}bd^2x^{13} + \frac{2}{9}bcdx^9 + \frac{1}{9}ad^2x^9 + \frac{1}{5}bc^2x^5 + \frac{2}{5}acdx^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*d*x^5 + a*c^2*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b\,d^2x^{13}}{13} + \frac{\left(a\,d^2 + 2bcd\right)x^9}{9} + \frac{\left(2acd + b\,c^2\right)x^5}{5} + a\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^2,x)
```

```
[Out] 1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

maxima [A] time = 0.60, size = 48, normalized size = 0.96

$$\frac{1}{13}bd^2x^{13} + \frac{1}{9}\left(2\,bcd + ad^2\right)x^9 + \frac{1}{5}\left(bc^2 + 2\,acd\right)x^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a*c^2*x
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^5\left(\frac{b\,c^2}{5}+\frac{2\,a\,d\,c}{5}\right)+x^9\left(\frac{a\,d^2}{9}+\frac{2\,b\,c\,d}{9}\right)+\frac{b\,d^2\,x^{13}}{13}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^2,x)
```

```
[Out] x^5*((b*c^2)/5 + (2*a*c*d)/5) + x^9*((a*d^2)/9 + (2*b*c*d)/9) + (b*d^2*x^13
)/13 + a*c^2*x
```

sympy [A] time = 0.08, size = 53, normalized size = 1.06

$$ac^2x+\frac{bd^2x^{13}}{13}+x^9\left(\frac{ad^2}{9}+\frac{2bcd}{9}\right)+x^5\left(\frac{2acd}{5}+\frac{bc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
+ b*c**2/5)
```

3.149 $\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

```
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx &= \int \left(ac+(bc+ad)x^4+bdx^8\right)\,dx\\ &= acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

fricas [A] time = 0.84, size = 26, normalized size = 0.93

$$\frac{1}{9}x^9db + \frac{1}{5}x^5cb + \frac{1}{5}x^5da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
[Out] 1/9*x^9*d*b + 1/5*x^5*c*b + 1/5*x^5*d*a + x*c*a
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{9}bdx^9 + \frac{1}{5}bcx^5 + \frac{1}{5}adx^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
[Out] 1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bd\,x^9}{9} + \frac{(ad + bc)\,x^5}{5} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c),x)
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

maxima [A] time = 0.49, size = 24, normalized size = 0.86

$$\frac{1}{9}bdx^9 + \frac{1}{5}(bc + ad)x^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^9}{9} + \left(\frac{a\,d}{5} + \frac{b\,c}{5}\right)\,x^5 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4),x)
```

```
[Out] x^5*((a*d)/5 + (b*c)/5) + a*c*x + (b*d*x^9)/9
```

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$acx + \frac{bdx^9}{9} + x^5\left(\frac{ad}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c),x)
```

```
[Out] a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

3.150 $\int \frac{a+bx^4}{c+dx^4}\,dx$

Optimal. Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{2\sqrt{2}\,c^{3/4}d^{5/4}}$$

```
[Out] b*x/d-1/4*(-a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*2
^(1/2)-1/4*(-a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)*2
^(1/2)+1/8*(-a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^
(3/4)/d^(5/4)*2^(1/2)-1/8*(-a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x
^2*d^(1/2))/c^(3/4)/d^(5/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{2\sqrt{2}\,c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*
c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2
*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^4}\,dx}{d}$$
$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d} - \frac{(bc-ad)\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d}$$
$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{3/2}} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{3/2}} + \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}\,c^{3/4}d^{5/4}}$$
$$= \frac{bx}{d} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}} - $$
$$= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\right.}{4\sqrt{2}\,c^{3/4}d^{5/4}}$$

Mathematica [A] time = 0.18, size = 196, normalized size = 0.88

$$\frac{\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+2\sqrt{2}(bc-ad)\,\mathrm{t}}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
 + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*
x^2])/(8*c^(3/4)*d^(5/4))
```

fricas [B] time = 1.32, size = 639, normalized size = 2.87

$$4\,d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{c^2d^4x\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{3}{4}}-c^2d^4\sqrt{\frac{c^2d^2\sqrt{-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c}{c^3d}}}{b^2}}}{b^3c^3-3\,ab^2c^2d+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(c^3*d^5))^(1/4)*arctan((c^2*d^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4) - c^2*d^4*sqrt((
c^2*d^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a
*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)) + d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x)
- d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)/(c^3*d^5))^(1/4)*log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

giac [A] time = 0.16, size = 245, normalized size = 1.10

$$\frac{bx}{d}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc-\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc-\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^3)
^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*
d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(
c/d))/(c*d^2)
```

maple [A] time = 0.05, size = 266, normalized size = 1.19

$$\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8c}+\frac{bx}{d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c),x)
```

```
[Out] b/d*x+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-1/4/d*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+1/8*(c/d)^(1/4)/c*2^(1/2
)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/
d)^(1/2)))*a-1/8/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b+1/4*(c/d)^(1/4)/c*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-1/4/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x+1)*b
```

maxima [A] time = 1.28, size = 212, normalized size = 0.95

$$\frac{bx}{d}-\frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(bc-ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(b\cdots}{\cdots}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2
)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^
(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*
c - a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/d
```

mupad [B] time = 1.48, size = 720, normalized size = 3.23

$$\frac{b\,x}{d}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}}\right)(a\cdots}{2(-c)^{3/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4),x)
```

```
[Out] (b*x)/d - (atan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d
^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-
c)^(3/4)*d^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^
```

```
2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4
*(-c)^(3/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4))))*(a*d - b*c)*1i)/(2*(-c)^(3/4)*d^(5/4)) - (atan((((x*
(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3)*(a*d
 - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)) + (
(x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a*c*d^3)*(
a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))
/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3
)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(
5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a
*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/
4)*d^(5/4))))*(a*d - b*c))/(2*(-c)^(3/4)*d^(5/4))
```

sympy [A] time = 0.66, size = 87, normalized size = 0.39

$$\frac{bx}{d}+\operatorname{RootSum}\left(256t^4c^3d^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{4tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/(a
*d - b*c) + x)))
```

3.151 $\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1\right.}{8\sqrt{2}\,c^{7/4}d^{5}}$$

```
[Out] -1/4*(-a*d+b*c)*x/c/d/(d*x^4+c)+1/16*(3*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2
)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)+1/16*(3*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1
/2)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)-1/32*(3*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x
*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)+1/32*(3*a*d+b*c)*ln(c
^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1\right.}{8\sqrt{2}\,c^{7/4}d^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c + 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4
)*d^(5/4)) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{1}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d} + \frac{(bc+3ad)\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} - \frac{(bc+3ad)}{16}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}} + \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\right.}{16\sqrt{2}\,c^{7/4}d^{5/4}}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} - \frac{(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}} + \frac{(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}} - \frac{(bc+3ad}{}$$

Mathematica [A] time = 0.20, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}\,x(bc-ad)}{c+dx^4} - \sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right) + \sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x\right.}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

fricas [B] time = 1.19, size = 711, normalized size = 2.90

$$\frac{4\left(cd^2x^4+c^2d\right)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{c^5d^4x\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{3}{4}}-c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^
2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*arctan(-(c^5*d^4*x*(-(b^
4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)
/(c^7*d^5))^(3/4) - c^5*d^4*sqrt((c^4*d^2*sqrt(-(b^4*c^4 + 12*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5)) + (b^2*c^2 +
 6*a*b*c*d + 9*a^2*d^2)*x^2)/(b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2))*(-(b^4*c^4
+ 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*
d^5))^(3/4))/(b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)) + (c*
d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3
*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3
*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) +
(b*c + 3*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*d*(-
(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d
^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4 + c^2
*d)
```

giac [A] time = 0.17, size = 266, normalized size = 1.09

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} bc + 3\left(cd^3\right)^{\frac{1}{4}} ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} bc + 3\left(cd^3\right)^{\frac{1}{4}} ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)^(
1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32*sqrt
(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/
4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

maple [A] time = 0.05, size = 295, normalized size = 1.20

$$\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32c^2} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*(a*d-b*c)/d/c*x/(d*x^4+c)+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x+1)*a+1/16/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
+1)*b+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a+1/16/c
/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+3/32/c^2*(c/d)^(1/
4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1
/2)*x+(c/d)^(1/2)))*a+1/32/c/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1
/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b
```

maxima [A] time = 1.16, size = 236, normalized size = 0.96

$$-\frac{(bc-ad)x}{4\left(cd^2x^4+c^2d\right)}+\frac{\frac{2\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(bc+3ad)\log\left(\sqrt{d}x^2+\sqrt{\cdots}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*c - a*d)*x/(c*d^2*x^4 + c^2*d) + 1/32*(2*sqrt(2)*(b*c + 3*a*d)*arct
an(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 3*a*d)*arctan(1/2*sqr
t(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*c + 3*a*d)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 3*a*d)*log(
sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d)
```

mupad [B] time = 1.52, size = 740, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}}\right)(3ad\cdots}{8(-c)^{7/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)
*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c
)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((
3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)
*1i)/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/
(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(
3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b
*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^
(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4))))*(3*a*d + b*c)*1i)/(8*(-c)^
(7/4)*d^(5/4)) + (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2)
 - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*
d + b*c))/(16*(-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d
^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(
5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d
 + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-
c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*
d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*
d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4))
))*(3*a*d + b*c))/(8*(-c)^(7/4)*d^(5/4)) + (x*(a*d - b*c))/(4*c*d*(c + d*x^
4))
```

sympy [A] time = 0.85, size = 112, normalized size = 0.46

$$\frac{x\,(ad-bc)}{4c^2d+4cd^2x^4}+\mathrm{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16\ldots}{3a\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**3*
d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

3.152 $\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx$

Optimal. Leaf size=273

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-}}{64\sqrt{2}\,c^{1}}$$

```
[Out] -1/8*(-a*d+b*c)*x/c/d/(d*x^4+c)^2+1/32*(7*a*d+b*c)*x/c^2/d/(d*x^4+c)+3/128*
(7*a*d+b*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)+3
/128*(7*a*d+b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/
2)-3/256*(7*a*d+b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(
11/4)/d^(5/4)*2^(1/2)+3/256*(7*a*d+b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2
)+x^2*d^(1/2))/c^(11/4)/d^(5/4)*2^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-}}{64\sqrt{2}\,c^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(8*c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^2*d*(c + d
*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt
[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sq
rt[2]*c^(11/4)*d^(5/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
```

```
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx = -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)\int \frac{1}{\left(c+dx^4\right)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{1}{c+dx^4}\,dx}{32c^2d}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{64c^{5/2}d} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}}{64c^{5/2}d}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{3/2}} + \frac{(3(bc+7ad))\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}+}}{128c^{5/2}d^{3}}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} - \frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{5/4}} + \frac{3(bc+7}{}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} - \frac{3(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{64\sqrt{2}\,c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\tan^{-1}\left(1\right.}{64\sqrt{2}\,c^{11/4}d^{5}}$$

Mathematica [A] time = 0.22, size = 243, normalized size = 0.89

$$\frac{-\frac{32c^{7/4}\sqrt[4]{d}\,x(bc-ad)}{\left(c+dx^4\right)^2} + \frac{8c^{3/4}\sqrt[4]{d}\,x(7ad+bc)}{c+dx^4} - 3\sqrt{2}\,(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right) + 3\sqrt{2}\,(7ad+bc)\log\left(\sqrt{2}\right.}{256c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] ((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b*
c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(
```

```
1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

fricas [B] time = 1.28, size = 787, normalized size = 2.88

$$\frac{4\left(bcd+7\,ad^{2}\right)x^{5}+12\left(c^{2}d^{3}x^{8}+2\,c^{3}d^{2}x^{4}+c^{4}d\right)\left(-\frac{b^{4}c^{4}+28\,ab^{3}c^{3}d+294\,a^{2}b^{2}c^{2}d^{2}+1372\,a^{3}bcd^{3}+2401\,a^{4}d^{4}}{c^{11}d^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{c^{8}d^{4}x(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 12*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(
-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*
a^4*d^4)/(c^11*d^5))^(1/4)*arctan(-(c^8*d^4*x*(-(b^4*c^4 + 28*a*b^3*c^3*d +
 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(3/4) -
 c^8*d^4*sqrt((c^6*d^2*sqrt(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^
2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5)) + (b^2*c^2 + 14*a*b*c*d +
49*a^2*d^2)*x^2)/(b^2*c^2 + 14*a*b*c*d + 49*a^2*d^2))*(-(b^4*c^4 + 28*a*b^3
*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))
^(3/4))/(b^3*c^3 + 21*a*b^2*c^2*d + 147*a^2*b*c*d^2 + 343*a^3*d^3)) + 3*(c^
2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^
2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3*d*
(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401
*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 3*(c^2*d^3*x^8 + 2*c^3*d
^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^
3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(-3*c^3*d*(-(b^4*c^4 + 28*a*
b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^
5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2
*c^3*d^2*x^4 + c^4*d)
```

giac [A] time = 0.19, size = 286, normalized size = 1.05

$$\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{2}}+\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right.}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{128\,c^{3}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^3)
^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)
^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3/256*
sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*c^2*x +
 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

maple [A] time = 0.05, size = 314, normalized size = 1.15

$$\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{128c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{128c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{256c^{3}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+21/128/c^3
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a+3/128/c^2/d*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+21/256/c^3*(c/d)^(1/4)*2^(1/2
)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2)))*a+3/256/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b+21/128/c^3*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a+3/128/c^2/d*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b
```

maxima [A] time = 1.21, size = 271, normalized size = 0.99

$$\frac{\left(bcd+7\,ad^{2}\right)x^{5}-\left(3\,bc^{2}-11\,acd\right)x}{32\left(c^{2}d^{3}x^{8}+2\,c^{3}d^{2}x^{4}+c^{4}d\right)}+\frac{3\left(\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\cdots\right)}{256\,c^{2}d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*
d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqr
t(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt
(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - s
qrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d
))) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

mupad [B] time = 1.58, size = 762, normalized size = 2.79

$$\frac{\frac{x^5\,(7\,a\,d+b\,c)}{32\,c^2}+\frac{x\,(11\,a\,d-3\,b\,c)}{32\,c\,d}}{c^2+2\,c\,d\,x^4+d^2\,x^8}-\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3\mathrm{i}}{128\,(-c)^{11/4}\,d^{5/4}}+\frac{\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d+\cdots}{256\cdots}\right.}{128\,(-c)^{11/4}\,d^{5\cdots}}}{\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^3+b\,c\,d^2\right)}{256\,(-c)^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,(-c)^{11/4}\,d^{5/4}}-\frac{3\left(\frac{9\,x\,\left(49\,a^2\,d^3+14\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{256\,c^4}+\frac{9\,(7\,a\,d\cdots}{25\cdots}\right.}{128\,(-c)^{11/4}\,d^{5\cdots}}}\right.\cdots}{64\,(-c)^{11/4}\,d^{5/4}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4)^3,x)
```

```
[Out] ((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*x
^8 + 2*c*d*x^4) - (atan(((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(2
56*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*
(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*c^2*
d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(
-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3*((9*x
*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*c)*(7*a
*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)
*d^(5/4)) - (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + (
9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4)*d^(5/4))
- (3*atan(((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((
7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c
*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*
d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x*(49*a^2*d^3 + b
^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i
)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) -
(((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*
(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(
-c)^(11/4)*d^(5/4))))*(7*a*d + b*c))/(64*(-c)^(11/4)*d^(5/4))
```

sympy [A] time = 1.03, size = 151, normalized size = 0.55

$$\frac{x^5\left(7ad^2+bcd\right)+x\left(11acd-3bc^2\right)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8}+\text{RootSum}\left(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d*
*2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 194481*
a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b**3*
c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c) + x
)))
```

3.153 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx$

Optimal. Leaf size=154

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5$$

```
[Out] a^2*c^4*x+2/5*a*c^3*(2*a*d+b*c)*x^5+1/9*c^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*x
^9+4/13*c*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^13+1/17*d^2*(a^2*d^2+8*a*b*c*d+6*
b^2*c^2)*x^17+2/21*b*d^3*(a*d+2*b*c)*x^21+1/25*b^2*d^4*x^25
```

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^4\,dx &= \int \left(a^2c^4+2ac^3(bc+2ad)x^4+c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^8+4cd\left(b^2c^2+3abcd\right.\right.\\ &= a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9+\frac{4}{13}cd\left(b^2c^2+3abcd\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 154, normalized size = 1.00

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

fricas [A] time = 0.61, size = 173, normalized size = 1.12

$$\frac{1}{25}x^{25}d^4b^2+\frac{4}{21}x^{21}d^3cb^2+\frac{2}{21}x^{21}d^4ba+\frac{6}{17}x^{17}d^2c^2b^2+\frac{8}{17}x^{17}d^3cba+\frac{1}{17}x^{17}d^4a^2+\frac{4}{13}x^{13}dc^3b^2+\frac{12}{13}x^{13}d^2c^2ba+\frac{4}{13}x^{13}d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*d^4*b^2 + 4/21*x^21*d^3*c*b^2 + 2/21*x^21*d^4*b*a + 6/17*x^17*d^2
*c^2*b^2 + 8/17*x^17*d^3*c*b*a + 1/17*x^17*d^4*a^2 + 4/13*x^13*d*c^3*b^2 +
12/13*x^13*d^2*c^2*b*a + 4/13*x^13*d^3*c*a^2 + 1/9*x^9*c^4*b^2 + 8/9*x^9*d*
c^3*b*a + 2/3*x^9*d^2*c^2*a^2 + 2/5*x^5*c^4*b*a + 4/5*x^5*d*c^3*a^2 + x*c^4
*a^2
```

giac [A] time = 0.15, size = 173, normalized size = 1.12

$$\frac{1}{25}\,b^2d^4x^{25}+\frac{4}{21}\,b^2cd^3x^{21}+\frac{2}{21}\,abd^4x^{21}+\frac{6}{17}\,b^2c^2d^2x^{17}+\frac{8}{17}\,abcd^3x^{17}+\frac{1}{17}\,a^2d^4x^{17}+\frac{4}{13}\,b^2c^3dx^{13}+\frac{12}{13}\,abc^2d^2x^{13}+\frac{4}{13}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2*
d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13 +
12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b*c^
3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a^2*c
^4*x
```

maple [A] time = 0.04, size = 163, normalized size = 1.06

$$\frac{b^2d^4x^{25}}{25}+\frac{\left(2ab\,d^4+4b^2c\,d^3\right)x^{21}}{21}+\frac{\left(a^2d^4+8abc\,d^3+6b^2c^2d^2\right)x^{17}}{17}+\frac{\left(4a^2c\,d^3+12ab\,c^2d^2+4b^2c^3d\right)x^{13}}{13}+\frac{\left(6a^2c^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^4,x)
```

```
[Out] 1/25*b^2*d^4*x^25+1/21*(2*a*b*d^4+4*b^2*c*d^3)*x^21+1/17*(a^2*d^4+8*a*b*c*d
^3+6*b^2*c^2*d^2)*x^17+1/13*(4*a^2*c*d^3+12*a*b*c^2*d^2+4*b^2*c^3*d)*x^13+1
/9*(6*a^2*c^2*d^2+8*a*b*c^3*d+b^2*c^4)*x^9+1/5*(4*a^2*c^3*d+2*a*b*c^4)*x^5+
a^2*c^4*x
```

maxima [A] time = 0.55, size = 158, normalized size = 1.03

$$\frac{1}{25}\,b^2d^4x^{25}+\frac{2}{21}\left(2\,b^2cd^3+abd^4\right)x^{21}+\frac{1}{17}\left(6\,b^2c^2d^2+8\,abcd^3+a^2d^4\right)x^{17}+\frac{4}{13}\left(b^2c^3d+3\,abc^2d^2+a^2cd^3\right)x^{13}+\frac{1}{9}\,(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

mupad [B] time = 0.07, size = 146, normalized size = 0.95

$$x^9\left(\frac{2\,a^2\,c^2\,d^2}{3}+\frac{8\,a\,b\,c^3\,d}{9}+\frac{b^2\,c^4}{9}\right)+x^{17}\left(\frac{a^2\,d^4}{17}+\frac{8\,a\,b\,c\,d^3}{17}+\frac{6\,b^2\,c^2\,d^2}{17}\right)+a^2\,c^4\,x+\frac{b^2\,d^4\,x^{25}}{25}+\frac{2\,a\,c^3\,x^5\,\left(2\,a\,d+b\,c\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^4,x)
```

```
[Out] x^9*((b^2*c^4)/9 + (2*a^2*c^2*d^2)/3 + (8*a*b*c^3*d)/9) + x^17*((a^2*d^4)/1
7 + (6*b^2*c^2*d^2)/17 + (8*a*b*c*d^3)/17) + a^2*c^4*x + (b^2*d^4*x^25)/25
+ (2*a*c^3*x^5*(2*a*d + b*c))/5 + (2*b*d^3*x^21*(a*d + 2*b*c))/21 + (4*c*d*
x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13
```

sympy [A] time = 0.12, size = 185, normalized size = 1.20

$$a^2c^4x+\frac{b^2d^4x^{25}}{25}+x^{21}\left(\frac{2abd^4}{21}+\frac{4b^2cd^3}{21}\right)+x^{17}\left(\frac{a^2d^4}{17}+\frac{8abcd^3}{17}+\frac{6b^2c^2d^2}{17}\right)+x^{13}\left(\frac{4a^2cd^3}{13}+\frac{12abc^2d^2}{13}+\frac{4b^2c^3d}{13}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21)
 + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(4*
a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*c**
2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*b*c*
*4/5)
```

3.154 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}$$

```
[Out] a^2*c^3*x+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^9+1/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^13+1/17*b*d^2*(2*a*d+3*b*c)*x^17+
1/21*b^2*d^3*x^21
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3\,dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^4+c\left(b^2c^2+6abcd+3a^2d^2\right)x^8+d\left(3b^2c^2+6abcd+a^2\right.\right.\\ &= a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{13}d\left(3b^2c^2+6abcd\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

fricas [A] time = 0.73, size = 132, normalized size = 1.08

$$\frac{1}{21}x^{21}d^3b^2+\frac{3}{17}x^{17}d^2cb^2+\frac{2}{17}x^{17}d^3ba+\frac{3}{13}x^{13}dc^2b^2+\frac{6}{13}x^{13}d^2cba+\frac{1}{13}x^{13}d^3a^2+\frac{1}{9}x^9c^3b^2+\frac{2}{3}x^9dc^2ba+\frac{1}{3}x^9d^2ca^2+\frac{2}{5}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^3*b^2 + 3/17*x^17*d^2*c*b^2 + 2/17*x^17*d^3*b*a + 3/13*x^13*d*c
^2*b^2 + 6/13*x^13*d^2*c*b*a + 1/13*x^13*d^3*a^2 + 1/9*x^9*c^3*b^2 + 2/3*x^
9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/5*x^5*c^3*b*a + 3/5*x^5*d*c^2*a^2 + x*c
^3*a^2
```

giac [A] time = 0.15, size = 132, normalized size = 1.08

$$\frac{1}{21}\,b^2d^3x^{21}+\frac{3}{17}\,b^2cd^2x^{17}+\frac{2}{17}\,abd^3x^{17}+\frac{3}{13}\,b^2c^2dx^{13}+\frac{6}{13}\,abcd^2x^{13}+\frac{1}{13}\,a^2d^3x^{13}+\frac{1}{9}\,b^2c^3x^9+\frac{2}{3}\,abc^2dx^9+\frac{1}{3}\,a^2cd^2x^9+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2*
d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + a^2
*c^3*x
```

maple [A] time = 0.04, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{21}}{21}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{17}}{17}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{13}}{13}+\frac{\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^9}{9}+a^2c^3x+\frac{\left(3a^2c^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^3,x)
```

```
[Out] 1/21*b^2*d^3*x^21+1/17*(2*a*b*d^3+3*b^2*c*d^2)*x^17+1/13*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^13+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/5*(3*a^2*c
^2*d+2*a*b*c^3)*x^5+a^2*c^3*x
```

maxima [A] time = 0.54, size = 124, normalized size = 1.02

$$\frac{1}{21}\,b^2d^3x^{21}+\frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17}+\frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13}+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+a^2c^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

mupad [B] time = 1.30, size = 116, normalized size = 0.95

$$x^9\left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right)+x^{13}\left(\frac{a^2\,d^3}{13}+\frac{6\,a\,b\,c\,d^2}{13}+\frac{3\,b^2\,c^2\,d}{13}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{21}}{21}+\frac{a\,c^2\,x^5\,\left(3\,a\,d+2\,b\,c\right)}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^3,x)
```

```
[Out] x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^13*((a^2*d^3)/13 +
(3*b^2*c^2*d)/13 + (6*a*b*c*d^2)/13) + a^2*c^3*x + (b^2*d^3*x^21)/21 + (a*c
^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^17*(2*a*d + 3*b*c))/17
```

sympy [A] time = 0.10, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{21}}{21}+x^{17}\left(\frac{2abd^3}{17}+\frac{3b^2cd^2}{17}\right)+x^{13}\left(\frac{a^2d^3}{13}+\frac{6abcd^2}{13}+\frac{3b^2c^2d}{13}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^5\left(\frac{3a^2c^2}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17)
 + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2*c
*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c**
3/5)
```

3.155 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

```
[Out] a^2*c^2*x+2/5*a*c*(a*d+b*c)*x^5+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+2/13*b*
d*(a*d+b*c)*x^13+1/17*b^2*d^2*x^17
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^2\,dx &= \int \left(a^2c^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{12}+b^2d^2x^{1}\right.\\ &= a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{13}bd(bc+ad)x^{13}+\frac{1}{17}b^{2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.00

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

fricas [A] time = 0.99, size = 91, normalized size = 1.11

$$\frac{1}{17}x^{17}d^2b^2+\frac{2}{13}x^{13}dcb^2+\frac{2}{13}x^{13}d^2ba+\frac{1}{9}x^9c^2b^2+\frac{4}{9}x^9dcba+\frac{1}{9}x^9d^2a^2+\frac{2}{5}x^5c^2ba+\frac{2}{5}x^5dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^2*b^2 + 2/13*x^13*d*c*b^2 + 2/13*x^13*d^2*b*a + 1/9*x^9*c^2*b^2
 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/5*x^5*d*c*a^2 +
x*c^2*a^2
```

giac [A] time = 0.17, size = 91, normalized size = 1.11

$$\frac{1}{17}\,b^2d^2x^{17}+\frac{2}{13}\,b^2cdx^{13}+\frac{2}{13}\,abd^2x^{13}+\frac{1}{9}\,b^2c^2x^9+\frac{4}{9}\,abcdx^9+\frac{1}{9}\,a^2d^2x^9+\frac{2}{5}\,abc^2x^5+\frac{2}{5}\,a^2cdx^5+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 +
a^2*c^2*x
```

maple [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{b^2d^2x^{17}}{17}+\frac{\left(2ab\,d^2+2b^2cd\right)x^{13}}{13}+\frac{\left(a^2d^2+4abcd+b^2c^2\right)x^9}{9}+a^2c^2x+\frac{\left(2a^2cd+2ab\,c^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^2,x)
```

```
[Out] 1/17*b^2*d^2*x^17+1/13*(2*a*b*d^2+2*b^2*c*d)*x^13+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+a^2*c^2*x
```

maxima [A] time = 0.69, size = 82, normalized size = 1.00

$$\frac{1}{17}\,b^2d^2x^{17}+\frac{2}{13}\left(b^2cd+abd^2\right)x^{13}+\frac{1}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^9+\frac{2}{5}\left(abc^2+a^2cd\right)x^5+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

mupad [B] time = 0.05, size = 75, normalized size = 0.91

$$x^9\left(\frac{a^2\,d^2}{9}+\frac{4\,a\,b\,c\,d}{9}+\frac{b^2\,c^2}{9}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^{17}}{17}+\frac{2\,a\,c\,x^5\,\left(a\,d+b\,c\right)}{5}+\frac{2\,b\,d\,x^{13}\,\left(a\,d+b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^2,x)
```

```
[Out] x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + a^2*c^2*x + (b^2*d^2*x^17
)/17 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^13*(a*d + b*c))/13
```

sympy [A] time = 0.09, size = 97, normalized size = 1.18

$$a^2c^2x+\frac{b^2d^2x^{17}}{17}+x^{13}\left(\frac{2abd^2}{13}+\frac{2b^2cd}{13}\right)+x^9\left(\frac{a^2d^2}{9}+\frac{4abcd}{9}+\frac{b^2c^2}{9}\right)+x^5\left(\frac{2a^2cd}{5}+\frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a*b
*c**2/5)
```

3.156 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right) dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

```
[Out] a^2*c*x+1/5*a*(a*d+2*b*c)*x^5+1/9*b*(2*a*d+b*c)*x^9+1/13*b^2*d*x^13
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2 \left(c+dx^4\right) dx &= \int \left(a^2c+a(2bc+ad)x^4+b(bc+2ad)x^8+b^2dx^{12}\right) dx\\ &= a^2cx+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{13}b^2dx^{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

fricas [A] time = 0.81, size = 50, normalized size = 1.00

$$\frac{1}{13}x^{13}db^2 + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9dba + \frac{2}{5}x^5cba + \frac{1}{5}x^5da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/5*x^5*c*b*a + 1/5*x^5*d
*a^2 + x*c*a^2
```

giac [A] time = 0.15, size = 50, normalized size = 1.00

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abdx^9 + \frac{2}{5}abcx^5 + \frac{1}{5}a^2dx^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*d
*x^5 + a^2*c*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b^2d\,x^{13}}{13} + \frac{\left(2abd + b^2c\right)x^9}{9} + \frac{\left(a^2d + 2abc\right)x^5}{5} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c),x)
```

```
[Out] 1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x
```

maxima [A] time = 0.48, size = 48, normalized size = 0.96

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}\left(b^2c + 2\,abd\right)x^9 + \frac{1}{5}\left(2\,abc + a^2d\right)x^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a
^2*c*x
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^5\left(\frac{d\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,d\,b}{9}\right)+\frac{b^2\,d\,x^{13}}{13}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4),x)
```

```
[Out] x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (b^2*d*x^13
)/13 + a^2*c*x
```

sympy [A] time = 0.08, size = 53, normalized size = 1.06

$$a^2cx+\frac{b^2dx^{13}}{13}+x^9\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5 +
 2*a*b*c/5)
```

3.157 $\int \frac{(a+bx^4)^2}{c+dx^4} \, dx$

Optimal. Leaf size=253

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2 \tan^{-1}\left(1\right.}{2\sqrt{2}\,c^{3/4}d^{9/4}}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/5*b^2*x^5/d+1/4*(-a*d+b*c)^2*arctan(-1+d^(1/4)*x*2^
(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+d^(1/4)*x*
2^(1/2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)+1/8*(-a*d+b*c)^2*
ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2 \tan^{-1}\left(1\right.}{2\sqrt{2}\,c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]
*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{c+dx^4}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^4\right)}\right)dx$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^4}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d^2}+\frac{(bc-ad)^2\int\frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{5/2}}+\frac{(bc-ad)^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{5/2}}-\frac{(bc\ldots}{\ldots}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\ldots\right.}{4\sqrt{2}\,c\ldots}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(b\ldots}{\ldots}$$

Mathematica [A] time = 0.13, size = 231, normalized size = 0.91

$$\frac{-40bc^{3/4}\sqrt[4]{d}\,x(bc-2ad)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x\ldots\right.}{40c^{3/4}d\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(b*
c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*d)^
2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b*c -
 a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^(3/4)
*d^(9/4))
```

fricas [B] time = 1.13, size = 1239, normalized size = 4.90

$$\frac{4\,b^2dx^5+20\,d^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{c^3d^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{c^2d^7x\left(-\frac{b^8c^8}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^2*d*x^5 + 20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan(-(c^2*d^7*x*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
*d^8)/(c^3*d^9))^(3/4) - c^2*d^7*sqrt((c^2*d^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9)) + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c^3*d^9))^(3/4))/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)) + 5*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 20*(b^2*c - 2*a*b
*d)*x)/d^2
```

giac [A] time = 0.20, size = 353, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\,\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd\right.\right.}{4\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d
^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
 - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3) + 1/5*(b
^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

maple [B] time = 0.05, size = 436, normalized size = 1.72

$$\frac{b^2x^5}{5d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8c}+\frac{2abx}{d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*c*x+1/8*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)
^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a^2-
1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(
c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a*b+1/8/d^2*(c/d)^(1/4)*c*2^(1/2)*ln((x^
2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)
))*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2-1/2/d*
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b+1/4/d^2*(c/d)^(1/4)
*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x+1)*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x+1)*b^2
```

maxima [A] time = 1.29, size = 286, normalized size = 1.13

$$\frac{b^2dx^5-5\left(b^2c-2\,abd\right)x}{5\,d^2}+\frac{\frac{2\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^2*d*x^5 - 5*(b^2*c - 2*a*b*d)*x)/d^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^2
```

mupad [B] time = 1.48, size = 1081, normalized size = 4.27

$$\frac{b^2\,x^5}{5\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{3/4}\,d^{9/4}}+\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4 \ldots\right.}{}\right.}{}}{\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(4\,a^2\,c\,d^3-8\,a\,b\,c^2\,d^2+4\,b^2\,c^3\,d\right)}{4\,{\left(-c\right)}^{3/4}\,d^{9/4}}\right)}{{\left(-c\right)}^{3/4}\,d^{9/4}}-\frac{{\left(a\,d-b\,c\right)}^2\,\left(\frac{x\,\left(a^4 \ldots\right.}{}\right.}{}}\right.}{2\,{\left(-c\right)}^{3/4}\,d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4),x)
```

```
[Out] (b^2*x^5)/(5*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d -
 ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*
d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4
*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c
)^(3/4)*d^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2
*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^
2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
c)^(3/4)*d^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3
 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8
*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)))/(((a*d - b
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/
(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
```

```
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a
*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d
^(9/4)))*1i)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2)/(2*(-c)^(3/4)*d^(9/4))
```

sympy [A] time = 1.12, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d}+x\left(\frac{2ab}{d}-\frac{b^2c}{d^2}\right)+\operatorname{RootSum}\left(256t^4c^3d^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] b**2*x**5/(5*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(256*_t**4*c**3*d**9 +
 a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d
**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d
**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*d**2
 - 2*a*b*c*d + b**2*c**2) + x)))
```

3.158 $\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^2} \, dx$

Optimal. Leaf size=291

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}+$$

```
[Out] b^2*x/d^2+1/4*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*
arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)-1/16*(-a*d+b*c
)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)
+1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^
(1/2))/c^(7/4)/d^(9/4)*2^(1/2)-1/32*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^(1/4)*d^(
1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/
4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b
*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^2}\,dx = \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{d^2\left(c+dx^4\right)^2}\right)dx$$

$$= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{\left(c+dx^4\right)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{c+dx^4}\,dx}{4cd^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d^2}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}+\sqrt{}}{c+d}}{8c^{3/2}d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{5/2}}-\frac{((bc-ad)(5bc+3ad))\int}{16c^{3/2}d^{5/2}}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(5b}{}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad)\tan^{-1}}{8\sqrt{2}\,c^{7/4}d^{9/4}}$$

Mathematica [A] time = 0.22, size = 298, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{c^{7/4}}-\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{c^{7/4}}+\frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b\right.}{c^{7}}}{32d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[2
]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (S
qrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

fricas [B] time = 1.27, size = 1335, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c*d*x^5 + 4*(c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*
c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(
c^7*d^9))^(1/4)*arctan((c^5*d^7*x*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a
^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c
^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/
4) - c^5*d^7*sqrt((c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*
b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*
b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)
*x^2)/(25*b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
9*a^4*d^4))*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c
^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4))/(125*b^6*c^6 - 150
*a*b^5*c^5*d - 165*a^2*b^4*c^4*d^2 + 172*a^3*b^3*c^3*d^3 + 99*a^4*b^2*c^2*d
^4 - 54*a^5*b*c*d^5 - 27*a^6*d^6)) + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 -
 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^
4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 8
1*a^8*d^8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d -
 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5
*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9
))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (c*d^3*x^4 + c^2*d^2)*(
-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d
^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 10
00*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c
^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a
^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) + 4*(5*b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*x^4 + c^2*d^2)
```

giac [A] time = 0.17, size = 376, normalized size = 1.29

$$\frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(cd^3\right)^{\frac{1}{4}}abcd - 3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3} - \frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(cd^3\right)^{\frac{1}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^
(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 -
 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((d*x^4 + c)*c*d^2)
```

maple [B] time = 0.06, size = 475, normalized size = 1.63

$$\frac{a^2x}{4\left(d\,x^4+c\right)c} - \frac{abx}{2\left(d\,x^4+c\right)d} + \frac{b^2cx}{4\left(d\,x^4+c\right)d^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{16c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{16c^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] b^2/d^2*x+1/4/c*x/(d*x^4+c)*a^2-1/2/d*x/(d*x^4+c)*a*b+1/4/d^2*c*x/(d*x^4+c)
*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2+1/8/d
/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b-5/16/d^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+3/32/c^2*(c/d)^(1/4)*2^(1/
2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c
/d)^(1/2)))*a^2+1/16/d/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a*b-5/32/d^2*(c/d)^(1
/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(
1/2)*x+(c/d)^(1/2)))*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x+1)*a^2+1/8/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*
a*b-5/16/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

maxima [A] time = 1.10, size = 319, normalized size = 1.10

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{4\left(cd^3x^4 + c^2d^2\right)} + \frac{b^2x}{d^2} - \frac{\frac{2\sqrt{2}\left(5\,b^2c^2 - 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x + \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}\left(5\,b^2c^2 - 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{\cdots}\right.}{2\cdots}\right.}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^4 + c^2*d^2) + b^2*x/d^2 - 1
/32*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqr
t(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*lo
g(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sq
rt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)
*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d^2)
```

mupad [B] time = 1.54, size = 1254, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*c*(c*d^2 + d^3*x^4)) +
 (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
+ 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 2
0*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5
*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*
a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9
/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)))/((((x*(9*a^4
*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/
(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b
*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/
4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*
c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d
^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a
*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(
```

```
-c)^(7/4)*d^(9/4)) + (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d
^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*
b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))
*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 2
5*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d
) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*
1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^
(9/4)))/((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
 + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 -
20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d
 + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a
^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)
*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/
4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4))))*(a*d
 - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4))
```

sympy [A] time = 1.98, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4c^2d^2+4cd^3x^4}+\text{RootSum}\left(65536t^4c^7d^9+81a^8d^8+216a^7bcd^7-324a^6b^2c^2d^6-984a^5b^3c^3d^5+64\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**3
*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7 -
 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**
4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c**7*d
 + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a*b*c*
d - 5*b**2*c**2) + x)))
```

3.159 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$

Optimal. Leaf size=349

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{\right.}}{128\sqrt{2}\,c^{11/4}d^{9/4}}$$

```
[Out] -1/8*(-a*d+b*c)*x*(b*x^4+a)/c/d/(d*x^4+c)^2-1/32*(-a*d+b*c)*(7*a*d+5*b*c)*x
/c^2/d^2/(d*x^4+c)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(-1+d^(1/4)
*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d+5*
b^2*c^2)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1/256
*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2
*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*l
n(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {413, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{(21a^2d^2+6abcd+5b^2c^2)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{\right.}}{128\sqrt{2}\,c^{11/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c +
7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) + ((
5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(64*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*
d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2} + \frac{\int \frac{a(bc+7ad)+b(5bc+3ad)x^4}{\left(c+dx^4\right)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{1}{c+dx^4}\,dx}{32c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{64c^{5/2}d^2} +$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}}{128c^{5/2}d^{5/2}}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)} - \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\log\left(\sqrt{c}-\sqrt{2}\right.}{128\sqrt{2}\,c^{11/4}d^{9/4}}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)} - \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{64\sqrt{2}\,c^{11/4}d^{9/4}}$$

Mathematica [A] time = 0.23, size = 319, normalized size = 0.91

$$\frac{-\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(9
*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(9
/4))
```

fricas [B] time = 1.40, size = 1411, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - 4*(c^2*d^4*x^8 + 2*
c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^
6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3
*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*
d^9))^(1/4)*arctan(-(c^8*d^7*x*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^
2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5
*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8
)/(c^11*d^9))^(3/4) - c^8*d^7*sqrt((c^6*d^4*sqrt(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2
*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x^2)/(25*b^4*c^4 + 60*a*b^3*c^3*d
 + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4))*(-(625*b^8*c^8 + 3
000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^
4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^
7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4))/(125*b^6*c^6 + 450*a*b^5*c^5
*d + 2115*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 8883*a^4*b^2*c^2*d^4 + 7
938*a^5*b*c*d^5 + 9261*a^6*d^6)) - (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*
(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c
^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c
^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2
*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*
c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*
c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(
-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^
5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^
```

```
2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2
*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*
c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*
c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2
)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)
```

giac [A] time = 0.21, size = 407, normalized size = 1.17

$$\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}abcd+21\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}+\frac{\sqrt{2}\left(5\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\,\left(cd^3\right)^{\frac{1}{4}}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b
*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*x^5 - 7*a
^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^4 + c)^2*c
^2*d^2)
```

maple [A] time = 0.06, size = 499, normalized size = 1.43

$$\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{128c^3}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{128c^3}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{256c^3}+\frac{3\left(\frac{c}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d-
5*b^2*c^2)/d^2/c*x)/(d*x^4+c)^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x-1)*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
```

```
^(1/4)*x-1)*a*b+5/128/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x-1)*b^2+21/256/c^3*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)
^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a^2+3/128/c^2/d*(c/d)^(1/4
)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/
2)*x+(c/d)^(1/2)))*a*b+5/256/c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*
2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b^2+21/128/
c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2+3/64/c^2/d*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a*b+5/128/c/d^2*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

maxima [A] time = 1.22, size = 361, normalized size = 1.03

$$-\frac{\left(9\,b^2c^2d-2\,abcd^2-7\,a^2d^3\right)x^5+\left(5\,b^2c^3+6\,abc^2d-11\,a^2cd^2\right)x}{32\left(c^2d^4x^8+2\,c^3d^3x^4+c^4d^2\right)}+\frac{2\,\sqrt{2}\left(5\,b^2c^2+6\,abcd+21\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] -1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^2
*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*sq
rt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
+ sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqr
t(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt
(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1
/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^2)
```

mupad [B] time = 1.66, size = 1401, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

```
[Out] - ((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 -
9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((((
((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d
^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2
```

```
*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*
b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2
*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4
)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*
c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)
*1i)/(128*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(2
1*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441
*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*
d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d
^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d
 + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 +
 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*
a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (atan((((((21*a^2*d^
2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(25
6*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2
 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2
+ 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*
b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4
)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d +
252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(
-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 +
 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4
 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(2
56*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4
)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*
a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 2
46*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^
2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2
+ 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4))
```

sympy [A] time = 5.85, size = 264, normalized size = 0.76

$$\frac{x^5\left(7a^2d^3+2abcd^2-9b^2c^2d\right)+x\left(11a^2cd^2-6abc^2d-5b^2c^3\right)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8}+\text{RootSum}\left(268435456t^4c^{11}d^9+194481a^8d^8+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6*
a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d**4
*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*a**
7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 + 1128
06*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c**6*d
```

```
**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c**3*d**
2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

3.160 $\int \frac{\left(c+dx^4\right)^4}{a+bx^4}\,dx$

Optimal. Leaf size=332

$$-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\tan^{-1}\left(1\right.}{2\sqrt{2}\,a^{3/4}b^{}}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/5*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^5/b^3+1/9*d^3*(-a*d+4*b*c)*x^9/b^2+1/13*d^4*x^13/b+1/4*(-a*
d+b*c)^4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)+1/4*
(-a*d+b*c)^4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)-1
/8*(-a*d+b*c)^4*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/
b^(17/4)*2^(1/2)+1/8*(-a*d+b*c)^4*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
b^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^{\cdots}\right.}{4\sqrt{2}\,a^{3/4}b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d
^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
- a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^4}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{b^3} + \frac{d^3(4bc-ad)x^8}{b^2} + \right.$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} +$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} +$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} +$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} +$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} +$$

Mathematica [A] time = 0.22, size = 322, normalized size = 0.97

$$-\frac{585\sqrt{2}(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}} + \frac{585\sqrt{2}(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}} - \frac{1170\sqrt{2}(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} +$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 9
36*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4*b
*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*d)^4*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (585*Sqrt
[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a
^(3/4))/(4680*b^(17/4))
```

fricas [B] time = 1.31, size = 2477, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*c
^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 2340*b^4*(-(b^16*c^16 - 16*a*b^15
*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^1
2*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9
*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^
10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^1
3 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)
*arctan(-(a^2*b^13*x*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^
2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5
 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 -
 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 +
1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16
*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4) - a^2*b^13*sqrt((a^2*b^8*sqrt
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17)) + (b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)*x^2)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))*(-(b^16*c^16 - 16*a*b^15*c^15*d +
120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 43
68*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 128
70*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*
a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^
14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4))/(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)) + 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 1
20*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 436
8*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 1287
0*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a
^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^1
4*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*log(a*b^4*
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
```

```
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*
a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a
^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a
^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11
*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b
^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*log(-a*b^4*(-
(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d
^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d
^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9
+ 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12
- 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d
^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*
d^3 - a^3*d^4)*x)/b^4
```

giac [B] time = 0.48, size = 617, normalized size = 1.86

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}} b^{4}c^{4} - 4\left(ab^{3}\right)^{\frac{1}{4}} ab^{3}c^{3}d + 6\left(ab^{3}\right)^{\frac{1}{4}} a^{2}b^{2}c^{2}d^{2} - 4\left(ab^{3}\right)^{\frac{1}{4}} a^{3}bcd^{3} + \left(ab^{3}\right)^{\frac{1}{4}} a^{4}d^{4}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{4ab^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3
)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d
^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1
/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)
^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1/
8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^
(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^
2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^13 + 260*b^12
*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*b^11*c*d^3*x^
5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^2*d^2*x + 2340
*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

maple [B] time = 0.05, size = 837, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a),x)
```

```
[Out] -4/5*d^3/b^2*x^5*a*c+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^2*x+1/4*(a/b)^(1/4)/a*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^4+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x
^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
)))*c^4+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^4+1/13*
d^4*x^13/b+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2*d^2-1/b^3*(a/b)^(1/4)
*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^3+3/2/b^2*(a/b)^(1/4)*a*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^2-1/b^3*(a/b)^(1/4)*a^2*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^3+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d^2-1/2/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x^2+(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*
c*d^3+6/5*d^2/b*x^5*c^2-d^4/b^4*a^3*x+4*d/b*c^3*x-1/9*d^4/b^2*x^9*a+4/9*d^3
/b*x^9*c+1/5*d^4/b^3*x^5*a^2+1/4/b^4*(a/b)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x-1)*d^4-1/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-
1)*c^3*d+1/8/b^4*(a/b)^(1/4)*a^3*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d^4-1/2/b*(a/b)^(1/4)*2^(
1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+
(a/b)^(1/2)))*c^3*d+1/4/b^4*(a/b)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)*d^4-1/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d
```

maxima [A] time = 1.44, size = 489, normalized size = 1.47

$$\frac{45\,b^3d^4x^{13} + 65\left(4\,b^3cd^3 - ab^2d^4\right)x^9 + 117\left(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4\right)x^5 + 585\left(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4\,a^2bcd^3\right.}{585\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/585*(45*b^3*d^4*x^13 + 65*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 117*(6*b^3*c^2*
d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/8*(2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
```

```
*sqrt(b))) + 2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(
sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

mupad [B] time = 1.51, size = 1822, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^4/(a + b*x^4),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^9*((a*d^4)/(9*b^2) - (4*c*d^3)/(9*b)) + x^5*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(5*b) + (6*c^2*d^2)/(5*b)) + (d^4*x^13)/(13*b) + (atan(((((4*x*(a^8*
d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7
))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^
2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a
)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*
c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(
21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - (4
*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 -
 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/
4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7
*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d -
 b*c)^4)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4*1i)/(2*(-a)^(3/4)*b^(17/4)
) + (atan(((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^
7*c^7*d - 8*a^7*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2
*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*
(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^
4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^
3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)))/((((4
```

```
*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7
*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1
i)/(4*(-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^4*(a^5*d^4 +
 a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)
^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4
)/(2*(-a)^(3/4)*b^(17/4))
```

sympy [A] time = 3.59, size = 435, normalized size = 1.31

$$x^9\left(-\frac{ad^4}{9b^2}+\frac{4cd^3}{9b}\right)+x^5\left(\frac{a^2d^4}{5b^3}-\frac{4acd^3}{5b^2}+\frac{6c^2d^2}{5b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)+\text{RootSum}\left(256t^4a^3b^{17}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*c
*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b*
*3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a**1
6*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b**3*c
**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 + 8008*
a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c**8*d*
*8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a**5*b**
11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d**3 + 12
0*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda(_t, _t*l
og(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)
```

3.161 $\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$

Optimal. Leaf size=288

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1\right.}{2\sqrt{2}\,a^{3/4}b^{}}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/5*d^2*(-a*d+3*b*c)*x^5/b^2+1/9*d^3*
x^9/b+1/4*(-a*d+b*c)^3*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(13/4
)*2^(1/2)+1/4*(-a*d+b*c)^3*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(1
3/4)*2^(1/2)-1/8*(-a*d+b*c)^3*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(
1/2))/a^(3/4)/b^(13/4)*2^(1/2)+1/8*(-a*d+b*c)^3*ln(a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\right.}{4\sqrt{2}\,a^{3/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b^
2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + (
(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{a+bx^4}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^4}{b^2}+\frac{d^3x^8}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^4\right)}\right.$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^4}\,dx}{b^3}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^3}+\frac{(bc-ad)}{}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{7/2}}+\frac{(bc}{}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\right.}{4\sqrt{2}\,a^{3/4}b^{13/4}}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc}{}$$

Mathematica [A] time = 0.17, size = 271, normalized size = 0.94

$$\frac{-72a^{3/4}b^{5/4}d^2x^5(ad-3bc)+40a^{3/4}b^{9/4}d^3x^9+360a^{3/4}\sqrt[4]{b}\,dx\left(a^2d^2-3abcd+3b^2c^2\right)-45\sqrt{2}\,(bc-ad)^3\log\left(-\sqrt{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^(
5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/
4))
```

fricas [B] time = 1.16, size = 1855, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 180*b^3*(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arctan((a^2*b^10*x*(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4) - a^2*b^10*sqrt((a^2*b^6*sq
rt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) + (b^6*c^6 - 6*a*b^5*c
^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5
*b*c*d^5 + a^6*d^6)*x^2)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6))*(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4))/(b^9*c^9 - 9*a*b^8*c^8*d +
 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^
4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d
^9)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(
a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b^3*(-(b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*x) + 180*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

giac [B] time = 0.17, size = 481, normalized size = 1.67

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^
4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3
)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^8*d^3*x^9
+ 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x
+ 45*a^2*b^6*d^3*x)/b^9
```

maple [B] time = 0.05, size = 627, normalized size = 2.18

$$\frac{d^{3}x^{9}}{9b}-\frac{a\,d^{3}x^{5}}{5b^{2}}+\frac{3c\,d^{2}x^{5}}{5b}+\frac{a^{2}d^{3}x}{b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/5*d^3/b^2*x^5*a+3/5*d^2/b*x^5*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/8/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d^3+3/8/b^2*(a/b)^(
1/4)*a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*
2^(1/2)*x+(a/b)^(1/2)))*c*d^2-3/8/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^2*d+1/8*
(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)
^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^3-1/4/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x-1)*d^3+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x-1)*c*d^2-3/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
```

```
-1)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3-1/4
/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^3+3/4/b^2*(a
/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^2-3/4/b*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3
```

maxima [A] time = 1.21, size = 385, normalized size = 1.34

$$\frac{5\,b^2d^3x^9+9\left(3\,b^2cd^2-abd^3\right)x^5+45\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{45\,b^3}+\frac{\frac{2\,\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a\right.}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right.}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/45*(5*b^2*d^3*x^9 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 45*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/8*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(b)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

mupad [B] time = 1.49, size = 1433, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^3/(a + b*x^4),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^5*((a*d^3)/(5*b^2)
- (3*c*d^2)/(5*b)) + (d^3*x^9)/(9*b) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2
*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^
(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d
 - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4
*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((((x*(a^6*
```

```
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b
^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d
- b*c)^3)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d
^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*
a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4))
))*(a*d - b*c)^3*1i)/(2*(-a)^(3/4)*b^(13/4)) - (atan(((((x*(a^6*d^6 + b^6*c
^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5
*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12
*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3
)/((-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3
 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*
d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))/(((
(x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3
 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13
/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b
^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/(
(-a)^(3/4)*b^(13/4))))*(a*d - b*c)^3)/(2*(-a)^(3/4)*b^(13/4))
```

sympy [A] time = 1.70, size = 303, normalized size = 1.05

$$x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\text{RootSum}\left(256t^4a^3b^{13}+a^{12}d^{12}-12a^{11}bcd^{11}+66a^{10}b^2c^2d^{10}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b
*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*a**5
*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 + 66*a*
*2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t*log(-
4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x
))) + d**3*x**9/(9*b)
```

3.162 $\int \frac{(c+dx^4)^2}{a+bx^4} \, dx$

Optimal. Leaf size=253

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2 \tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}b^{9/}}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/5*d^2*x^5/b+1/4*(-a*d+b*c)^2*arctan(-1+b^(1/4)*x*2^(
1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*arctan(1+b^(1/4)*x*2
^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*ln(-a^(1/4)*b^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)+1/8*(-a*d+b*c)^2*l
n(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2 \tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}b^{9/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^2}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)}{b^2} + \frac{d^2x^4}{b} + \frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^4\right)}\right) dx$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2\int \frac{1}{a+bx^4}\,dx}{b^2}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^2} + \frac{(bc-ad)^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^2}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{5/2}} + \frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{5/2}} - \frac{(bc-}{}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{(bc-ad)^2\log\left(\sqrt{a}+\sqrt{}\right.}{4\sqrt{2}\,a^{3/4}}$$

$$= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}} - \frac{(bc-}{}$$

Mathematica [A] time = 0.13, size = 231, normalized size = 0.91

$$\frac{8a^{3/4}b^{5/4}d^2x^5 - 40a^{3/4}\sqrt[4]{b}\,dx(ad-2bc) - 5\sqrt{2}\,(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right) + 5\sqrt{2}\,(bc-ad)^2\log\left(\right.}{40a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*(b
*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a*d)
^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(b*c
- a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a^(3/4
)*b^(9/4))
```

fricas [B] time = 1.12, size = 1240, normalized size = 4.90

$$4\,bd^2x^5 + 20\,b^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^7x\left(-\frac{b}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*d^2*x^5 + 20*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*arctan(-(a^2*b^7*x*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
*d^8)/(a^3*b^9))^(3/4) - a^2*b^7*sqrt((a^2*b^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9)) + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(a^3*b^9))^(3/4))/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)) + 5*b^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 20*(2*b*c*d - a*d
^2)*x)/b^2
```

giac [A] time = 0.18, size = 353, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(ab^3\right)^{\frac{1}{4}}abcd + \left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2 - 2\,\left(ab^3\right)^{\frac{1}{4}}abcd + \left(\right.\right.}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d +
(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/5*(b
^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

maple [B] time = 0.05, size = 436, normalized size = 1.72

$$\frac{d^2x^5}{5b}-\frac{a\,d^2x}{b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a),x)
```

```
[Out] 1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*c*x+1/8/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x^2+(
a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*
d^2-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x
^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^
2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)
))*c^2+1/4/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^2-1/
2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d+1/4*(a/b)^(1/4)
/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2+1/4/b^2*(a/b)^(1/4)*a*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^2-1/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x+1)*c*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x+1)*c^2
```

maxima [A] time = 1.09, size = 287, normalized size = 1.13

$$\frac{bd^2x^5+5\left(2\,bcd-ad^2\right)x}{5\,b^2}+\frac{\frac{2\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/5*(b*d^2*x^5 + 5*(2*b*c*d - a*d^2)*x)/b^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

mupad [B] time = 1.47, size = 1081, normalized size = 4.27

$$\frac{d^2\,x^5}{5\,b}-x\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{(-a)^{3/4}\,b^{9/4}}+\frac{(a\,d-b\,c)^2\left(\frac{x\left(\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3\,b\,d^2-8\,a^2\,b^2\,c\,d+4\,a\,b^3\,c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}}-\frac{(a\,d-b\,c)^2\left(\frac{x\left(\cdots\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{2\,(-a)^{3/4}\,b^{9/\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^2/(a + b*x^4),x)
```

```
[Out] (d^2*x^5)/(5*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b -
 ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*
b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4
*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a
)^(3/4)*b^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3
*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*
d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
a)^(3/4)*b^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2
 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8
*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)))/(((a*d - b
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/
(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
```

```
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a
*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b
^(9/4)))*1i)/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2)/(2*(-a)^(3/4)*b^(9/4))
```

sympy [A] time = 1.08, size = 187, normalized size = 0.74

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\text{RootSum}\left(256t^4a^3b^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a*
*7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**
4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c*
*7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b*
*2*c**2) + x))) + d**2*x**5/(5*b)
```

3.163 $\int \frac{c+dx^4}{a+bx^4}\,dx$

Optimal. Leaf size=223

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}b^{5/4}}$$

```
[Out] d*x/b+1/4*(-a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)+1/4*(-a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)-1/8*(-a*d+b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^
(3/4)/b^(5/4)*2^(1/2)+1/8*(-a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x
^2*b^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^4}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b} + \frac{(bc-ad)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{3/2}} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{3/2}} - \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} +$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{}\right.}{4\sqrt{2}\,a^{3/4}b^{5/}}$$

Mathematica [A] time = 0.15, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b}\,dx-\sqrt{2}\,(bc-ad)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+\sqrt{2}\,(bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(8*a^(3/4)*b^(5/4))
```

fricas [B] time = 1.32, size = 639, normalized size = 2.87

$$-\frac{4\,b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b^4x\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{3}{4}}-a^2b^4\sqrt{\frac{a^2b^2\sqrt{-\frac{b^4c^4-4\,ab^3c^3d+6}{}}}{}}}{b^3c^3-3\,ab^2c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(a^3*b^5))^(1/4)*arctan((a^2*b^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(
(a^2*b^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(a^3*b^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x)
 - b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

giac [A] time = 0.17, size = 245, normalized size = 1.10

$$\frac{dx}{b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc-\left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc-\left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)
^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*
b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b^2)
```

maple [A] time = 0.05, size = 266, normalized size = 1.19

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{dx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\,\mathrm{arc}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a),x)
```

```
[Out] 1/b*d*x-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d+1/4*(a/
b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-1/8/b*(a/b)^(1/4)*2^(1
/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))*d+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c-1/4/b*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)*c
```

maxima [A] time = 1.13, size = 212, normalized size = 0.95

$$\frac{dx}{b}+\frac{\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\sqrt{2}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b*
c - a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

mupad [B] time = 0.22, size = 720, normalized size = 3.23

$$\frac{dx}{b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}+\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}}{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}-\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}\right)(a}{2(-a)^{3/4}b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)/(a + b*x^4),x)
```

```
[Out] (d*x)/b - (atan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2
*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-
a)^(3/4)*b^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*
```

```
b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4
*(-a)^(3/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4))))*(a*d - b*c)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (atan((((x*
(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c)*(a*d
 - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)) + (
(x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a*b^3*c)*(
a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))
/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c
)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(
5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a
*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/
4)*b^(5/4))))*(a*d - b*c))/(2*(-a)^(3/4)*b^(5/4))
```

sympy [A] time = 0.61, size = 87, normalized size = 0.39

$$\operatorname{RootSum}\left(256t^4a^3b^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(-\frac{4tab}{ad - bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d - b*
c) + x))) + d*x/b
```

3.164 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\text{ta}}{2\sqrt{}}$$

```
[Out] 1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-
a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\text{ta}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}(bc-ad)} - \frac{b^{3/4}\int}{4}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3}}{}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)} - \frac{d^{3/}}{}$$

Mathematica [A] time = 0.14, size = 340, normalized size = 0.76

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

fricas [B] time = 1.36, size = 1356, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4))^(3/4)*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4)))/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5 - 3*a*b^
2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3*a
*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt((d^2*x^2 +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))/d^2))/d^2) + 1/4*(-b^
3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*
d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d
+ 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3
*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
+ (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
 a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^
3 + a^4*c^3*d^4))^(1/4))
```

giac [A] time = 0.21, size = 437, normalized size = 0.97

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^{2}d\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^{2}-\sqrt{2}acd\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\,\mathrm{arcta}}{2\left(\sqrt{2}\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1
/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
```

```
)/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/
4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c
 - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)
^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2
)*a*c*d)
```

maple [A] time = 0.06, size = 320, normalized size = 0.71

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)a}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\,\mathrm{arcta}}{4\left(ad-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(
1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d/(a*d-b*c)*(c/d)^(1/4)/
c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*
ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)
^(1/2)))-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 1.43, size = 365, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{3}{4}}\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\right.}}{a^{\frac{3}{4}}}}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
```

```
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

mupad [B] time = 2.76, size = 6153, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a
^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3
*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)
*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*
c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^
11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7
 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10)
+ x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6
*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^
3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7
*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^
4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^
4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*
a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*
d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 +
 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c
^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^
7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d
^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11
 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2
*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c
^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
```

*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i)/(((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^

4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*
b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10
*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^
4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d
^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*
a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^
6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7
x)(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^
2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^
4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((
-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2
*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 102
4*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 +
256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*
d^3))^(1/4)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1
536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*2i - 2*atan((b^3*d^3*x - (12
8*b^10*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^
5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*
a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3)
 - (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^
3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(2
56*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4
- 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*
b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a
^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256
*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3
) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3
*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))/((-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*
b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*
d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*
c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^
2*d^2 - 4*a*b^3*c*d^3)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^
3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) - 2*atan((b^3*d^3
*x - (128*a^7*d^10*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^
7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^

```
3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d
^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*
c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (
768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3
+ 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c
^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b
^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3
*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7
+ 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*
c^6*d))^(1/4)*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2
560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*
b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*
a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 +
2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.165 $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

Optimal. Leaf size=513

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan}{2\sqrt{2}}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(d*x^4+c)+1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(
1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a
^(1/4))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*d+7*b*c)*arctan(-1+
d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(3/4)*(-3*a*
d+7*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1
/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-a*d
+b*c)^2*2^(1/2)+1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/32*d^(3/4)*(-3*a*d+7*b*c)*ln(-c^(1/4)*d^(
1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/32*d^(3/
4)*(-3*a*d+7*b*c)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)
/(-a*d+b*c)^2*2^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*
c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c
 - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*
(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c
```

```
 - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[
2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\int \frac{4bc-3ad-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^2\int \frac{1}{a+bx^4}\,dx}{(bc-ad)^2}-\frac{(d(7bc-3ad))\int \frac{1}{c+dx^4}\,dx}{4c(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)^2}+\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)^2}-\frac{(d(7bc-3ad))\int \sqrt{\ldots}}{8c^{3/2}(bc-a\ldots}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)^2}+\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)^2}-\frac{b^{7/4}\int \ldots}{4\sqrt{\ldots}}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}-\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\ldots\right.}{4\sqrt{2}\,a^{3/4}(\ldots}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7\ldots}{\ldots}$$

Mathematica [A] time = 0.33, size = 498, normalized size = 0.97

$$\frac{8a^{3/4}c^{3/4}dx(ad-bc)-2\sqrt{2}\,a^{3/4}d^{3/4}\left(c+dx^4\right)(3ad-7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+2\sqrt{2}\,a^{3/4}d^{3/4}\left(c+dx^4\right)(3ad-7bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^
4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
 d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)
*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*S
qrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*a^(3/4)*
d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*c^(7/4)*(b*
c - a*d)^2*(c + d*x^4))
```

fricas [B] time = 46.36, size = 3299, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b
^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a
^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^
7 + a^8*c^7*d^8))^(1/4)*arctan(((b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9
*d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*
d^6)*x*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 75
6*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^
2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^
6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4) - (b^6*c^11 - 6*a*b^5
*c^10*d + 15*a^2*b^4*c^9*d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*
a^5*b*c^6*d^5 + a^6*c^5*d^6)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 264
6*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*
d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^
5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4)
```

```
*sqrt(((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x^2 + (b^4*c^8 - 4*a*b^3
*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^1
2*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*
a^7*b*c^8*d^7 + a^8*c^7*d^8)))/(49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4))
)/(343*b^3*c^3*d^2 - 441*a*b^2*c^2*d^3 + 189*a^2*b*c*d^4 - 27*a^3*d^5)) + 1
6*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^
5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^
10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*a
rctan(-((a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^
3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8 -
8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*
d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))
^(3/4)*x - (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3
*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8
 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c
^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^
8))^(3/4)*sqrt((b^4*x^2 + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^
2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*
a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*
d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8)))/b^4))/b^5) + 4*(-b^
7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3
+ 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c
*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(b^2
*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5
*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8
*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - 4*(
-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d
^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*
b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(
b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*
b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6
- 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) +
((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^
3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15
- 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c
^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*
c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c
^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 7
56*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d
^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a
^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - ((b*c^2*d - a*c*d
^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*
a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d
```

```
+ 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*
b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*l
og(-(7*b*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b
^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 +
81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c
^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 -
8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 +
b*c^3 - a*c^2*d)
```

giac [A] time = 0.19, size = 667, normalized size = 1.30

$$\frac{\left(ab^3\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^2c^2 - 2\sqrt{2}\,a^2bcd + \sqrt{2}\,a^3d^2\right)} + \frac{\left(ab^3\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^2c^2 - 2\sqrt{2}\,a^2bcd + \sqrt{2}\,a^3d^2\right)} + \frac{\left(ab^3\right)^{\frac{1}{4}} b \log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\ldots}\right.}{4\left(\sqrt{2}\,ab^2c^2 - 2\sqrt{2}\,a^2bcd + \sqrt{2}\,a^3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b
^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sq
rt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^(1/4
)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqr
t(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2
)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*
a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^(1
/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/16*(7*(c
*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sq
rt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/
4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

maple [A] time = 0.06, size = 550, normalized size = 1.07

$$\frac{a\,d^2x}{4\left(ad-bc\right)^2\left(d\,x^4+c\right)c} - \frac{bdx}{4\left(ad-bc\right)^2\left(d\,x^4+c\right)} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;a\,d^2 \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16\left(ad-bc\right)^2c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;a\,d^2 \arctan\left(\frac{\sqrt{2}\,x\ldots}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{16\left(ad-bc\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^2/(a*d-b*c)^2/c*x/(d*x^4+c)*a-1/4*d/(a*d-b*c)^2*x/(d*x^4+c)*b+3/16*d^
2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-7/1
6*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+3/3
2*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a-7/32*d/(a*d-b*c)^2/c*(
c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1
/4)*2^(1/2)*x+(c/d)^(1/2)))*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-7/16*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+1/8*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)
*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)
```

maxima [A] time = 1.57, size = 481, normalized size = 0.94

$$-\frac{dx}{4\left(\left(bc^2d-acd^2\right)x^4+bc^3-ac^2d\right)}+\frac{\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\cdots\right.}{a^{\frac{3}{4}}}\cdots}{8\left(b^2c^2-2\,abcd+a^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*sqrt(2)*b^2*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2
*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/32*(2*sq
rt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2
)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b
*c*d - 3*a*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(
3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)
```

mupad [B] time = 4.00, size = 21975, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^2),x)
```

```
[Out] 2*atan(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^
2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a
^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 458752
0*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 5
24288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c
^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^
8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b
^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008
*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28
*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a
^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*
c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*
a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 45875
20*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 -
524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 -
 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +
 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10
 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d
^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3
*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*
a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 +
 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10
*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912
*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 +
 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5
 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c
^9*d)))*1i) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8
 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
```

b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c
^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6553
6*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^1
3*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b
^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*
a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^
8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2
*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65
536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c
^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5
*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(2867
2*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 9011
2*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 32
9728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^1
6*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 280985
6*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^1
0 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b
^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 46694
4*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*
d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^
2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7
- 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(
b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*
c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-(81*a^4*d^7 + 2401*b^4*
c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(655
36*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^
13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*
b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81
*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 7
56*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7
+ 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^1
1*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c
^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*
d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7
- a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65
536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c

^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5
*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(
81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 -
756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^
7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c
^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7
*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3
*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376
*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 1146
88*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b
^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^
15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b
^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 1451
6224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*
d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11
*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64
*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^
3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i)*1i + (x*(81*a^4*b^9*d
^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*
a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^
2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))
+ (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*
d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c
^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*
b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*
a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
+ 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*
d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^1
2*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b
^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28*b^11*
c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9
*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)
+ (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2
*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*
c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4
*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288
*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d
^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^
7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c
^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6
*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*
a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672
*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253
952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 +
3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2

*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^
15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 33628
16*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d
^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*
b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864
*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*
a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))
)*1i)*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 -
756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d)))))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4) - atan(((-(81*a^4*d^7 + 240
1*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6
)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*
b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 367001
6*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*
((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d
^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^
8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b
^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a
*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10
*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7
 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^
4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6
5536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*
c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^
5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-
(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5
- 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d
^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*
c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^
7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^
3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 22937
6*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114
688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(
b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c
^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*
b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 145
16224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7
*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^1
1*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(
b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*

c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*

a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18
35008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16
+ 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (19
71*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*
c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*
a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 45875
20*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 -
524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 -
4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +
28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10
- 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d
^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3
*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*
a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 +
3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10
*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912
*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 +
36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 +
15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*
d)))) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756
*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*
a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^
4 - 6*a*b^5*c^9*d))) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^
4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7
*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^
12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*
b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3
- 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*
c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2
- 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^1
0*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7
*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9
)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2
- 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 +
2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8
- 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d
^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*
c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 -
4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^1
5 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3

670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*2i - atan(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7

- a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i - (x*(81*a^4*b^9*d^11
+ 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*
b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^
4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (-
b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6
*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^
5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 +
256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^
5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^
2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 -
2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a
^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b
*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d
+ 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 1
4336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28
672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90
112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 +
329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12
- 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^
3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b
^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809
856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d
^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9
*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466
944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d
^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2
*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (21
45*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)
/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i + (x*(81*a^4
*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 +
2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 +
15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9
*d))))/((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a
^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*
b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*
a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14
336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*
a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*
b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3
+ 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2
048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4
*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c
^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))
^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^1
2*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*

c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))))*(-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14

336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*2i +
2*atan(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*
a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8
*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256
*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 1
4336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168
*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3
*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^
3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 -
2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^
4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*
c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7)
)^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^
12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9
*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b
^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 -
a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 -
524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12
*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6
*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 1
7833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4
*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7*d^10)/16 + 28*
b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^
2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c
^6*d)) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 75
6*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6
*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d
^4 - 6*a*b^5*c^9*d))) - (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^
7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*
d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1
/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*
b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3
*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^1
1*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336
*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9
*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^
8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 +
17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 204
8*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78
848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 -
229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11
+ 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^
14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*

b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 346931
2*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9
 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b
^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 26091
52*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*
1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20
*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7
*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9
)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*
d^2 - 3*a*b^2*c^6*d)) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^
12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 +
a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 1
5*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^
8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 179
20*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^
10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^
7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4
- 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*
((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*
c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3
*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d
^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^
6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^
2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^1
4*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*
b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a
^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*
a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*
d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^
13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^
5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 -
 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c
^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13
*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b
^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i
+ (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (67
5*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3
 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i + (x*(81*a^4*b^9*d^11 + 3185*b^13*c
^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)
1i)/(64(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 2
0*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (-b^7/(256*a^11
*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*
a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*
b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*
c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 1

```
7920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*
a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*
c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^
4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4
)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b
^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*
c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*
c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*
c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^
8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10
*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c
^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5
+ 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*
c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 2419097
6*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13
 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*
c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5
*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -
 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b
^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(
b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i - (x*(81*a^4*b
^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2
790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
)))))*(-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5
*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^
3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4) + (d*x)/(4*c*(
c + d*x^4)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.166 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=407

$$-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{\right.}}{16\sqrt{2}\,a^{7/4}b^{21/4}}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/5*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^5/b^4+1/9*d^4*(-2*a*d+5*b*c)*x^9/b^3+1/13*
d^5*x^13/b^2+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^4+a)+1/16*(-a*d+b*c)^4*(17*a*d+3
*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21/4)*2^(1/2)+1/16*(-
a*d+b*c)^4*(17*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(21
/4)*2^(1/2)-1/32*(-a*d+b*c)^4*(17*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1/2)+1/32*(-a*d+b*c)^4*(17*a*d+3*b
*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(21/4)*2^(1
/2)
```

Rubi [A] time = 0.40, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{\right.}}{16\sqrt{2}\,a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d
)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^
4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d
)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^5}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{b^4}+\frac{d^4(5b\ldots}{}\right.$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-\ldots}{9b\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-\ldots}{9b\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-\ldots}{9b\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-\ldots}{9b\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-\ldots}{9b\ldots}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-\ldots}{9b\ldots}$$

Mathematica [A] time = 0.45, size = 391, normalized size = 0.96

$$-\frac{585\sqrt{2}\,(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{585\sqrt{2}\,(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{7/4}}-\frac{1170\sqrt{2}\,(bc-ad)^4(17\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*b^
(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(b*c
 - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c - a*
d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (5
85*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(18720*b
^(21/4))
```

<u>fricas</u> [B] time = 1.01, size = 3222, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 20
8*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a*b
^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 + 23
40*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(1/4)*arctan(-(a^5*b^16*x*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b
^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a
^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 -
500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10
*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 54
8231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^
5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 909
4830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21
))^(3/4) - a^5*b^16*sqrt((a^4*b^10*sqrt(-(81*b^20*c^20 + 540*a*b^19*c^19*d
- 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d
^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13
*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 1748735
```

56*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c
^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 26604
0144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^
3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^2
0)/(a^7*b^21)) + (9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2 + 40*a
^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^
4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b*c*d^9
 + 289*a^10*d^10)*x^2)/(9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2
 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*
a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b
*c*d^9 + 289*a^10*d^10))*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(3/4))/(27*b^15*c^15 + 135*a*b^14*c^14*d - 1125*a^2*b^13*c^13*d^2 - 1945*a
^3*b^12*c^12*d^3 + 25095*a^4*b^11*c^11*d^4 - 42141*a^5*b^10*c^10*d^5 - 1319
45*a^6*b^9*c^9*d^6 + 774675*a^7*b^8*c^8*d^7 - 1837935*a^8*b^7*c^7*d^8 + 270
0885*a^9*b^6*c^6*d^9 - 2702799*a^10*b^5*c^5*d^10 + 1889685*a^11*b^4*c^4*d^1
1 - 914675*a^12*b^3*c^3*d^12 + 293505*a^13*b^2*c^2*d^13 - 56355*a^14*b*c*d^
14 + 4913*a^15*d^15)) + 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b
^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*
b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 61372
00*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^
9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250
*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d
^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*
a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 835
21*a^20*d^20)/(a^7*b^21))^(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^
19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c
^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7
*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 17
4873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*
b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 -
266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b
^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^2
0*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2
 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 +
a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 157
80*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 -
 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^
12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 36090

```
0280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c
^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 12083
6285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*
d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(-a^2*
b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^
3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960
920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^
8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*
a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^
13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*
a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18
- 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 +
 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 +
 17*a^5*d^5)*x) + 2340*(b^5*c^5 - 5*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 90*a
^3*b^2*c^2*d^3 + 65*a^4*b*c*d^4 - 17*a^5*d^5)*x)/(a*b^6*x^4 + a^2*b^5)
```

giac [B] time = 0.18, size = 798, normalized size = 1.96

$$\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^5c^5+5\left(ab^3\right)^{\frac{1}{4}}ab^4c^4d-50\left(ab^3\right)^{\frac{1}{4}}a^2b^3c^3d^2+90\left(ab^3\right)^{\frac{1}{4}}a^3b^2c^2d^3-65\left(ab^3\right)^{\frac{1}{4}}a^4bcd^4+17\left(ab^3\right)^{\frac{1}{4}}\right.}{16\,a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a
*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)
^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b
^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 9
0*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)
^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*
c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3
 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5
*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*
(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(
1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) + 1/4*
(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x
+ 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*(45*b^24*d^5*x^1
3 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^2*d^3*x^5 - 1170*
```

```
a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d^2*x - 11700*a*b^2
3*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*x)/b^26
```

maple [B] time = 0.06, size = 1118, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^5/(b*x^4+a)^2,x)
```

```
[Out] 5/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^4*d-65/16/b^4*a^2*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^4+45/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^3+5/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)*c^4*d-65/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)*c*d^4+45/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)*c^2*d^3+5/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)*c^4*d-65/32/b^4*a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)
*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d^4+45/16/b^3*a*
(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(
1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^2*d^3+1/13*d^5*x^13/b^2+2*d^3/b^2*x^5*c^2-4*
d^5/b^5*a^3*x+10*d^2/b^2*c^3*x+1/4/a*x/(b*x^4+a)*c^5-2/9*d^5/b^3*x^9*a+5/9*
d^4/b^2*x^9*c+3/5*d^5/b^4*x^5*a^2-5/2/b^3*a^2*x/(b*x^4+a)*c^2*d^3+5/2/b^2*a
*x/(b*x^4+a)*c^3*d^2+17/16/b^5*a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)*d^5-25/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*c^3*d^2+17/32/b^5*a^3*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d^5-25/16/b^2*(a/b)^(1
/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(
1/2)*x+(a/b)^(1/2)))*c^3*d^2+17/16/b^5*a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)*d^5-25/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)*c^3*d^2+5/4/b^4*a^3*x/(b*x^4+a)*c*d^4-1/4/b^5*a^4*x/(b*x^4+a)*d^5
-5/4/b*x/(b*x^4+a)*c^4*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x-1)*c^5+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^5+3/16/a^2*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^5-2*d^4/b^3*x^5*a*c+15*d^4/b^4
*a^2*c*x-20*a/b^3*c^2*d^3*x
```

maxima [A] time = 1.45, size = 644, normalized size = 1.58

$$\frac{\left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)x}{4\left(ab^6x^4 + a^2b^5\right)} + \frac{45\,b^3d^5x^{13} + 65\left(5\,b^3cd^4 - 2\,ab^2d^5\right)x^9 + 117$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^13 + 6
5*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 +
3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4
- 4*a^3*d^5)*x)/b^5 + 1/32*(2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*
b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*
x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b
^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b
*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(1/4)))/(a*b^5)
```

mupad [B] time = 1.71, size = 2490, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^5/(a + b*x^4)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^9*((2*a*d^5)/(9*b^3) - (5*c*d^4)/(9*b^2)) + x^5*((2*a*((2*a*d^5)/b^3
- (5*c*d^4)/b^2))/(5*b) - (a^2*d^5)/(5*b^4) + (2*c^2*d^3)/b^2) + (d^5*x^13)
/(13*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3
 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(4*a*(a*b^5 + b^6*x^4)) + (atan(((((x*(2
89*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 301
0*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^
7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/
(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*
(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b
^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^
4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 221
0*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 +
 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a
```

```
^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(1
6*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^
8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 +
14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a
*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*
c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*
b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d
 + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 2
75*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b
^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c
^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(
17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c
^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b
*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d +
3*b*c)*1i)/(8*(-a)^(7/4)*b^(21/4)) + (atan(((((x*(289*a^10*d^10 + 9*b^10*c^
10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548
*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8
*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*
c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3
*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))
*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) + (((x*(289*a^10*
d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^
6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^
3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b
^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*
c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^
(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)))/
((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d
^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 -
13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*
c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c
^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d
^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2
 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770
*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*
c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(1
7*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c
^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d +
 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(8*(
-a)^(7/4)*b^(21/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.167 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=357

$$-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{\right.}}{16\sqrt{2}\,a^{7/4}b^{17/4}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/5*d^3*(-a*d+2*b*c)*x^5/b^3+1/9*
d^4*x^9/b^2+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^4+a)+1/16*(-a*d+b*c)^3*(13*a*d+3*
b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/4)*2^(1/2)+1/16*(-a
*d+b*c)^3*(13*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(17/
4)*2^(1/2)-1/32*(-a*d+b*c)^3*(13*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+1/32*(-a*d+b*c)^3*(13*a*d+3*b*
c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/
2)
```

Rubi [A] time = 0.37, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)
/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((
b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b
*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sq
rt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^4}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)}{b^4}+\frac{2d^3(2bc-ad)x^4}{b^3}+\frac{d^4x^8}{b^2}+\frac{(bc-ad)^3(bc+3ad)+4bd(bc \cdots}{b^4\left(a+bx^4\right)^2}\cdots\right.$$

$$=\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{\int\frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^4}{\left(a+bx^4\right)^2}\,dx}{b^4}$$

$$=\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}+\frac{\left((bc-ad)^3(3bc\cdots\right.}{\cdots}$$

$$=\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}+\frac{\left((bc-ad)^3(3bc\cdots\right.}{\cdots}$$

$$=\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}+\frac{\left((bc-ad)^3(3bc\cdots\right.}{\cdots}$$

$$=\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}-\frac{(bc-ad)^3(3bc\cdots}{\cdots}$$

$$=\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}-\frac{(bc-ad)^3(3bc\cdots}{8\cdots}$$

Mathematica [A] time = 0.35, size = 341, normalized size = 0.96

$$\frac{\frac{45\sqrt{2}(ad-bc)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{45\sqrt{2}(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{90\sqrt{2}(ad-bc)^3(13ad+3b\cdots}{a^{7/\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(
2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(
a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c)
+ a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

fricas [B] time = 1.04, size = 2580, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*(
30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 180*(a*b^5*x^4 +
a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 830
4*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4
34808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^
5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^
14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)
*arctan((a^5*b^13*x*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^1
4*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c
^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^
8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*
a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 +
 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b
^17))^(3/4) - a^5*b^13*sqrt((a^4*b^8*sqrt(-(81*b^16*c^16 + 432*a*b^15*c^15*
d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d
^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^
9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6
*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 465150
4*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28
561*a^16*d^16)/(a^7*b^17)) + (9*b^8*c^8 + 24*a*b^7*c^7*d - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + 169*a^8*d^8)*x^2)/(9*b^8*c^8 + 24*a*
b^7*c^7*d - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4
 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + 169*a^8*
d^8))*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a
^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4348
```

```
08*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 48
10608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d
^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*
b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(3/4))/(
27*b^12*c^12 + 108*a*b^11*c^11*d - 666*a^2*b^10*c^10*d^2 - 1124*a^3*b^9*c^9
*d^3 + 8901*a^4*b^8*c^8*d^4 - 7848*a^5*b^7*c^7*d^5 - 34860*a^6*b^6*c^6*d^6
+ 113688*a^7*b^5*c^5*d^7 - 161451*a^8*b^4*c^4*d^8 + 132924*a^9*b^3*c^3*d^9
- 65754*a^10*b^2*c^2*d^10 + 18252*a^11*b*c*d^11 - 2197*a^12*d^12)) - 45*(a*
b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14
*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^
11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8
*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a
^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 +
1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^
17))^(1/4)*log(a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*
c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^1
1*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8
*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 114861
60*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^1
3 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^
7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b
*c*d^3 - 13*a^4*d^4)*x) + 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*
b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b
^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*
a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391
2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^1
2 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c
*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*log(-a^2*b^4*(-(81*b^16*c^16 + 4
32*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*
a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 77
2112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9
723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^
4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^1
5*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*
d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4
*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*
x^4 + a^2*b^4)
```

giac [B] time = 0.17, size = 642, normalized size = 1.80

$$\frac{\sqrt{2}\left(3\,\left(ab^3\right)^{\frac{1}{4}}b^4c^4+4\,\left(ab^3\right)^{\frac{1}{4}}ab^3c^3d-30\,\left(ab^3\right)^{\frac{1}{4}}a^2b^2c^2d^2+36\,\left(ab^3\right)^{\frac{1}{4}}a^3bcd^3-13\,\left(ab^3\right)^{\frac{1}{4}}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x\right.}{2}\right.}{16\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a
*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/
4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*
d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a
*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a
*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^
3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c
^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13
*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b
^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^
3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^16*c*d^3
*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c*d^3*x + 135*a^
2*b^14*d^4*x)/b^18
```

maple [B] time = 0.06, size = 885, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a)^2,x)
```

```
[Out] 1/4/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3*d+9/8/b^3*a
*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d^3+1/8/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/
b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^
3*d+9/4/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^3+1/4
/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d+9/4/b^3*a*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^3+1/9*d^4*x^9/b^2+1/4
/a*x/(b*x^4+a)*c^4-2/5*d^4/b^3*x^5*a+4/5*d^3/b^2*x^5*c+6/b^2*c^2*d^2*x+3*a^
2/b^4*d^4*x-13/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-
1)*d^4-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d^2
-1/b^3*a^2*x/(b*x^4+a)*c*d^3+3/2/b^2*a*x/(b*x^4+a)*c^2*d^2-13/16/b^4*a^2*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^4-15/8/b^2*(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^2-13/32/b^4*a^2*(a/b)^(1/4)*2
^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2)))*d^4-15/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/
2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^2*d^2+1/4/b^4*
a^3*x/(b*x^4+a)*d^4-1/b*x/(b*x^4+a)*c^3*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arct
```

```
an(2^(1/2)/(a/b)^(1/4)*x-1)*c^4+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^4-8
*d^3/b^3*a*c*x+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
*c^4
```

maxima [A] time = 1.45, size = 521, normalized size = 1.46

$$\frac{\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)x}{4\left(ab^5x^4 + a^2b^4\right)} + \frac{5\,b^2d^4x^9 + 18\left(2\,b^2cd^3 - abd^4\right)x^5 + 45\left(6\,b^2c^2d^2 - 8\,abcd^3 + \right.}{45\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^4 + a^2*b^4) + 1/45*(5*b^2*d^4*x^9 + 18*(2*b^2*c*d^3 - a*b*d^4)
*x^5 + 45*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/32*(2*sqrt(2
)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^4*c^4 + 4*a*b
^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*
c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^
(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d -
 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^4)
```

mupad [B] time = 0.30, size = 2043, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^4/(a + b*x^4)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^5*((2*a*d^4)/(5*b^3) - (4*c*d^3)/(5*b^2)) + (d^4*x^9)/(9*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(4*
a*(a*b^4 + b^5*x^4)) + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c
^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 +
 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((
```

a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^
8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*
a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))
/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30
*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a
)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4))
- (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3
+ 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*
a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*
c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*
c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(
7/4)*b^(17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(8*(-a)^(7/4)*b^(17/4))
+ (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*
c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^
6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d
+ 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36
*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/
(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d
- b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a^8*d^8 + 9*b^8*c^8 -
164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5
*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4
*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^
2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) - (((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*
(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(17/4))

sympy [A] time = 47.54, size = 471, normalized size = 1.32

$$x^5\left(-\frac{2ad^4}{5b^3}+\frac{4cd^3}{5b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)+\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{4a^2b^4+4ab^5x^4}+\mathrm{RootSum}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

```
[Out] x**5*(-2*a*d**4/(5*b**3) + 4*c*d**3/(5*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*c
*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b*
*2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(4*a**2*b**4 + 4*a*b**5*x**4) +
 RootSum(65536*_t**4*a**7*b**17 + 28561*a**16*d**16 - 316368*a**15*b*c*d**1
5 + 1577784*a**14*b**2*c**2*d**14 - 4651504*a**13*b**3*c**3*d**13 + 8923164
*a**12*b**4*c**4*d**12 - 11486160*a**11*b**5*c**5*d**11 + 9723912*a**10*b**
6*c**6*d**10 - 4810608*a**9*b**7*c**7*d**9 + 617958*a**8*b**8*c**8*d**8 + 7
72112*a**7*b**9*c**9*d**7 - 434808*a**6*b**10*c**10*d**6 + 20400*a**5*b**11
*c**11*d**5 + 45724*a**4*b**12*c**12*d**4 - 8304*a**3*b**13*c**13*d**3 - 23
76*a**2*b**14*c**14*d**2 + 432*a*b**15*c**15*d + 81*b**16*c**16, Lambda(_t,
 _t*log(-16*_t*a**2*b**4/(13*a**4*d**4 - 36*a**3*b*c*d**3 + 30*a**2*b**2*c*
*2*d**2 - 4*a*b**3*c**3*d - 3*b**4*c**4) + x))) + d**4*x**9/(9*b**2)
```

3.168 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$

Optimal. Leaf size=317

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/5*d^3*x^5/b^2+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a)
+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)
/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1+b^(1/4)*x*2^(1/2)/
a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/32*(-a*d+b*c)^2*(3*a*d+b*c)*ln(-a^(1/4)
*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+3/32*(-a*d
+b*c)^2*(3*a*d+b*c)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/
4)/b^(13/4)*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^
3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3
*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\left(a+bx^4\right)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{\left(a+bx^4\right)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{a+bx^4}\,dx}{4ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^3}+\frac{\left(3(bc-\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{7/2}}+\frac{\left(3(\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{\right.}}{16\sqrt{2}a^{7/4}b^{13/4}}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-}{}$$

Mathematica [A] time = 0.25, size = 301, normalized size = 0.95

$$\frac{-\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{30\sqrt{2}(bc-ad)^2(3ad+bc)\tan^{-}}{a^{7/4}}}{160b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
- a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(b*
c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[2]*(
b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

fricas [B] time = 0.90, size = 1938, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 60*(a*b^4*x
^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
)*arctan(-(a^5*b^10*x*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2
 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6
*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^
3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^1
3))^(3/4) - a^5*b^10*sqrt((a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*
a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*
c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8
- 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^
12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3
*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x^2)/(b^6*c
^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2
*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6))*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2
*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7
*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1
932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*
d^12)/(a^7*b^13))^(3/4))/(b^9*c^9 + 3*a*b^8*c^8*d - 12*a^2*b^7*c^7*d^2 - 20
*a^3*b^6*c^6*d^3 + 78*a^4*b^5*c^5*d^4 - 6*a^5*b^4*c^4*d^5 - 188*a^6*b^3*c^3
*d^6 + 252*a^7*b^2*c^2*d^7 - 135*a^8*b*c*d^8 + 27*a^9*d^9)) + 15*(a*b^4*x^4
 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*
b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d
^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 14
58*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*
log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
```

```
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) - 15*(a*b^4*x^
4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3
*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*
d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1
458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)
*log(-3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*
a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c
^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9
+ 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1
/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) + 20*(b^3*c^
3 - 3*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 9*a^3*d^3)*x)/(a*b^4*x^4 + a^2*b^3)
```

giac [A] time = 0.18, size = 496, normalized size = 1.56

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} b^3c^3 + \left(ab^3\right)^{\frac{1}{4}} ab^2c^2d - 5\left(ab^3\right)^{\frac{1}{4}} a^2bcd^2 + 3\left(ab^3\right)^{\frac{1}{4}} a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^4} + \frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} b^3 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)
^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d
- 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
+ (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)
*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/4*(b^3
*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a*b^3)
 + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

maple [B] time = 0.06, size = 669, normalized size = 2.11

$$\frac{d^3x^5}{5b^2} - \frac{a^2d^3x}{4\left(bx^4+a\right)b^3} + \frac{3ac\,d^2x}{4\left(bx^4+a\right)b^2} + \frac{c^3x}{4\left(bx^4+a\right)a} - \frac{3c^2dx}{4\left(bx^4+a\right)b} - \frac{2a\,d^3x}{b^3} + \frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16b^3} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a)^2,x)
```

```
[Out] 1/5*d^3*x^5/b^2-2*a/b^3*d^3*x+3/b^2*c*d^2*x-1/4/b^3*a^2*x/(b*x^4+a)*d^3+3/4
/b^2*a*x/(b*x^4+a)*c*d^2-3/4/b*x/(b*x^4+a)*c^2*d+1/4/a*x/(b*x^4+a)*c^3+9/16
/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^3-15/16/b^2*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^2+3/16/b/a*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3+9/32/b^3*a*(a/b)^(1/4)*2^(1/2)*ln((x^
2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)
))*d^3-15/32/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1
/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d^2+3/32/b/a*(a/b)^(1/4)*2^
(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x
+(a/b)^(1/2)))*c^2*d+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/
2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^3+9/16/b^3*a*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^3-15/16/b^2*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^2+3/16/b/a*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)*c^3
```

maxima [A] time = 1.31, size = 405, normalized size = 1.28

$$\frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x}{4\left(ab^4x^4 + a^2b^3\right)} + \frac{bd^3x^5 + 5\left(3\,bcd^2 - 2\,ad^3\right)x}{5\,b^3} + \frac{3\left(\frac{2\sqrt{2}\left(b^3c^3 + ab^2c^2d - 5\,a^2bcd^2 + 3\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\cdots\right.}{2\sqrt{\cdots}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^4 + a^2*
b^3) + 1/5*(b*d^3*x^5 + 5*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3/32*(2*sqrt(2)*(b
^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^
3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 + a*b^2*c^2*d
- 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2
```

```
 + 3*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)))/(a*b^3)
```

mupad [B] time = 1.53, size = 1616, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^3/(a + b*x^4)^2,x)
```

```
[Out] (d^3*x^5)/(5*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a*(a*b^3 + b^4*x^4)) + (atan((((a*d
 - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*
a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*
a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b
^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^
(13/4)) + ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b
^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^
5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b
^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(1
6*(-a)^(7/4)*b^(13/4)))/((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 +
b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*
b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(
36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*
b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) - (3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x
*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*
c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(
3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(
16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d +
 b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (3*atan(((3*(a*d - b*c)^2*(3*a*d + b*c)
*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^
4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - ((a*d - b*c)
^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2
)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) + (3*(a*d - b*c)^
2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*
c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3)
 + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d -
 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)))/(
((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2
 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5)
)/(4*a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*
a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7
/4)*b^(13/4)) - ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9
*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d -
 30*a^5*b*c*d^5))/(4*a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 +
```

```
12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))
*3i)/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*
b^(13/4))
```

sympy [A] time = 6.99, size = 337, normalized size = 1.06

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x\left(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3\right)}{4a^2b^3+4ab^4x^4}+\text{RootSum}\left(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(4*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t**
4*a**7*b**13 + 6561*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b**2
*c**2*d**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 + 2656
8*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5*b**7*c**7*d*
*5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9*d**3 - 1134*a**2*b**10
*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**12*c**12, Lambda(_t, _t*log(16*_t
*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b**3*c**3)
 + x))) + d**3*x**5/(5*b**2)
```

3.169 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2} dx$

Optimal. Leaf size=291

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}$$

```
[Out] d^2*x/b^2+1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*
arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/16*(-a*d+b*c
)*(5*a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)
-1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))/a^(7/4)/b^(9/4)*2^(1/2)+1/32*(-a*d+b*c)*(5*a*d+3*b*c)*ln(a^(1/4)*b^(
1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c
 + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/
4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b
*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{b^2\left(a+bx^4\right)^2}\right)dx$$

$$= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{\left(a+bx^4\right)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}+\frac{((bc-ad)(3bc+5ad))\int \frac{1}{a+bx^4}\,dx}{4ab^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}+\frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b^2}+\frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}+}{a+}}{8a^{3/2}b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}+\frac{((bc-ad)(3bc+5ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{5/2}}+\frac{((bc-ad)(3bc+5ad))\int}{16a^{3/2}b^{5/}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}-\frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(3b}{}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}-\frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(3bc+5ad)\tan^{-}}{8\sqrt{2}\,a^{7/4}b^{9/4}}$$

Mathematica [A] time = 0.23, size = 297, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\left(-5a^2d^2+2abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{2\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right.}{a^{7/4}}}{32b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[2
]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*
a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) +
(Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

fricas [B] time = 1.08, size = 1335, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b*d^2*x^5 - 4*(a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^
7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(
a^7*b^9))^(1/4)*arctan((a^5*b^7*x*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2
*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3
*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/
4) - a^5*b^7*sqrt((a^4*b^4*sqrt(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9)) + (9*b
^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4)
*x^2)/(9*b^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 2
5*a^4*d^4))*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3
*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2
*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/4))/(27*b^6*c^6 + 54*a
*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4
 + 150*a^5*b*c*d^5 - 125*a^6*d^6)) - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 +
216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c
^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62
5*a^8*d^8)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 3
24*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b
^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9
))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + (a*b^3*x^4 + a^2*b^2)*(
-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3
+ 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a
^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(81*b^8*c^8 + 216
*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + 4*(b^2*
c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^4 + a^2*b^2)
```

giac [A] time = 0.17, size = 376, normalized size = 1.29

$$\frac{d^2x}{b^2}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}abcd-5\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d
 - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/
(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^
(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2
 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 +
 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((b*x^4 + a)*a*b^2)
```

maple [B] time = 0.06, size = 475, normalized size = 1.63

$$\frac{a\,d^2x}{4\left(b\,x^4+a\right)b^2}+\frac{c^2x}{4\left(b\,x^4+a\right)a}-\frac{cdx}{2\left(b\,x^4+a\right)b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a)^2,x)
```

```
[Out] 1/b^2*d^2*x+1/4/b^2*a*x/(b*x^4+a)*d^2-1/2/b*x/(b*x^4+a)*c*d+1/4/a*x/(b*x^4+
a)*c^2-5/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^2+1/8
/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d+3/16/a^2*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2-5/32/b^2*(a/b)^(1/4)*2^(
1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+
(a/b)^(1/2)))*d^2+1/16/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d+3/32/a^2*(a/b)^
(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2
^(1/2)*x+(a/b)^(1/2)))*c^2-5/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)*d^2+1/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)*c*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2
```

maxima [A] time = 1.26, size = 319, normalized size = 1.10

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x}{4\left(ab^3x^4+a^2b^2\right)}+\frac{d^2x}{b^2}+\frac{\frac{2\sqrt{2}\left(3\,b^2c^2+2\,abcd-5\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(3\,b^2c^2+2\,abcd-5\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{\cdots}\right.}{2\cdots}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^4 + a^2*b^2) + d^2*x/b^2 + 1
/32*(2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + 2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sq
rt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

mupad [B] time = 0.30, size = 1254, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^2/(a + b*x^4)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*a*(a*b^2 + b^3*x^4)) +
 (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*
a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9
/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^
4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/
(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b
^2*c*d))/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)))/(((a*d - b*c
)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^
3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/
4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26
*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*
c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4
)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4))))*(a*d - b*c)*(5*a*d + 3*b*c)*1i)/(8*(
```

```
-a)^(7/4)*b^(9/4)) + (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 +
9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b
) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*
1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d
 + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d
 - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 -
20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^
(9/4)))/(((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*
b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5
*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*
b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(2
5*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^
3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8
*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4))))*(a*d
 - b*c)*(5*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(9/4))
```

sympy [A] time = 2.17, size = 219, normalized size = 0.75

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4a^2b^2+4ab^3x^4}+\text{RootSum}\left(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + RootS
um(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*b*
*2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*a**
3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**8*c*
*8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b**2*c*
*2) + x))) + d**2*x/b**2
```

3.170 $\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(\right.}{8\sqrt{2}\,a^{7/4}}$$

```
[Out] 1/4*(-a*d+b*c)*x/a/b/(b*x^4+a)+1/16*(a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)
/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*(a*d+3*b*c)*arctan(1+b^(1/4)*x*2^(1/
2)/a^(1/4))/a^(7/4)/b^(5/4)*2^(1/2)-1/32*(a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/32*(a*d+3*b*c)*ln(a^
(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(\right.}{8\sqrt{2}\,a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c + a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(5/4)) + ((3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx = \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} + \frac{(3bc+ad)\int \frac{1}{a+bx^4}\,dx}{4ab}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} + \frac{(3bc+ad)\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b} + \frac{(3bc+ad)\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}} - \frac{(3bc+ad)}{1\cdots}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\cdots\right.}{16\sqrt{2}\,a^{7/4}b^{5/4}}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} - \frac{(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}} - \frac{(3bc+ad\cdots}{}$$

Mathematica [A] time = 0.17, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}\,x(ad-bc)}{a+bx^4} - \sqrt{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right) + \sqrt{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{\cdots}\right.}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

fricas [B] time = 0.88, size = 711, normalized size = 2.90

$$\frac{4\left(ab^2x^4+a^2b\right)\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5b^4x\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{3}{4}}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^
2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*arctan(-(a^5*b^4*x*(-(81
*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)
/(a^7*b^5))^(3/4) - a^5*b^4*sqrt((a^4*b^2*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3
*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5)) + (9*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^2)/(9*b^2*c^2 + 6*a*b*c*d + a^2*d^2))*(-(81*b^4*c
^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*
b^5))^(3/4))/(27*b^3*c^3 + 27*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)) + (a*
b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*
a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4 + 108*a*b^3
*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) +
(3*b*c + a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54
*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*b*(-
(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d
^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4 + a^2
*b)
```

giac [**A**] time = 0.17, size = 266, normalized size = 1.09

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}b^{2}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}b^{2}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^3)
^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32*sqrt
(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

maple [**A**] time = 0.05, size = 295, normalized size = 1.20

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\cdots}{16a^{2}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/a/b*x/(b*x^4+a)+1/16/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)*c+1/32/a/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d+3/32/a^2*(a/b)^(1/4)*2^(1/2)
*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))*c+1/16/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d+
3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c
```

maxima [A] time = 1.35, size = 236, normalized size = 0.96

$$\frac{(bc-ad)x}{4\left(ab^2x^4+a^2b\right)}+\frac{\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc+ad)\log\left(\sqrt{b}x^2+\sqrt{\cdots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c - a*d)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(3*b*c + a*d)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c + a*d)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [B] time = 1.53, size = 740, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}+\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}}{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)1i}{16(-a)^{7/4}b^{5/4}}-\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)1i}{16(-a)^{7/4}b^{5/4}}}\right)(ad\cdots}{8(-a)^{7/4}b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)/(a + b*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)
*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((
a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)
*1i)/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/
(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(
a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b
^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^
(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c)*1i)/(8*(-a)^
(7/4)*b^(5/4)) + (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2)
 - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d
+ 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c
*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(
5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2
 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-
a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*
c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^
2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4))
))*(a*d + 3*b*c))/(8*(-a)^(7/4)*b^(5/4)) - (x*(a*d - b*c))/(4*a*b*(a + b*x^
4))
```

sympy [A] time = 0.97, size = 112, normalized size = 0.46

$$\frac{x\left(-ad+bc\right)}{4a^2b+4ab^2x^4}+\operatorname{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16t\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5 +
 a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*d
+ 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

3.171 $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=513

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc}{}$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)+1/16*b^(3/4)*(-7*a*d+3*b*c)*arctan(-1+b^(1/4
)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/16*b^(3/4)*(-7*a*d+3*b*
c)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/4*d^(
7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/4*
d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/
32*b^(3/4)*(-7*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2)
)/a^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/32*b^(3/4)*(-7*a*d+3*b*c)*ln(a^(1/4)*b^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(7/4)*
ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)^2*2^(
1/2)+1/8*d^(7/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/
(-a*d+b*c)^2*2^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (b^(3/4)*(
3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(b*c - a*d)^2) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d
)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) +
```

```
(d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{-3bc+4ad-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^2\int \frac{1}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(3bc-7ad))\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^2\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)^2} + \frac{d^2\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)^2} + \frac{(b(3bc-7ad))\int \cdots}{8a^{3/2}(bc-\cdots}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,(bc-ad)^2} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,(bc-ad)^2} - \frac{d^{7/4}\int \cdots}{4\sqrt{\cdots}}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3b\cdots}{\cdots}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} - \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\tan^{-\cdots}}{8\sqrt{2}\,a^{7/4}(bc\cdots}$$

Mathematica [A] time = 0.35, size = 499, normalized size = 0.97

$$8a^{3/4}bc^{3/4}x(bc-ad)-8\sqrt{2}\,a^{7/4}d^{7/4}\left(a+bx^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+8\sqrt{2}\,a^{7/4}d^{7/4}\left(a+bx^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)-4\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a
*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/4)
*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*
b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4)*(b*c - a*
d)^2*(a + b*x^4))
```

fricas [B] time = 36.68, size = 3299, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/16*(4*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*
b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(
a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 +
70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*
c*d^7 + a^15*d^8))^(1/4)*arctan(((a^5*b^6*c^6 - 6*a^6*b^5*c^5*d + 15*a^7*b^
4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4 - 6*a^10*b*c*d^5 + a^11
*d^6)*x*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^
6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 2
8*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4) - (a^5*b^6*c^6 - 6*a
^6*b^5*c^5*d + 15*a^7*b^4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4
 - 6*a^10*b*c*d^5 + a^11*d^6)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^
5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7
*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 5
6*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4
```

```
)*sqrt(((9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)*x^2 + (a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)*sqrt(-(81*b^7*
c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^
4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^
5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6
 - 8*a^14*b*c*d^7 + a^15*d^8)))/(9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)
))/(27*b^5*c^3 - 189*a*b^4*c^2*d + 441*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)) -
16*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d
^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b
*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*
arctan(-((b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*(-d^7/(b^8*c^11 - 8*
a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4
 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)
)^(3/4)*x - (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*
d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*(-d^7/(b^8*c^11 -
 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*
d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d
^8))^(3/4)*sqrt((d^4*x^2 + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4
*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2
*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5
 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)))/d^4))/d^5) - 4*(-d
^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 7
0*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d
^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*log(d^
2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^
8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^
7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + 4*
(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3
+ 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^
4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*log
(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5
*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8
*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) +
 ((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*
d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*
c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*
b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x + (a^2*b^2*c^2 - 2*a^3*b*c*d +
a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*
b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c
^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 +
28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - ((a*b^2*c - a^2*
b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*
c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c
```

```
^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*
a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*
log(-(3*b^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^
7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*
a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*
b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d
^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 +
 a^2*b*c - a^3*d)
```

giac [A] time = 0.22, size = 667, normalized size = 1.30

$$\frac{\left(cd^3\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right)} + \frac{\left(cd^3\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right)} + \frac{\left(cd^3\right)^{\frac{1}{4}} d \log\left(x^2 + \sqrt{2}\, x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right.}{4\left(\sqrt{2}\, b^2c^3 - 2\sqrt{2}\, abc^2d + \sqrt{2}\, a^2cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c*d
^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^(1/4
)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(
2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*
a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt
(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2
)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(3*(a*b^3)^(1
/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/16*(3*(a
*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/
4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

maple [A] time = 0.06, size = 550, normalized size = 1.07

$$\frac{b^2cx}{4\left(ad-bc\right)^2\left(b\,x^4+a\right)a} - \frac{bdx}{4\left(ad-bc\right)^2\left(b\,x^4+a\right)} - \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, bd \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16\left(ad-bc\right)^2 a} - \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, bd \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right.}{16\left(ad-bc\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d^2/(a*d-b*c)^2*(c/d
)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d^2/(a*d-b*c)^2*(c/d)
^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/4*b/(a*d-b*c)^2*x/(b*x^4
+a)*d+1/4*b^2/(a*d-b*c)^2/a*x/(b*x^4+a)*c-7/16*b/(a*d-b*c)^2/a*(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c-7/16*b/(a*d-b*c)^2/a*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-7/32*b/(a*d-b*c)^2/a*(a/b
)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2)))*d+3/32*b^2/(a*d-b*c)^2/a^2*(a/b)^(1/4)*2^(1/2)*ln(
(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1
/2)))*c
```

maxima [A] time = 1.46, size = 470, normalized size = 0.92

$$\frac{\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-}{32\left(ab^2c^2-2a^2bcd+a^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a
^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*
sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3
*b*c - 7*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(
1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)
+ 1/4*b*x/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d) + 1/8*(2*sqrt(2)*d^2*
arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(
2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1
```

```
/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4
)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
```

mupad [B] time = 3.82, size = 21975, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)),x)
```

```
[Out] 2*atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b
^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*
a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458752
0*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 -
524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (67
5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)
*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*
c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*
b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 183
5008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4
- 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7
))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^1
4 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8
*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^
5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5
*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*
(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 4
66944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10
*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8
*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2
809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2
*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d
^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i) + (x*(3
185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d
^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d
 + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c
*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^
5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a
^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520
*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5
24288*a^14*b*c*d^7))^(1/4) - ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b
```

^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4))/(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a

^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(
65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6
*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^
12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(30
72*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114
688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 -
229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11
- 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^
3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^1
4*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152
*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8
- 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^1
0*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 346931
2*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^
6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*
b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i)*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4
*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1
i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*
a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 240
1*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d
)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*
b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016
*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)
+ ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2
*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^
7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11
*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288
*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^
9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(
a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 +
2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^
3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a
^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670
016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/
4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c
^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*
b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^
11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5242
88*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28
672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 +
329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10
+ 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d
^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536
*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*
a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 +

17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i)*1i - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) - atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))*1i - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(6

4*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b
^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4
*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65
536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c
^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12
*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) - ((-
(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2
- 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7
*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*
c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14
*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*
d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 -
a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b
^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(6553
6*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6
*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b
^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81
*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 7
56*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d
+ 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4
*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*
c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^
13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^
8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a
^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7
*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*
d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c
^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*
a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 -
14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*
c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^1
0*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3
*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))*1i + (x*(3185*a^4*b^7*d^13 +
81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*
c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^
4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^
7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*
a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1
835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^
4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d
^7))^(1/4))/(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*
a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52
4288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4
587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d
^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 -

(675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)
/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*
c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*
b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 183
5008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4
- 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7
))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^1
4 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8
*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^
5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5
*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*
(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 4
66944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10
*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8
*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2
809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2
d^15))/(64(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2
- 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) - (x*(3185*a^
4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 +
2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*
a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))
)*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*
d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7
*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*
b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*
a^14*b*c*d^7))^(1/4) + ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d
^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^
8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c
^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13
*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4
*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^
8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) +
(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^
2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c
^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^
4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^
14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3
+ 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c
^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*
d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^
2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14

*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^
7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a
^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 2867
2*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*
d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^
13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5
*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24
190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^
5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^
13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) + (
x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8
*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c
^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9
*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^
2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 5242
88*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458
7520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
 - 524288*a^14*b*c*d^7))^(1/4)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^
3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 655
36*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a
^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183
5008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*2i - atan(((-d^7/(256*b
^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 143
36*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a
^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c
^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3
 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 20
48*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b
*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7
*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(
3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^
2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b
^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^1
4*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^
12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9
*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^
12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*
b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^
16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*
c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512
*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11
 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7
*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5
*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 -

6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) - (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))/((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))

^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10
*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*
c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7
*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 -
a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 -
524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5
 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^1
3*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 145162
24*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^1
3 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 +
 a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 +
15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7
)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^
2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) - (x*
(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c
*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5
*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b
*c*d^5))) + (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 71
68*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*
a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(
256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2
- 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7
168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*
a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8
*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6
 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048
*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b
^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10
d))^(1/4)(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*
c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b
^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11
*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^
3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^1
7 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12
*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6
*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14
516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4
*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d
^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^
3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4
*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^
8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) +
 (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b
^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5
*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a

^9*b*c*d^5)))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*2i + 2*atan(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) - (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10

+ 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^
13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*
a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a
^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 +
17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c
^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*
a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*
1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20
*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*
d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10
)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^
2*d - 3*a^6*b*c*d^2)) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^1
0*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))/((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^
8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 179
20*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*
b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*
d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4
- 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*
((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*
c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6
*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^
11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^
3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^
2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b
^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*
b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^1
0*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*
a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^
2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13
*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b
^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 2419
0976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*
d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13
*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b
^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i
+ ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145
*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3
 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c
^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)
1i)/(64(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 2
0*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (-d^7/(256*b^8*
c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*
a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*

```
b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*
d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 1
7920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*
a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^
4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^
4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4
)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b
^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*
c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c
^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*
c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^
9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*
b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2
*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16
+ 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^1
1*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^
7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 +
3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^
3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5
*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 -
 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^
9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(
a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4
*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2
790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 1
5*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5
)))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2
*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^
3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4) - (b*x)/(4*a*(
a + b*x^4)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.172 $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=596

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc}{}$$

```
[Out] 1/4*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^4+c)+1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)
/(d*x^4+c)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4)
)/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+b^(1/4
)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(7/4)*(-3*a*d+11*b
*c)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*
d^(7/4)*(-3*a*d+11*b*c)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b
*c)^3*2^(1/2)-1/32*b^(7/4)*(-11*a*d+3*b*c)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*b^(7/4)*(-11*a*d+3*b*c
)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^
(1/2)-1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^
2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/32*d^(7/4)*(-3*a*d+11*b*c)*ln(c^(
1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d
)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c -
11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*
(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3
```

```
) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-3bc+4ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-4\left(3b^2c^2-8abcd+3a^2d^2 \cdots}{\left(a+bx^4\right)\left(c+ \cdots\right.}}{16ac(bc- \cdots}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^2(3bc-11ad)\right)\int \cdots}{4a(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^2(3bc-11ad)\right)\int \cdots}{8a^{3/2}(bc-a \cdots}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^{3/2}(3bc-11ad)\right)\int \cdots}{16a^{3/2}(bc \cdots}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{b^{7/4}(3bc-11ad)\log \cdots}{16\sqrt{2} \cdots}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{b^{7/4}(3bc-11ad)\tan \cdots}{8\sqrt{2}\,a^{7/4}(bc \cdots}$$

Mathematica [A] time = 6.19, size = 629, normalized size = 1.06

$$\frac{b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11a \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (b^2*x)/(4*a*(-(b*c) + a*d)^2*(a + b*x^4)) + (d^2*x)/(4*c*(b*c - a*d)^2*(c
+ d*x^4)) - (b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/
4)*x)/(Sqrt[2]*a^(1/4))])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(-3*
b*c + 11*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(8
```

```
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(-(Sqrt[
2]*c^(1/4)) + 2*d^(1/4)*x)/(Sqrt[2]*c^(1/4))])/(8*Sqrt[2]*c^(7/4)*(-(b*c) +
 a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*x)
/(Sqrt[2]*c^(1/4))])/(8*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) + (b^(7/4)*(-3*b*
c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(
-(b*c) + a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 967, normalized size = 1.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*
b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(3*(a*b^3)^(1/4)
*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2
)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^
(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*
a^3*c^2*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*s
qrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/16*
(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(
2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/16*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3
)^(1/4)*a*b*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^
3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3)
+ 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(x^2 + sqrt(2)*x
```

```
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt
(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) - 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3
*(c*d^3)^(1/4)*a*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^
2*d^3) + 1/4*(b^2*c*d*x^5 + a*b*d^2*x^5 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^8
+ b*c*x^4 + a*d*x^4 + a*c)*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

maple [A] time = 0.07, size = 784, normalized size = 1.32

$$\frac{a d^3 x}{4(ad-bc)^3\left(d x^4+c\right)c}-\frac{b^3 c x}{4(ad-bc)^3\left(b x^4+a\right)a}+\frac{b^2 d x}{4(ad-bc)^3\left(b x^4+a\right)}-\frac{b d^2 x}{4(ad-bc)^3\left(d x^4+c\right)}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,\cdots}{16(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^3/(a*d-b*c)^3/c*x/(d*x^4+c)*a-1/4*d^2/(a*d-b*c)^3*x/(d*x^4+c)*b+3/16*
d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-1
1/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*
b+3/32*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*
x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a-11/32*d^2/(a*d-b*
c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-
(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+1/4*b^2/(a*d-b*c)^3*x/(b*x^4+a
)*d-1/4*b^3/(a*d-b*c)^3/a*x/(b*x^4+a)*c+11/32*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)
*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2
)*x+(a/b)^(1/2)))*d-3/32*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a
/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c
+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*d-3/16*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x-1)*c+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)*d-3/16*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)*c
```

maxima [A] time = 1.29, size = 670, normalized size = 1.12

$$\frac{\left(\frac{2\sqrt{2}\,(3\,bc-11\,ad)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,bc-11\,ad)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(3\,bc-11\,ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{32\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(3*b*c - 11*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c^3 - 3*a^2*b^2*c^2*d +
 3*a^3*b*c*d^2 - a^4*d^3) + 1/4*((b^2*c*d + a*b*d^2)*x^5 + (b^2*c^2 + a^2*d
^2)*x)/((a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^8 + a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 +
 a^4*c*d^3)*x^4) + 1/32*(2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2
)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*s
qrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sq
rt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 + sqr
t(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 -
 3*a*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d
^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

mupad [B] time = 5.62, size = 37266, normalized size = 62.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^5
*(a*d + b*c))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^4*(a*d + b*
c) + b*d*x^8) - atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^
```

12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920
*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c
^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*
b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d
^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^1
2*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*
a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^
11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b
^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*
a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32
 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5
*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*
d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)
 + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c
^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432
*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 +
32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^
13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a
^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^1
5*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 49
5*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^1
1*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c
^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 653
4*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^
12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*
c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264
*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 -
 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18
d))^(1/4)(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b
^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483
072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^1
2*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448307
2*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16

+ 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d
^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 +
28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*
c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8
*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c
^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a
^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^
16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^
2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 92
4*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^1
3*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*
b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7
+ 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7
 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b
^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*
c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512
*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 +
 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c
^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*
c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*
a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d
^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*
b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*
c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 2
8*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^
12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920
*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c
^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*
b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*
d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096
*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^1
5*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 -
3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 900772659
2*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^
10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 -
 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b
^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^
12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a
^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^

6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7
*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972
*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19
 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7
*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 3244032
0*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10
- 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^1
8*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*
b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1
993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^1
0*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 11151
36*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 +
 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d -
 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c
^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (x*(9801*a^8*b^9*
d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16
+ 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*
c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024
*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 +
 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a
^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b
^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*a^4*d^1
1 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a
^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^1
1 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^
8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 519045
12*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11
 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^
3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11
 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8
*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5190451
2*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d
^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*
c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (
60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9
*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c
^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56
*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*
b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7

+ 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 114
03264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c
^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 779
6490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^
10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c
^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16
+ 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096
*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^
21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^1
2*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b
^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^
11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^
8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641
*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^
10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 43253
76*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^
4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^
5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 432537
6*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4
- 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^1
6*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^1
0*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^1
2 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*
b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056
*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8
- 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^
9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))) +
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^1
5*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^
12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^
7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) +
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*
a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 3
2440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^1
3*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^
9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13

```
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((
(891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 -
(3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^
5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 +
(31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*
d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^
4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14
641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c
*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 43
25376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15
*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7
*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 432
5376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*
c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394
368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*
c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 +
 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 36354785
28*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*
b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^1
8 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a
^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/
(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d
^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 -
792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a
^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^
4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1
188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^
8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a
^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5
1904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^1
0*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*
b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 111513
6*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9
 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12
*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 +
 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^
6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 +
 a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 -
 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2
*c^6*d^6))) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7
*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^
14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 10015
20*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c
^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
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495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a
^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2
*c^6*d^10)))))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6
534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*
d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^
9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 605552
64*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8
 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^
18*d))^(1/4)*2i + 2*atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*
c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 6553
6*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 1441
7920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d
^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b
^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 78643
2*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536
*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417
920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^
5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^
4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432
*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (
3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^
9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (3306
9*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 +
a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 -
56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*
c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*
a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12
 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^
16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a
^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 1
4417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d
))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762
752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c
^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10
168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*
a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12
*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17
 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*
a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22
 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*
a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c
^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d
^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 6
6*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c

^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536
*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417
920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^
5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^
4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432
*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 +
292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^1
5*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a
^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d
^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*
b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^
19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*
b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 -
56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i) - (x*(9801*a^8*b^9*d^17 + 9
801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 100152
0*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13
- 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*
c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^1
0*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11
*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8
- 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b
^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10
)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376
*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4
- 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*
c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*
a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^
4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)
/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*
a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 -
51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c
^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a
^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 +
(891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16
+ (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*
b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)
/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
+ 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b
^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15
972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c
^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*
d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*
b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 3244
0320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^
10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*

a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^
7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 779649024
0*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^1
5*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^
14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796
490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*
b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 1
1403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c
^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10
*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*
d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 -
220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^
4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)
/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*
a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 -
51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c
^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a
^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 4
5056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d
^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b
^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 +
4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8
*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^
18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8
*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d
^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i) +
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^1
5*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^
12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^
7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/
((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*
a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 3
2440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^1
3*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^
9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((

((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i)*1i - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*

d^9 + 66*a^14*b^2*c^6*d^10))) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*
a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19
+ 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
- 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*
c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320
*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 -
786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a
*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 +
65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 -
14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c
^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*
a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 -
786432*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/
64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*
c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 -
(33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c
^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*
d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^1
0*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 +
6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^
7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*
b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 6055
5264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d
^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*
c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 +
98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*
b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^1
0 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 36354
78528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^1
3*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^
8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 5103
94368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^
3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12
- 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7
*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*
c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d
^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a
*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 +
65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 -
14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c
^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*
a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 -
786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*
d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^
15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 199

3728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*
c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136
*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3
072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8
*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8
*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i)*1i + (x*(9801*a^8*b^9
*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16
 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13
*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(102
4*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11
+ 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*
a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*
b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*a^4*d^
11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*
a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^
11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b
^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904
512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^
9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4) - atan(((-(81*
b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 -
 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^1
1*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320
*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 -
 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*
c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b
^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 -
1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11
*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*
a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 -
51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c
^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^
11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1
188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*
c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a
^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 5
1904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^
3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(58982
4*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d
^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 434484019
2*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*
c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13
 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 101683
69152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*
b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 +
 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*

c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a
^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*
c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*
d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
+ ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*
c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 78643
2*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*
c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*
a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)
*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d
^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^
14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1
993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9
*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*
a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4
*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6
*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 +
28*a^10*b^2*c^6*d^6)) + ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3
105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9
)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069
*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*
c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^
7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d
^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^1
0 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c
^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a
^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^
15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 4
95*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^
11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*
c^6*d^10))) + (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 65
34*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c
^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^
9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 6055526
4*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8
- 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d
^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 653
4*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^
12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9
*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264
*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 -
14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^
11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534
*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^1

2 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*
c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*
a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 -
14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^1
1))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 9876
2752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*
c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 1
0168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528
*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^1
2*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^1
7 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368
*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^2
2 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^
5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^1
3*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*
a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c
*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a
^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 1441792
0*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5
+ 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4
*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a
^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 29
2864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*
d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^1
1*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^1
3 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^
7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19
*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*
c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 5
6*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - ((891*a^8*b^7*d^15)/64 + (891*b
^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31
509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c
^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(
a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*
b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5
 + 28*a^10*b^2*c^6*d^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 1
49094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11
 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*
c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d
^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*
a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b
^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^
7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 1597
2*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^1

2 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^
2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b
^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 324403
20*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10
 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972
*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12
 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2
 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^
7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 3244032
0*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10
- 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*
a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12
+ 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2
- 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7
*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320
*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 -
 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*
b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 +
1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^
8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^
13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 +
 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 77964902
40*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*
c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403
264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a
^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d
^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 9
24*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^
13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d
^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536
*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^1
0*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 519045
12*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7
 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2
*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^
5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 27
45344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^1
3*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 459571
2*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^
15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*
c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^
7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70
*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + ((891*a^8*b
^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*
b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16

+ (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6
*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11
*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
- 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (x*(9801*a^8*b^9*d^17 + 9801
*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a
^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 -
3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^1
6 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c
^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^
5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 2
20*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*b^11*c^4 + 14641*a^4*
b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(
65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^
9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 5
1904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^
5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^1
7*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b
^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(6
5536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9
*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51
904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5
*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17
*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^
7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65
536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*
b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 519
04512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*
d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*
b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 -
11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^
20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 +
7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 900772659
2*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^
13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d
^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 176691
6096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^
4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4
*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a
^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^
7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^
8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 1
4641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10
*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4
325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^
8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^

14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 43
25376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*
d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16
*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712
*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11
*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a
^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 4
5056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*
d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))
 - ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/1
6 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^1
2*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/1
6 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c
^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^
8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (x*(9801*a^8*b
^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^
16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^
13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024
*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 +
 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a
^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b
^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*b^11*c^
4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a
*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*
d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b
^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 519045
12*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9
 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*2i + 2*atan(((-
(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d
^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8
*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 3244
0320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d
^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*
b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(
81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^
2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*
b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440
320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^
6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b
^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(8
1*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2
 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b
^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 324403
20*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6

- 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^
3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(5
89824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^
19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 43448
40192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b
^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*
d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10
168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a
^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^
20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*
b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d
- 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a
^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b
^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*
d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a
^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12
- 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^
9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^
13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14
417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11)
)^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17
*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072
*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d
^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a
^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 +
292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19
))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*
a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7
*d^5 + 28*a^10*b^2*c^6*d^6))*1i + (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d
^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b
^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/
32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b
^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c
^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28
*a^10*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*
b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 348460
2*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14
+ 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^
5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^1
3*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
- 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*
a^14*b^2*c^6*d^10))) + (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c
*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a
^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 1441792
0*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5

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+ 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4
*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a
^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*
d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^
7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920
*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 +
 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*
c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^
18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d
^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7
*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*
a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 +
60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c
^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^1
8*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d
^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*
a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15
*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3
635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592
*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^1
0*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 -
510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^
5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 159
72*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^
12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d
^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*
b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440
320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^1
0 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c
^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^
8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 -
 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b
^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 111
5136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18
 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d
 - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4
*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (((891*a^8*b^7*d
^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*
c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (
60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9
*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11
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*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
- 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^17 + 9801
*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a
^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 -
3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^1
6 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c
^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^
5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 2
20*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*b^11*c^4 + 14641*a^4*
b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(
65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^
9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 5
1904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^
5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^1
7*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b
^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(6
5536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9
*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51
904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5
*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17
*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^
7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65
536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*
b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 519
04512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*
d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*
b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 -
11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^
20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 +
7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 900772659
2*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^
13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d
^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 176691
6096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^
4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(
a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 6
6*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9
*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4
*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4
+ 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b
^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d
+ 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8
*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512
*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 +
4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^

19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b
^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595
712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c
^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 274534
4*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17
- 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c
^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7
*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^
6))*1i + (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7
*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*
a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*
d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 -
8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*
d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i -
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*
a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8
*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9
*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
) - (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9
*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 7864
32*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3
+ 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6
*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920
*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4
)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*
c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 78643
2*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*
c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*
a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)
*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c
^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432
*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c
^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a
^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484

```
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*
c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*
a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^
2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 +
6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12
*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*
b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555
264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^
8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c
*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^
6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4
483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*
c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448
3072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d
^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^
4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b
^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (((891*a^8*b^7*d^15)/64 + (891*b^15
*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509
*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*
d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/
(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6
*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^
5 + 28*a^10*b^2*c^6*d^6))*1i - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 -
149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11
 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12
*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 159
72*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^
12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d
^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*
b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440
320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^1
0 - 786432*a^18*b*c*d^11))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.173 $\int \frac{\left(a-bx^4\right)^{5/2}}{c-dx^4} \, dx$

Optimal. Leaf size=321

$$\frac{\sqrt[4]{a}\, b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\left(47a^2d^2-56abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^3\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\right.\right.}{2\sqrt[4]{b}\,cd^3\sqrt{a-bx^4}}$$

```
[Out] 1/7*b*x*(-b*x^4+a)^(3/2)/d-1/21*b*(-13*a*d+7*b*c)*x*(-b*x^4+a)^(1/2)/d^2+1/
21*a^(1/4)*b^(3/4)*(47*a^2*d^2-56*a*b*c*d+21*b^2*c^2)*EllipticF(b^(1/4)*x/a
^(1/4),I)*(1-b*x^4/a)^(1/2)/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*E
llipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)
^(1/2)/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*EllipticPi(b
^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/
4)/c/d^3/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {416, 528, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}\, b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\left(47a^2d^2-56abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}\,(7bc-13ad)}{21d^2}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] -(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^4)^(3/2))/
(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*d^3*Sqrt[a - b*x^4
]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(S
qrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(
1/4)*c*d^3*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{5/2}}{c-dx^4}\,dx = \frac{bx\left(a-bx^4\right)^{3/2}}{7d} - \frac{\int \frac{\sqrt{a-bx^4}\left(a(bc-7ad)-b(7bc-13ad)x^4\right)}{c-dx^4}\,dx}{7d}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx\left(a-bx^4\right)^{3/2}}{7d} + \frac{\int \frac{a\left(7b^2c^2-16abcd+21a^2d^2\right)-b\left(21b^2c^2-56abcd+47a^2d^2\right)}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{21d^2}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx\left(a-bx^4\right)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{d^3} + \frac{\left(b\left(21b^2c^2-\right.\right.}{}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx\left(a-bx^4\right)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^3} - \frac{(bc-ad)^3}{}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx\left(a-bx^4\right)^{3/2}}{7d} + \frac{\sqrt[4]{a}\,b^{3/4}\left(21b^2c^2-56abcd+47a^2d^2\right)\sqrt{1-\frac{b}{}}}{21d^3\sqrt{a-bx^4}}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx\left(a-bx^4\right)^{3/2}}{7d} + \frac{\sqrt[4]{a}\,b^{3/4}\left(21b^2c^2-56abcd+47a^2d^2\right)\sqrt{1-\frac{b}{}}}{21d^3\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.82, size = 290, normalized size = 0.90

$$\frac{x\left(-\frac{bx^4\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} + \frac{25a^2c\left(21a^2d^2-16abcd+7b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1\right.\right.}\right.}{105d^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] (x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) - (b*(21*b^2*c^2 - 56*a*b
*c*d + 47*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^
```

```
4)/a, (d*x^4)/c])/c + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*Appel
lF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2,
 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*
x^4)/c])))))/(105*d^2*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

maple [C] time = 0.31, size = 408, normalized size = 1.27

$$-\frac{\sqrt{-b\,x^4+a}\;b^2x^5}{7d}-\frac{\left(\frac{\left(-\frac{5a\,b^2}{7d}+\frac{(3ad-bc)b^2}{d^2}\right)a}{3b}-\frac{\left(3a^2d^2-3abcd+b^2c^2\right)b}{d^3}\right)\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}}+\frac{\left(-\frac{5a}{7}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

```
[Out] -1/7*b^2/d*x^5*(-b*x^4+a)^(1/2)+1/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/d*a)/b*x*(
-b*x^4+a)^(1/2)-(-b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+1/3*(b^2/d^2*(3*a*d-b
*c)-5/7*b^2/d*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-1/a^(1/2)*b^(1/2)*x^2)^(
```

```
1/2)*(1+1/a^(1/2)*b^(1/2)*x^2)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*
b^(1/2))^(1/2)*x,I)+1/8/d^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((
a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/
c*(1-1/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*b^(1/2)*x^2)^(1/2)/(-b*x^4+a
)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,a^(1/2)/b^(1/2)*_alpha^2/c*d
,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*
d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-bx^4 + a\right)^{\frac{5}{2}}}{dx^4 - c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a - b\,x^4\right)^{5/2}}{c - d\,x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(5/2)/(c - d*x^4),x)
```

```
[Out] int((a - b*x^4)^(5/2)/(c - d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2\sqrt{a - bx^4}}{-c + dx^4}\, dx - \int \frac{b^2x^8\sqrt{a - bx^4}}{-c + dx^4}\, dx - \int \left(-\frac{2abx^4\sqrt{a - bx^4}}{-c + dx^4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqrt
(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-c
+ d*x**4), x)
```

3.174 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

Optimal. Leaf size=277

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(3bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^2\sqrt{a-bx^4}}+\cdots$$

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-1/3*a^(1/4)*b^(3/4)*(-5*a*d+3*b*c)*EllipticF(b^(
1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+
b*c)^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-
b*x^4/a)^(1/2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+b*c)^2*Elli
pticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/
2)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {416, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(3bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd^2\sqrt{a-bx^4}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*d^2*Sqrt[a - b*x^4])
 + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sq
rt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/
4)*c*d^2*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{3/2}}{c-dx^4}\,dx = \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\int \frac{a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{3d}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} + \frac{(bc-ad)^2\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} + \frac{(bc-ad)^2\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} - \frac{\Big(b(3bc-}{}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}\,b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\left((bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\right)\int}{2cd^2\sqrt{a-}}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}\,b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\,(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\,\Pi}{2\sqrt[4]{b}\,cd}$$

Mathematica [C] time = 0.46, size = 341, normalized size = 1.23

$$-\frac{x\left(\frac{5\left(5ac\left(3a^2d-abdx^4+b^2x^4\left(dx^4-c\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4\left(a-bx^4\right)\left(c-dx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{\left(dx^4-c\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)} + \frac{bx^4\sqrt{1-}}{}\right.}{15d\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] -1/15*(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1,
 9/4, (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4*(
-c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(a
- b*x^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x^4)*(
5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*Appel
lF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4
, (b*x^4)/a, (d*x^4)/c])))))/(d*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

maple [C] time = 0.29, size = 311, normalized size = 1.12

$$\frac{\sqrt{-b\,x^4+a}\;bx}{3d}-\frac{\left(\frac{ab}{3d}-\frac{(2ad-bc)b}{d^2}\right)\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}}+\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-\frac{2\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-(-b*(2*a*d-b*c)/d^2+1/3*b/d*a)/(1/a^(1/2)*b^(1/2
))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(
-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)+1/8/d^3*sum((-a^2*
d^2+2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_al
pha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2)
)^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^
2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2
*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^4\right)^{3/2}}{c-d\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(3/2)/(c - d*x^4),x)
```

```
[Out] int((a - b*x^4)^(3/2)/(c - d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \left(-\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4*sqrt(a -
b*x**4)/(-c + d*x**4), x)
```

3.175 $\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-}{2}$$

```
[Out] a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/d/(-b*x^4+
a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/
2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)-1/2*a^
(1/4)*(-a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/
2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {406, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,(bc-}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{d} + \frac{(-bc+ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{d}$$

$$= \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd} + \frac{(-bc+ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd} + \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{d\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}} + \frac{\left((-bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2cd\sqrt{a-bx^4}} + \frac{\left((-bc\right.}{}$$

$$= \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\,(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd\sqrt{a-bx^4}} -$$

Mathematica [C] time = 0.21, size = 155, normalized size = 0.65

$$-\frac{5acx\sqrt{a-bx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\left(c-dx^4\right)\left(2x^4\left(bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
)/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

maple [C] time = 0.31, size = 259, normalized size = 1.08

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;b\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;d}+\frac{(-ad+bc)\left(-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{RootOf}\left(d\_Z^4-c\right)^3 d\,\mathrm{EllipticPi}\left(\sqrt{\cdots}\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;c\cdots}\right.}{8d^2\,\mathrm{RootOf}\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

```
[Out] b/d/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b
^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,
I)+1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*
_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)
*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alph
a^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-bx^4 + a}}{dx^4 - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a - b\,x^4}}{c - d\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(c - d*x^4),x)
```

```
[Out] int((a - b*x^4)^(1/2)/(c - d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{a - bx^4}}{-c + dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

3.176 $\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}$$

```
[Out] 1/2*a^(1/4)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I
)*(1-b*x^4/a)^(1/2)/b^(1/4)/c/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c
/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {409, 1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(
1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c}$$

$$= \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c\sqrt{a-bx^4}} + \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2c\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-\right.}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.10, size = 156, normalized size = 0.96

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\left(dx^4-c\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^4
]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*
x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[
5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{-bx^4+a}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

maple [C] time = 0.31, size = 183, normalized size = 1.13

$$-\frac{-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\mathrm{RootOf}\left(d\,\_Z^4-c\right)^3 d\,\mathrm{EllipticPi}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,\frac{\mathrm{RootOf}\left(d\,\_Z^4-c\right)^2\sqrt{a}\,d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,c}-\frac{\mathrm{arctanh}\left(\frac{-2\,\mathrm{RootOf}\left(d\,\_Z^4-c\right)^2 b\,x^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}}{8d\,\mathrm{RootOf}\left(d\,\_Z^4-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

```
[Out] -1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^
2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_a
lpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)
/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*a^(1/2)/b
^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-bx^4+a}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)),x)
```

```
[Out] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

3.177 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

Optimal. Leaf size=281

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}\sqrt{a-bx^4}\,(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}\,(bc-ad)}-\frac{\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)}-$$

```
[Out] 1/2*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1/2*b^(3/4)*EllipticF(b^(1/4)*x/a^(1/
4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*E
llipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)
^(1/2)/b^(1/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c
/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {414, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}\sqrt{a-bx^4}\,(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}\,(bc-ad)}-\frac{\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c - a*d)*Sqrt[a - b*x
^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sq
rt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c -
a*d)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{\int \frac{bc-2ad-bdx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} - \frac{d\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)} - \frac{d\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)} + \frac{\left(b\sqrt{1-\right.}{2a(b}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\left(d\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{}{\left(1-\right.}}{2c(bc-ad)\sqrt{}}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\right.}{2\sqrt[4]{b}\,c(bc}$$

Mathematica [C] time = 0.32, size = 381, normalized size = 1.36

$$\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(bdx^4\sqrt{1-\frac{bx^4}{a}}\left(dx^4-c\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-5c\left(2ad-2bc+bdx^4\right)\right)+2bx^5\left(c-\right.}{10ac\sqrt{a-bx^4}\left(dx^4-c\right)(ad-bc)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c + 2
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqrt[1
 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*Appe
llF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

maple [C] time = 0.36, size = 301, normalized size = 1.07

$$-\frac{bx}{2\left(ad-bc\right)\sqrt{-\left(x^4-\frac{a}{b}\right)b}\;a}-\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;b\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)}{2\left(ad-bc\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;a}-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{RootOf}(d\_}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] -1/2*b/a*x/(a*d-b*c)/(-(x^4-a/b)*b)^(1/2)-1/2/a*b/(a*d-b*c)/(1/a^(1/2)*b^(1
/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)
/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-1/8*sum(1/(a*d-b
*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((
a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/
c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+
a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*a^(1/2)/b^(1/2)/c*
d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4
*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^4\right)^{3/2}\,\left(c-d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)),x)
```

```
[Out] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*s
qrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

3.178 $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx$

Optimal. Leaf size=334

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(5bc-11ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)^2}$$

```
[Out] 1/6*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(3/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*
c)^2/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-11*a*d+5*b*c)*EllipticF(b^(1/4)*x/a^(1
/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*
d^2*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x
^4/a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*d^2*Ellipti
cPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/
b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(5bc-11ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{b}\,c\sqrt{a-bx^4}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7/4)*(b*c - a*d)^2*S
qrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(
b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi
[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx = \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{\int \frac{5bc-6ad-5bdx^4}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2-bd(5bc-11}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{12a^2(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{(bc-ad)^2} + \frac{(b(5}{}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)^2} + \frac{d^2}{}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\,F\Big(}{12a^{7/4}(bc-ad)^2\sqrt{}}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\,F\Big(}{12a^{7/4}(bc-ad)^2\sqrt{}}$$

Mathematica [C] time = 0.85, size = 422, normalized size = 1.26

$$\frac{x\left(\frac{bdx^4\sqrt{1-\frac{bx^4}{a}}\,(11ad-5bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} - \frac{5\left(2bx^4\left(dx^4-c\right)\left(13a^2d-ab\left(7c+11dx^4\right)+5b^2cx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right.\right.\right.}{\left(a-bx^4\right)\left(dx^4-c\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right.\right.}\right.}{60a^2\sqrt{a-bx^4}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*x^
4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^8))
*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4)*(1
3*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2,
9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x
^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^
4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^
4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(60*a^2*(
b*c - a*d)^2*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

maple [C] time = 0.32, size = 361, normalized size = 1.08

$$-\frac{d\left(-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\mathrm{RootOf}\left(d\_Z^4-c\right)^3 d\,\mathrm{EllipticPi}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,\frac{\mathrm{RootOf}\left(d\_Z^4-c\right)^2\sqrt{a}\,d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,c}-\frac{\mathrm{arctanh}\left(\frac{-2\,\mathrm{RootOf}\left(d\_Z^4-c\right)^2 b\,x^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right)}{8\,(ad-bc)^2\,\mathrm{RootOf}\left(d\_Z^4-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

```
[Out] -1/6/a*x/b/(a*d-b*c)*(-b*x^4+a)^(1/2)/(x^4-a/b)^2-1/12*b/a^2*x*(11*a*d-5*b*
c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/12/a^2*b*(11*a*d-5*b*c)/(a*d-b*c)^2/(
1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2
)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-1/
8*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alp
ha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2
+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*
a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a-b\,x^4\right)}^{5/2}\,\left(c-d\,x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)),x)
```

```
[Out] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] Timed out
```

3.179 $\int \frac{\left(a+bx^4\right)^{3/2}}{c+dx^4}\,dx$

Optimal. Leaf size=926

$$\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}\,\sqrt{-c}\,d^2(bc+ad)\sqrt{bx^4+a}}+\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(}{}$$

```
[Out] -1/4*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a
)^(1/2))/(-c)^(3/4)/d^(7/4)-1/4*(a*d-b*c)^(3/2)*arctan(x*(a*d-b*c)^(1/2)/(-
c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/d^(7/4)+1/3*b*x*(b*x^4+a)^(1/2
)/d-1/6*b^(3/4)*(-5*a*d+3*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2
)/a^(1/4)/d^2/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*arctan(b^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1
/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d^2/
(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arctan(b^(1/4)
*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/
b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)
^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*(cos(2*ar
ctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1
/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a
^(1/4)/d^2/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)+1/8*(-a*d+b*c)^2*(cos(2*arc
tan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticP
i(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2)
)^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(
b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(
1/2)/a^(1/4)/b^(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 1.66, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 523, 220, 409, 1217, 1707}

$$\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}\,\sqrt{-c}\,d^2(bc+ad)\sqrt{bx^4+a}}+\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(7/4)) - ((-(b*c) +
 a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^
4])])/(4*(-c)^(3/4)*d^(7/4)) - (b^(3/4)*(3*b*c - 5*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*
Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a
]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt
[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqr
t[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/2}}{c+dx^4}\,dx = \frac{bx\sqrt{a+bx^4}}{3d} + \frac{\int \frac{-a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{3d}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}\,d^2\sqrt{a+bx^4}} + \frac{(bc-ad)}{}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}\,d^2\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}\left(\sqrt{b}\right.\right.}{}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} - \frac{(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} - \frac{b}{}$$

Mathematica [C] time = 0.52, size = 346, normalized size = 0.37

$$\frac{x\left(\frac{5\left(2bx^4\left(a+bx^4\right)\left(c+dx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5ac\left(3a^2d+abdx^4+b^2x^4\left(c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+\frac{bx}{}\right.}{15d\sqrt{a+bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
-((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(3*a^2*d + a*b*d*x^4 + b^2*x^4*
(c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x
^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
 b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(15*d*Sqrt
[a + b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

maple [C] time = 0.48, size = 322, normalized size = 0.35

$$\frac{\sqrt{b\,x^4+a}\;bx}{3d}+\frac{\left(-\frac{ab}{3d}+\frac{(2ad-bc)b}{d^2}\right)\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}}-\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(2\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/2)/(d*x^4+c),x)
[Out] 1/3*b*x*(b*x^4+a)^(1/2)/d+((2*a*d-b*c)*b/d^2-1/3*a*b/d)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8/d^3*sum((-a^2*d^2+
2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2
*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{3/2}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/2)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(3/2)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/2)/(c + d*x**4), x)
```

3.180 $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

Optimal. Leaf size=881

$$-\frac{(bc-ad)\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{8\sqrt[4]{a}\,\sqrt[4]{b}\,cd(bc+ad)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}\,(bc-ad\cdots}{\cdots}$$

```
[Out] 1/4*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))*(-a*d+b*c
)^(1/2)/(-c)^(3/4)/d^(3/4)-1/4*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/
(b*x^4+a)^(1/2))*(a*d-b*c)^(1/2)/(-c)^(3/4)/d^(3/4)+1/2*b^(3/4)*(cos(2*arct
an(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(
sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4
+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(b*x^4+a)^(1/2)-1/4*b^(1/4)*(-
a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b
^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(-c)^(1/2)/(b*x^4+a)^(1/2)-1/8*(-a*d+b
*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^
(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x
^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*
b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(-a*d+b
*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/
4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a
^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+
x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2
*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/d/(a*d+b*c)/(b*x^4+a)^(1/2)-1/4*b^(1/4
)*(-a*d+b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c)^(1/2))*((b*x^4+a)/(a^(1/2)+x
^2*b^(1/2))^2)^(1/2)/a^(1/4)/d/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {406, 220, 409, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{8\sqrt[4]{a}\,\sqrt[4]{b}\,cd(bc+ad)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}\,(bc-ad\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(4*(-c)^(3/4)*d^(3/4)) - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(3/4)) + (b^
(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4])
 - (b^(1/4)*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d*(b*c + a*d)*Sqrt[a + b*x^4]
) - (b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(b*c - a*d)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] -
Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
```

```
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{d}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\sqrt{a+bx^4}} - \frac{(bc-ad)\int \frac{1}{\left(1-\frac{\sqrt{d}\,x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd} - \frac{(bc

```
[Out] (5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^4 + a}}{dx^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

maple [C] time = 0.33, size = 273, normalized size = 0.31

$$\frac{\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,b\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,d} - \frac{(-ad+bc)\left(\frac{2\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\mathrm{RootOf}\left(d\_Z^4+c\right)^3 d\,\mathrm{EllipticPi}\left(\sqrt{\cdots}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,c}\cdots\right)}{8d^2\,\mathrm{RootOf}\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(d*x^4+c),x)
```

```
[Out] b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b
^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I
)-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_a
lpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2)
)^(1/2)*_alpha^3*d/c*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^
```

```
2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*b^(1/2))^(1/2)*x,I*_alpha^
2*a^(1/2)/b^(1/2)/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
),_alpha=RootOf(_Z^4*d+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^4 + a}}{dx^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^4 + a}}{d\,x^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^4}}{c + dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

3.181 $\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=742

$$-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)F\left(2\tan\right.}{4\sqrt[4]{a}\sqrt{a+bx^4}\,(ad+bc)}$$

```
[Out] -1/4*d^(1/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/
(-c)^(3/4)/(-a*d+b*c)^(1/2)-1/4*d^(1/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)
/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(1/2)+1/8*(cos(2*arctan(b^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*
arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/
2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(
-c)^(1/2)-a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1
/4)/b^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4
)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(
-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^
(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/
c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c)^(
1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(a*d+b*c)/(b*x^4+a)
^(1/2)+1/4*b^(1/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {409, 1217, 220, 1707}

$$-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)F\left(2\tan\right.}{4\sqrt[4]{a}\sqrt{a+bx^4}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] -(d^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])
/(4*(-c)^(3/4)*Sqrt[b*c - a*d]) - (d^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((
```

```
-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*Sqrt[-(b*c) + a*d]) + (b
^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(4*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4
)*c*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqr
t[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d
)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d)*Sqrt[a + b*x^
4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c}$$

$$= \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}\,c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2c(bc+ad)} - \frac{\left(\sqrt{a}\right.}{}$$

$$= -\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}} + \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}\right.}{4}$$

Mathematica [C] time = 0.03, size = 161, normalized size = 0.22

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\left(c+dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(Sqrt[a +
 b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

maple [C] time = 0.33, size = 191, normalized size = 0.26

$$\frac{\frac{2\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\mathrm{RootOf}\left(d\_Z^4+c\right)^3 d\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,\frac{i\,\mathrm{RootOf}\left(d\_Z^4+c\right)^2\sqrt{a}\,d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,c}-\frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(d\_Z^4+c\right)^2 b\,x^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}}{8d\,\mathrm{RootOf}\left(d\_Z^4+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x)
```

```
[Out] 1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+
2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alph
a^3*d/c*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b
*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*b^(1/2))^(1/2)*x,I*_alpha^2*a^(1/2)/b^(
1/2)/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=Root
Of(_Z^4*d+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/2)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(1/2)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^4}\left(c + dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

3.182 $\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=913

$$-\frac{d\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{8\sqrt[4]{a}\,\sqrt[4]{b}\,c(bc-ad)(bc+ad)\sqrt{bx^4+a}}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-}}{\sqrt[4]{-c}\,\sqrt[4]{d}}\right.}{4(-c)^{3/4}(bc-ad}$$

```
[Out] 1/4*d^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(
-c)^(3/4)/(-a*d+b*c)^(3/2)-1/4*d^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/
d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a*d-b*c)^(3/2)+1/2*b*x/a/(-a*d+b*c)/(b
*x^4+a)^(1/2)+1/4*b^(3/4)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
5/4)/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/
2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d
^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(-a^2
*d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/8*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1
/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2
)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^
(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(-a^2*
d^2+b^2*c^2)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)+a^(1/2)*d^(1/2)/(-c
)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(-a*d+b*c)/(a*d+
b*c)/(b*x^4+a)^(1/2)-1/4*b^(1/4)*d*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a^(1/2)*(-c)^(1/2)*d^(1/2
))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/c/(-a^2*d^2+b^2*c^2)/(
b*x^4+a)^(1/2)
```

Rubi [A] time = 1.13, antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {414, 523, 220, 409, 1217, 1707}

$$-\frac{d\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{8\sqrt[4]{a}\,\sqrt[4]{b}\,c(bc-ad)(bc+ad)\sqrt{bx^4+a}}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-}}{\sqrt[4]{-c}\,\sqrt[4]{d}}\right.}{4(-c)^{3/4}(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + (d^(5/4)*ArcTan[(Sqrt[b*c - a*d]*
x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(3/2))
- (d^(5/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/(4*(-c)^(3/4)*(-(b*c) + a*d)^(3/2)) + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b
] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b^2*c^2 - a^2*d^2)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*
Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*
c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*S
qrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c
*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} - \frac{\int \frac{-bc+2ad-bdx^4}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \ldots d$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \ldots$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}} + \ldots$$

Mathematica [C] time = 0.28, size = 331, normalized size = 0.36

$$\frac{x\left(\frac{5\left(2bx^4\left(c+dx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5ac\left(2ad-b\left(2c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)} - \frac{bdx^4\sqrt{\frac{bx^4}{a}+1}\,F_1\left(\frac{5}{4};\frac{1}{2}\ldots\right.}{c}\ldots\right.}{10a\sqrt{a+bx^4}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (x*(-((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c) + (5*(5*a*c*(2*a*d - b*(2*c + d*x^4))*AppellF1[1/4, 1/2,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c + d*x^4)*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])))))/(10*a*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

maple [C] time = 0.29, size = 313, normalized size = 0.34

$$-\frac{bx}{2\left(ad-bc\right)\sqrt{\left(x^4+\frac{a}{b}\right)b}\;a}-\frac{\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;b\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;x,i\right)}{2\left(ad-bc\right)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\;\sqrt{b\,x^4+a}\;a}+\frac{2\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{RootOf}\Big(d\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x)
[Out] -1/2*b/a*x/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2/a*b/(a*d-b*c)/(I/a^(1/2)*b^(1/
2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/
(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)+1/8*sum(1/(a*d-b*c
)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d
-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(-
I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1
/2)*EllipticPi((I/a^(1/2)*b^(1/2))^(1/2)*x,I*_alpha^2*a^(1/2)/b^(1/2)/c*d,(
-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d+
c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{3/2}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(3/2)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(3/2)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{3}{2}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

3.183 $\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=976

$$-\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(bc-ad)^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(ad-bc)^{5/2}}+\frac{\sqrt[4]{b}\left(\sqrt{b}\,c-\sqrt{a}\,\sqrt{-c}\,\sqrt{d}\right)\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}}{4\sqrt[4]{a}\,c(bc-ad)^2(bc+ad)\sqrt{bx^4+}}$$

```
[Out] 1/6*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/2)-1/4*d^(9/4)*arctan(x*(-a*d+b*c)^(1/2)/
(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(-a*d+b*c)^(5/2)-1/4*d^(9/4)
*arctan(x*(a*d-b*c)^(1/2)/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(a
*d-b*c)^(5/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/2
4*b^(3/4)*(-11*a*d+5*b*c)*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*b^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(
9/4)/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*
x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)
^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)-a^(1/
2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/c/(
-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*d^2*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)
/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(b^(1/2)*(-c)^(1/2)+
a^(1/2)*d^(1/2))^2*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)-a
^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*b^(1/4)*d^2*(cos(2*arctan(b^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(c*b^(1/2)+a
^(1/2)*(-c)^(1/2)*d^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4
)/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^(1/2)
```

Rubi [A] time = 1.68, antiderivative size = 976, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21,

$\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {414, 527, 523, 220, 409, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(bc-ad)^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(ad-bc)^{5/2}}+\frac{\sqrt[4]{b}\left(\sqrt{b}\,c-\sqrt{a}\,\sqrt{-c}\,\sqrt{d}\right)\left(\sqrt{b}\,x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}\,x^2+\sqrt{a}\right)^2}}\,F}{4\sqrt[4]{a}\,c(bc-ad)^2(bc+ad)\sqrt{bx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a + b*x^4]) - (d^(9/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(5/2)) - (d^(9/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(
-c)^(3/4)*(-(b*c) + a*d)^(5/2)) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqr
t[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (
(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*
Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx = \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} - \frac{\int \frac{-5bc+6ad-5bdx^4}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2+bd(5bc-1}{\sqrt{a+bx^4}\left(c+dx^4\right)}}{12a^2(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{(bc-ad)^2} + \frac{(b(5}{}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}\right.}{24a^{9/4}(b}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}\right.}{24a^{9/4}(b}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} - \frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\,\sqrt[4]{d}\,\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{5/2}} -$$

Mathematica [C] time = 0.99, size = 429, normalized size = 0.44

$$-\frac{x\left(\frac{5\left(2bx^4\left(c+dx^4\right)\left(13a^2d+ab\left(11dx^4-7c\right)-5b^2cx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5ac\left(12a^3d^2-a^2bd\left(24c+dx^4\right)+ab^2\left(12c^2\right.\right.}{\left(a+bx^4\right)\left(c+dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\right.\right.}\right.}{60a^2\sqrt{a+bx^4}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] -1/60*(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(5*a*c*(12*a^3*d^2 + 5*b^3*c*x
```

```
^4*(2*c + d*x^4) - a^2*b*d*(24*c + d*x^4) + a*b^2*(12*c^2 - 15*c*d*x^4 - 11
*d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4
*(c + d*x^4)*(13*a^2*d - 5*b^2*c*x^4 + a*b*(-7*c + 11*d*x^4))*(2*a*d*Appell
F1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Appe
llF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(a^2*(b*c - a*d)^2*Sqrt[a + b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

maple [C] time = 0.34, size = 371, normalized size = 0.38

$$\frac{d\left(\frac{2\sqrt{-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{i\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\mathrm{RootOf}\left(d\_Z^4+c\right)^3 d\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x,\frac{i\,\mathrm{RootOf}\left(d\_Z^4+c\right)^2\sqrt{a}\,d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,c}-\frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(d\_Z^4+c\right)^2 b\,x^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right)}{8\left(ad-bc\right)^2\mathrm{RootOf}\left(d\_Z^4+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/2)/(d*x^4+c),x)
```

```
[Out] -1/6/a*x/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b/a^2*x*(11*a*d-5*b*c
)/(a*d-b*c)^2/((x^4+a/b)*b)^(1/2)-1/12/a^2*b*(11*a*d-5*b*c)/(a*d-b*c)^2/(I/
a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*
x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x,I)+1/8*d
*sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2
*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(
1/2)/(b*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*b^(1/2))^(1/2)*x,I*_alpha^2*a^(1
/2)/b^(1/2)/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alp
ha=RootOf(_Z^4*d+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{5/2}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/2)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(5/2)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{5}{2}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)
```

3.184 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

Optimal. Leaf size=426

$$-\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{84cd^3}+\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)F\Big(}{84cd^4\sqrt{a-bx^4}}$$

```
[Out] 1/28*b*(-7*a*d+11*b*c)*x*(-b*x^4+a)^(3/2)/c/d^2-1/4*(-a*d+b*c)*x*(-b*x^4+a)
^(5/2)/c/d/(-d*x^4+c)-1/84*b*(21*a^2*d^2-122*a*b*c*d+77*b^2*c^2)*x*(-b*x^4+
a)^(1/2)/c/d^3+1/84*a^(1/4)*b^(3/4)*(21*a^3*d^3+349*a^2*b*c*d^2-553*a*b^2*c
^2*d+231*b^3*c^3)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^4/(-
b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(1/4)*x
/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/
d^4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*EllipticPi(b^(
1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)
/c^2/d^4/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {413, 528, 523, 224, 221, 409, 1219, 1218}

$$-\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{84cd^3}+\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(349a^2bcd^2+21a^3d^3-553ab^2c^2d+231b^3c^3\right)F\Big(}{84cd^4\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(84*c*d^3) +
 (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) - ((b*c - a*d)*x*(a -
b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a*b
^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcS
in[(b^(1/4)*x)/a^(1/4)], -1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c -
a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*Sq
rt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a
]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{7/2}}{\left(c-dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a-bx^4\right)^{5/2}}{4cd\left(c-dx^4\right)} - \frac{\int \frac{\left(a-bx^4\right)^{3/2}\left(-a(bc+3ad)+b(11bc-7ad)x^4\right)}{c-dx^4}\,dx}{4cd}$$

$$= \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5/2}}{4cd\left(c-dx^4\right)} + \frac{\int \frac{\sqrt{a-bx^4}\left(-a\left(11b^2c^2-14abcd-21a^2d^2\right)+b(7\ldots}{c-dx^4}}{28cd^2}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a\ldots\right.}{4cd\left(c-\ldots\right.}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a\ldots\right.}{4cd\left(c-\ldots\right.}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a\ldots\right.}{4cd\left(c-\ldots\right.}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a\ldots\right.}{4cd\left(c-\ldots\right.}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a\ldots\right.}{4cd\left(c-\ldots\right.}$$

Mathematica [C] time = 0.92, size = 477, normalized size = 1.12

$$-\frac{bx^5\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(2x^5\left(bx^4-a\right)\left(21a^3d^3-63a^2bcd^2+a\cdots\right.\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/420*(b*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*x^
5*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5
*c*(5*a*c*x*(-84*a^4*d^3 + 29*a^2*b^2*c*d^2*x^4 + 21*a^3*b*d^3*x^4 + a*b^3*
c*d*x^4*(111*c - 104*d*x^4) + b^4*c*x^4*(-77*c^2 + 44*c*d*x^4 + 12*d^2*x^8)
)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(-a + b*x^4)*(-6
3*a^2*b*c*d^2 + 21*a^3*d^3 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-77*c^2
+ 44*c*d*x^4 + 12*d^2*x^8))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d
*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((c - d*
x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^3*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{7}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

maple [C] time = 0.29, size = 540, normalized size = 1.27

$$-\frac{\sqrt{-b\,x^4+a}\;b^3x^5}{7d^2}+\frac{\left(\frac{\left(\frac{5a\,b^3}{7d^2}-\frac{2(2ad-bc)b^3}{d^3}\right)a}{3b}+\frac{\left(6a^2d^2-8abcd+3b^2c^2\right)b^2}{d^4}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)b}{4c\,d^4}\right)\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}\cdots}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^(1/2)
/(d*x^4-c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7
*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1
/4*b/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c+1/3*(-2*b^3/d^3*(2
*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)
*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1
/a^(1/2)*b^(1/2))^(1/2)*x,I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2
*b^2*c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*ar
ctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(
1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a
^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))
^(1/2)*x,_alpha^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)
*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{7}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^4\right)^{7/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(7/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(7/2)/(c - d*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.185 $\int \frac{\left(a-bx^4\right)^{5/2}}{\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=365

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\left(-3a^2d^2-26abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(3ad+7bc)(bc-ad)^2\Pi\left(-\frac{\sqrt{}}{\sqrt{}}\right.}{8\sqrt[4]{b}\,c^2d^3\sqrt{a-bx^4}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(-b*x^4+a)^(3/2)/c/d/(-d*x^4+c)+1/12*b*(-3*a*d+7*b*c)*x*(
-b*x^4+a)^(1/2)/c/d^2-1/12*a^(1/4)*b^(3/4)*(-3*a^2*d^2-26*a*b*c*d+21*b^2*c^
2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^3/(-b*x^4+a)^(1/2)+
1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2
)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4+a)^(
1/2)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^
(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^3/(-b*x^4+
a)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {413, 528, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\left(-3a^2d^2-26abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(3ad+7bc)(bc-ad)^2\Pi\left(-\frac{\sqrt{}}{\sqrt{}}\right.}{8\sqrt[4]{b}\,c^2d^3\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x*(a - b*x^
4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d -
 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c
 - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqr
t[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
```

```
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{5/2}}{\left(c-dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a-bx^4\right)^{3/2}}{4cd\left(c-dx^4\right)} - \frac{\int \frac{\sqrt{a-bx^4}\left(-a(bc+3ad)+b(7bc-3ad)x^4\right)}{c-dx^4}\,dx}{4cd}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{3/2}}{4cd\left(c-dx^4\right)} + \frac{\int \frac{-a\left(7b^2c^2-6abcd-9a^2d^2\right)+b\left(21b^2c^2-26abcd-3a^2\right.}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{12cd^2}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{3/2}}{4cd\left(c-dx^4\right)} + \frac{\left((bc-ad)^2(7bc+3ad)\right)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{4cd^3}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{3/2}}{4cd\left(c-dx^4\right)} + \frac{\left((bc-ad)^2(7bc+3ad)\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-b}}}{8c^2d^3}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{3/2}}{4cd\left(c-dx^4\right)} - \frac{\sqrt[4]{a}\,b^{3/4}\left(21b^2c^2-26abcd-3a^2d^2\right)\sqrt{1}}{12cd^3\sqrt{a-bx}}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{3/2}}{4cd\left(c-dx^4\right)} - \frac{\sqrt[4]{a}\,b^{3/4}\left(21b^2c^2-26abcd-3a^2d^2\right)\sqrt{1}}{12cd^3\sqrt{a-bx}}$$

Mathematica [C] time = 0.67, size = 396, normalized size = 1.08

$$-\frac{bx^5\sqrt{1-\frac{bx^4}{a}}\left(3a^2d^2+26abcd-21b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(2x^5\left(a-bx^4\right)\left(3a^2d^2-6abcd+b^2c\left(7c-4dx^4\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\right.\right.}{\left(dx^4-c\right)\left(2x^4\left(2adF\right.\right.}}{60c^2d^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] -1/60*(b*(-21*b^2*c^2 + 26*a*b*c*d + 3*a^2*d^2)*x^5*Sqrt[1 - (b*x^4)/a]*App
ellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*a*c*x*(12*a^3*d^2 +
 2*a*b^2*c*d*x^4 - 3*a^2*b*d^2*x^4 + b^3*c*x^4*(-7*c + 4*d*x^4))*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(a - b*x^4)*(-6*a*b*c*d + 3*
a^2*d^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/
a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-
c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4,
 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^2*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

maple [C] time = 0.34, size = 412, normalized size = 1.13

$$\frac{\sqrt{-b\,x^4+a}\;b^2x}{3d^2}+\frac{\left(-\frac{a\,b^2}{3d^2}+\frac{(3ad-2bc)b^2}{d^3}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)b}{4c\,d^3}\right)\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}}-\frac{\left(a^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/3*b^2
/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*b/d^3*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^
2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^
(1/2)*b^(1/2))^(1/2)*x,I)-1/32/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^
2*d+7*b^3*c^3)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*
x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*
_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/
2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*a^(1/2)
/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^4\right)^{5/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(5/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(5/2)/(c - d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a-bx^4\right)^{\frac{5}{2}}}{\left(-c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
[Out] Integral((a - b*x**4)**(5/2)/(-c + d*x**4)**2, x)
```

3.186 $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2} \, dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}\, b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(ad+3bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)(ad+bc)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^2\sqrt{a-bx^4}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(-b*x^4+a)^(1/2)/c/d/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*(a*d+
3*b*c)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d^2/(-b*x^4+a)^(1
/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*
d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)^(1/
2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^
(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/d^2/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {413, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}\, b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(ad+3bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-ad)(ad+bc)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d^2\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a - b*x^4])/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(3*
b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/
(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*
x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a
*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*S
qrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^
4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}\,dx = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} - \frac{\int \frac{-a(bc+3ad)+b(3bc+ad)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c} + \frac{(b(3bc+ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a}}}{8c^2}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\sqrt[4]{a}\,b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\sqrt[4]{a}\,b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \frac{3\sqrt[4]{a}\left(a^2-\frac{b^2c^2}{d^2}\right.}{}$$

Mathematica [C] time = 0.44, size = 342, normalized size = 1.11

$$\frac{x\left(\frac{5c\left(-5ac\left(4a^2d-abdx^4+b^2cx^4\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-2x^4\left(a-bx^4\right)(ad-bc)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}-bx^4\sqrt{1-\frac{bx^4}{a}}\right.}{20c^2d\sqrt{a-bx^4}\left(dx^4-c\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*(-(b*(3*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + (5*c*(-5*a*c*(4*a^2*d + b^2*c*x^4 - a*b
*d*x^4)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 2*(-(b*c) + a*d)
*x^4*(a - b*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/(5*a*c*AppellF1[1/4
, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9
/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^
4)/c]))))/(20*c^2*d*Sqrt[a - b*x^4]*(-c + d*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

maple [C] time = 0.34, size = 329, normalized size = 1.06

$$\frac{\left(\frac{b^2}{d^2}+\frac{(ad-bc)b}{4c\,d^2}\right)\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\ \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{(ad-bc)\sqrt{-b\,x^4+a}\,x}{4\left(d\,x^4-c\right)cd}-\frac{3\left(a^2d^2-b^2c^2\right)\left(-\frac{2\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/d/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+(b^2/d^2+1/4*b/d^2*(a*d-b*c
)/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*
b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x
,I)-3/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arcta
nh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a
^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1
/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1
```

```
/2)*x,_alpha^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^
(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a-b\,x^4\right)^{3/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(3/2)/(c - d*x^4)^2,x)
[Out] int((a - b*x^4)^(3/2)/(c - d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a-bx^4\right)^{\frac{3}{2}}}{\left(-c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
[Out] Integral((a - b*x**4)**(3/2)/(-c + d*x**4)**2, x)
```

3.187 $\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-3ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-}{8}$$

```
[Out] 1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*EllipticF(b^(1/4)*x
/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)
*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/
a)^(1/2)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)*EllipticPi
(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(
1/4)/c^2/d/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {412, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-3ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(bc-}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a
]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sqrt[a - b*x^4]) - (a^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqr
t[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a -
 b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2
*d*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx = \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} - \frac{\int \frac{-3a+bx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c}$$

$$= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} + \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd} + \frac{(-bc+3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4cd}$$

$$= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} - \frac{(bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d} - \frac{(bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d} + \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right.}{4cd}$$

$$= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} + \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\left((bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{b}{}}}}{8c^2d\sqrt{a-bx^4}}$$

$$= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)} + \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\,(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin\right.}{8\sqrt[4]{b}\,c^2d\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.13, size = 233, normalized size = 0.84

$$\frac{x\left(-\frac{75a^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}\left(c-dx^4\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c^2}-\frac{5\left(a-bx^4\right)}{c}\right)}{20\sqrt{a-bx^4}\left(dx^4-c\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*((-5*(a - b*x^4))/c + (b*x^4*Sqrt[1 - (b*x^4)/a]*(c - d*x^4)*AppellF1[5/
4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c^2 - (75*a^2*AppellF1[1/4, 1/2, 1,
5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a - b*x^4
]*(-c + d*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4+a}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

maple [C] time = 0.31, size = 294, normalized size = 1.07

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,b\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,x,i\right)}{4\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,cd}-\frac{\sqrt{-b\,x^4+a}\,x}{4\left(d\,x^4-c\right)c}-\frac{(3ad-bc)\left(-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\mathrm{RootOf}\left(d\_Z^4\right.}{\sqrt{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/c/d/(1/a^(1/2)*b^(1/2))^(1/2)*(-1
/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1
/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-1/32/c/d^2*sum((3*a*d-b*c)/_al
pha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c
)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(-1/a
^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2
)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*a^(1/2)/b^(1/2)/c*d,(-1/a
^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4+a}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-b\,x^4}}{{\left(c-d\,x^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(1/2)/(c - d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^4}}{\left(-c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(-c + d*x**4)**2, x)
```

3.188 $\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}\,(bc-ad)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(}{}$$

```
[Out] -1/4*d*x*(-b*x^4+a)^(1/2)/c/(-a*d+b*c)/(-d*x^4+c)-1/4*a^(1/4)*b^(3/4)*Ellip
ticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1
/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(
1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1
/8*a^(1/4)*(-3*a*d+5*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1
/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {414, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}\,(bc-ad)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{bx^4}{a}}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] -(d*x*Sqrt[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b^(3/4)*Sqr
t[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*(b*c - a*
d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Elliptic
Pi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sq
rt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx = -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-bdx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} - \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4c(bc-ad)} + \frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} + \frac{(5bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)} + \frac{(5bc-3ad)\int \frac{}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right.}}{8c^2(bc-ad}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} - \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\left((5bc-3ad)\sqrt{1\right.}{8c^2(b}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)} - \frac{\sqrt[4]{a}\,b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(5bc-3ad)}{8}$$

Mathematica [C] time = 0.28, size = 386, normalized size = 1.25

$$\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(bdx^4\sqrt{1-\frac{bx^4}{a}}\left(dx^4-c\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-5c\left(-4ad+4bc+bdx^4\right)\right)+2dx^5\left(b\right.}{20c^2\sqrt{a-bx^4}\left(dx^4-c\right)(bc-ad)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(4*b*c - 4*
a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2
, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*d*x^5*(5*c*(a - b*x^4) + b*x^4*Sqrt[1
- (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])
*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4
, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(20*c^2*(b*c - a*d)*Sqrt[a - b*x^4]*
(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^
4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/
4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

maple [C] time = 0.34, size = 322, normalized size = 1.04

$$\frac{\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\,\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;b\,\mathrm{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;x,i\right)}{4\left(ad-bc\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\;\sqrt{-b\,x^4+a}\;c}-\frac{\sqrt{-b\,x^4+a}\;dx}{4\left(ad-bc\right)\left(d\,x^4-c\right)c}-\frac{\left(3ad-5bc\right)\left(-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\;\mathrm{Ro}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*d/(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/(a*d-b*c)/c/(1/a^(1/2
)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)
^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-1/32/c/d*s
um((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2
*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2
)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alp
ha^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4 + a}\left(dx^4 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a - b\,x^4}\,\left(c - d\,x^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2),x)
[Out] int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - bx^4}\,\left(-c + dx^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
[Out] Integral(1/(sqrt(a - b*x**4)*(-c + d*x**4)**2), x)
```

3.189 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

Optimal. Leaf size=362

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c\sqrt{a-bx^4}\,(bc-ad)^2}-\frac{3\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,(3bc-ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)^2}-3\sqrt[4]{a}\,d\sqrt{1}$$

```
[Out] 1/4*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-1/4*d*x/c/(-a*d+b*c)/
(-d*x^4+c)/(-b*x^4+a)^(1/2)+1/4*b^(3/4)*(a*d+2*b*c)*EllipticF(b^(1/4)*x/a^(
1/4),I)*(1-b*x^4/a)^(1/2)/a^(3/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^(1/
4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(
1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^(
1/4)*d*(-a*d+3*b*c)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^
(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c\sqrt{a-bx^4}\,(bc-ad)^2}-\frac{3\sqrt[4]{a}\,d\sqrt{1-\frac{bx^4}{a}}\,(3bc-ad)\Pi\left(-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2\sqrt{a-bx^4}\,(bc-ad)^2}-3\sqrt[4]{a}\,d\sqrt{1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c
 - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c + a*d)*Sqrt[1 - (b*x
^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*a^(3/4)*c*(b*c - a*d)
^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]
)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-5bdx^4}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{\int \frac{-2\left(2b^2c^2-8abcd+3a^2d^2\right)+\cdots}{\sqrt{a-bx^4}\left(c-d\cdots\right.}}{8ac(bc-a\cdots}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{(3d(3bc-ad))\int \frac{\cdots}{\sqrt{a-b\cdots}}}{4c(bc-ad)^{\cdots}}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{(3d(3bc-ad))\int \frac{\cdots}{\left(1-\frac{\sqrt{\cdots}}{\cdots}\right.}}{8c^2(bc-a\cdots}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx\cdots}{a}}\cdots}{4a^{3/4}c(bc-\cdots}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx\cdots}{a}}\cdots}{4a^{3/4}c(bc-\cdots}$$

Mathematica [C] time = 0.52, size = 374, normalized size = 1.03

$$\frac{x\left(\frac{c\left(25ac\left(4a^2d^2-abd\left(8c+dx^4\right)+2b^2c\left(2c-dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-10x^4\left(-a^2d^2+abd^2x^4-2b^2c\left(c-dx^4\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a}\cdots\right.\right.\right)}{\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}\cdots\right.}{20ac^2\sqrt{a-bx^4}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*(-(b*d*(2*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
(b*x^4)/a, (d*x^4)/c]) + (c*(25*a*c*(4*a^2*d^2 + 2*b^2*c*(2*c - d*x^4) - a*
b*d*(8*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 10*x^
4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a,
 (d*x^4)/c])))/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(20*a*c^2*(b*c -
a*d)^2*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4 + a\right)^{\frac{3}{2}}\left(dx^4 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

maple [C] time = 0.35, size = 375, normalized size = 1.04

$$\frac{b^2 x}{2\left(ad-bc\right)^2 \sqrt{-\left(x^4-\frac{a}{b}\right) b\ a}}-\frac{\sqrt{-b\,x^4+a}\ d^2 x}{4\left(ad-bc\right)^2 \left(d\,x^4-c\right) c}+\frac{\left(\frac{b^2}{2(ad-bc)^2 a}+\frac{bd}{4(ad-bc)^2 c}\right)\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\ \sqrt{\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}\ \mathrm{Ellipti}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\ \sqrt{-b\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

```
[Out] 1/2*b^2/a*x/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/4*d^2/(a*d-b*c)^2/c*x*(-b*x^
4+a)^(1/2)/(d*x^4-c)+(1/2/a*b^2/(a*d-b*c)^2+1/4*d*b/(a*d-b*c)^2/c)/(1/a^(1/
2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1
)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-3/32/c*su
m((a*d-3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*
_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)
*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alph
a^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^4\right)^{3/2}\,\left(c-d\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x)
```

```
[Out] int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.190 $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=439

$$\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{12a^2c\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}}{}$$

```
[Out] 1/12*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(3/2)-1/4*d*x/c/(-a*d+b*
c)/(-b*x^4+a)^(3/2)/(-d*x^4+c)+1/12*b*(-3*a^2*d^2-17*a*b*c*d+5*b^2*c^2)*x/a
^2/c/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-3*a^2*d^2-17*a*b*c*d+5*b^
2*c^2)*EllipticF(b^(1/4)*x/a^(1/4),I)*(1-b*x^4/a)^(1/2)/a^(7/4)/c/(-a*d+b*c
)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1/4)*x/a
^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/c^2/(-
a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*EllipticPi(b^(1
/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)*(1-b*x^4/a)^(1/2)/b^(1/4)/
c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{12a^2c\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c\sqrt{a-bx^4}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,d^2\sqrt{1-\frac{bx^4}{a}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) + (b*(5*b^2*
c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) -
 (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + (b^(3/4)*(5*b^2*c^
2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(12*a^(7/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^
2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a - b*x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-9bdx^4}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-2\left(10b^2c^2-24a}{(a}}{2}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx\right.}$$

Mathematica [C] time = 0.88, size = 382, normalized size = 0.87

$$\frac{x\left(\frac{bdx^4\sqrt{1-\frac{bx^4}{a}}\left(3a^2d^2+17abcd-5b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a^2c^2}+5\left(\frac{2b^3c}{a^2-abx^4}+\frac{5b^3c}{a^2}+\frac{5\left(-9a^3d^3+36a^2bcd^2-17ab^2c^2d+5b^3c^3\right)F_1\left(\right.}{a\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a}\right.\right.\right.}\right.\right.}{60\sqrt{a-bx^4}\,(bc-ad)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*((b*d*(-5*b^2*c^2 + 17*a*b*c*d + 3*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*Appe
llF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(a^2*c^2) + 5*((5*b^3*c)/a^2
- (17*b^2*d)/a - (2*b^2*d)/(a - b*x^4) + (2*b^3*c)/(a^2 - a*b*x^4) - (3*a*d
^3)/(c^2 - c*d*x^4) + (3*b*d^3*x^4)/(c^2 - c*d*x^4) + (5*(5*b^3*c^3 - 17*a*
b^2*c^2*d + 36*a^2*b*c*d^2 - 9*a^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/
a, (d*x^4)/c])/(a*(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
(d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c]
+ b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))))/(60*(b*c - a*d
)^3*Sqrt[a - b*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

maple [C] time = 0.35, size = 484, normalized size = 1.10

$$-\frac{\sqrt{-b\,x^4+a}\;b\,d^3x}{4\left(a^2d^2-2abcd+b^2c^2\right)(ad-bc)\left(bd\,x^4-bc\right)c}+\frac{(17ad-5bc)\,b^2x}{12\,(ad-bc)^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}\;a^2}-\frac{d\,(3ad-13bc)\left(-\frac{2\sqrt{-\frac{\sqrt{b}\,x^2}{\sqrt{a}}+1}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*b*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(b*d*
x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-a/b)^2+1/12*b^2/a^2*x*(1
7*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d^2/(a^2*d^2-2*a*b*c*d
+b^2*c^2)/(a*d-b*c)/c+1/12/a^2*b^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/2)*b
^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)-1/32/c*d*sum(
(3*a*d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2
*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2
)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alp
ha^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*d-c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\,x^4\right)^{5/2}\left(c-d\,x^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x)
[Out] int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - bx^4\right)^{\frac{5}{2}}\left(-c + dx^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
[Out] Integral(1/((a - b*x**4)**(5/2)*(-c + d*x**4)**2), x)
```

3.191 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] 1/4*arctan(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)/c*2^(
1/2)+1/4*arctanh(a^(1/4)*b^(1/4)*x*2^(1/2)/(b*x^4+a)^(1/2))/a^(1/4)/b^(1/4)
/c*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{c}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,c}\end{aligned}$$

Mathematica [C] time = 0.17, size = 155, normalized size = 1.50

$$\frac{5ax\sqrt{a+bx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)}{c\left(a-bx^4\right)\left(2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)+5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (b*x^4)/a]
)/(c*(a - b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (b*x^4)/a]
+ 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), (b*x^4)/a] + AppellF
1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), (b*x^4)/a])))
```

fricas [B] time = 2.75, size = 315, normalized size = 3.06

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^4+a}\,c\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}bcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}{\sqrt{b}}}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*arctan(((1/4)^(1/4)*sqrt(b*x^4 + a)*c*(1/(
a*b*c^4))^(1/4) - (2*(1/4)^(3/4)*a*b*c^3*(1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*
b*c*x^2*(1/(a*b*c^4))^(1/4))/sqrt(b))/x) + 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1
/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*
x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b
*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^
3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(
b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

maple [A] time = 0.21, size = 103, normalized size = 1.00

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2(ab)^{\frac{1}{4}}x}\right)}{4\,(ab)^{\frac{1}{4}}\,c}+\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)}{8\,(ab)^{\frac{1}{4}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x)
```

```
[Out] -1/4/c*2^(1/2)/(a*b)^(1/4)*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4)
)+1/8/c*2^(1/2)/(a*b)^(1/4)*ln((1/2*(b*x^4+a)^(1/2)*2^(1/2)/x+(a*b)^(1/4))/
(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x-(a*b)^(1/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^4+a}}{a\,c-b\,c\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4),x)
[Out] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
[Out] -Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

3.192 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4} \, dx$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}-\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

```
[Out] 1/2*arctan(b^(1/4)*x*(a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^(1/2))/a^(1/4
)/b^(1/4)/c+1/2*arctanh(b^(1/4)*x*(a^(1/2)-x^2*b^(1/2))/a^(1/4)/(-b*x^4+a)^
(1/2))/a^(1/4)/b^(1/4)/c
```

Rubi [A] time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {405}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}-\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^
(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*Sqr
t[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}+\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x\left(\sqrt{a}-\sqrt{b}\,x^2\right)}{\sqrt[4]{a}\,\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,c}$$

Mathematica [C] time = 0.17, size = 155, normalized size = 1.34

$$\frac{5ax\sqrt{a-bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)}{c\left(a+bx^4\right)\left(5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)-2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((b*x^4)/a)]
)/(c*(a + b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((b*x^4)/a)]
- 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, -((b*x^4)/a)] + AppellF
1[5/4, 1/2, 1, 9/4, (b*x^4)/a, -((b*x^4)/a)])))
```

fricas [B] time = 1.92, size = 339, normalized size = 2.92

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\sqrt{-\frac{1}{b}}\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bcx^2\sqrt{-\frac{1}{b}}+\sqrt{-bx^4+a}\,c\right)\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*arctan((2*(1/4)^(3/4)*a*b*c^3*sqrt(-1/b)*
(-1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*(b*c*x^2*sqrt(-1/b) + sqrt(-b*x^4 + a)*c
)*(-1/(a*b*c^4))^(1/4))/x) - 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(
1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-
1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*
x^2)/(b*x^4 + a)) + 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)
*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4
)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^
4 + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

maple [A] time = 0.22, size = 158, normalized size = 1.36

$$\frac{\arctan\left(-\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}x}+1\right)}{4\,(ab)^{\frac{1}{4}}\,c}-\frac{\arctan\left(\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}x}+1\right)}{4\,(ab)^{\frac{1}{4}}\,c}-\frac{\ln\left(\frac{-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\frac{-b\,x^4+a}{2x^2}+\sqrt{ab}}{\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\frac{-b\,x^4+a}{2x^2}+\sqrt{ab}}\right)}{8\,(ab)^{\frac{1}{4}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x)
```

```
[Out] -1/8/c/(a*b)^(1/4)*ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a
*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a*b)^(1/2)))
-1/4/c/(a*b)^(1/4)*arctan(1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+1)+1/4/c/(a*b)^(
1/4)*arctan(-1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^4}}{b\,c\,x^4+a\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4),x)
```

```
[Out] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

3.193 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4} \, dx$

Optimal. Leaf size=211

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}}{}$$

```
[Out] 1/4*b*x*(b*x^4+a)^(3/4)/d-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^
4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x
^4+a)^(1/4))/c^(3/4)/d^2-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^
4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*
x^4+a)^(1/4))/c^(3/4)/d^2
```

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {416, 530, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2) - (b^(3/4)*(4*b*c - 7*a*d
)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
```

```
, n}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{7/4}}{c+dx^4}\,dx = \frac{bx\left(a+bx^4\right)^{3/4}}{4d} + \frac{\int \frac{-a(bc-4ad)-b(4bc-7ad)x^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4d}$$

$$= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{(b(4bc-7ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{d^2}$$

$$= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{(b(4bc-7ad))\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^2} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{c-(bc-a}\right.}{d^2}$$

$$= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{(b(4bc-7ad))\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{(b(4bc-7ad))\,\text{Subst}\left(\int \right.}{8d^2}$$

$$= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}(4b}{}$$

Mathematica [C] time = 0.63, size = 364, normalized size = 1.73

$$\frac{5\sqrt[4]{c}\left(4a^2d\sqrt[4]{a+bx^4}\,\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+4b^2c^{3/4}x^5\sqrt[4]{bc-ad}+4abc^{3/4}x\sqrt[4]{bc-ad}+a\sqrt[4]{a+bx^4}\,(bc-4ad)\log\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (4*b*(b*c - a*d)^(1/4)*(-4*b*c + 7*a*d)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[
5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 5*c^(1/4)*(4*a*b*c^(3/4)*(b
*c - a*d)^(1/4)*x + 4*b^2*c^(3/4)*(b*c - a*d)^(1/4)*x^5 + 2*a*(-(b*c) + 4*a
*d)*(a + b*x^4)^(1/4)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/
4))] + a*(b*c - 4*a*d)*(a + b*x^4)^(1/4)*Log[c^(1/4) - ((b*c - a*d)^(1/4)*x
)/(b + a*x^4)^(1/4)] - a*b*c*(a + b*x^4)^(1/4)*Log[c^(1/4) + ((b*c - a*d)^(
1/4)*x)/(b + a*x^4)^(1/4)] + 4*a^2*d*(a + b*x^4)^(1/4)*Log[c^(1/4) + ((b*c
- a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(80*c*d*(b*c - a*d)^(1/4)*(a + b*x^4)^
(1/4))
```

fricas [B] time = 9.03, size = 2381, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")

```
[Out] 1/16*(4*(b*x^4 + a)^(3/4)*b*x + 16*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*arctan(-(c*d^2*x*sqrt(((b^7*c^8*d^
4 - 7*a*b^6*c^7*d^5 + 21*a^2*b^5*c^6*d^6 - 35*a^3*b^4*c^5*d^7 + 35*a^4*b^3*
c^4*d^8 - 21*a^5*b^2*c^3*d^9 + 7*a^6*b*c^2*d^10 - a^7*c*d^11)*x^2*sqrt((b^7
*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3
*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8)) + (b^10
*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4
*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*
d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*sqrt(b*x^4 + a))/x^2
)*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*
a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^
(1/4) + (b^5*c^6*d^2 - 5*a*b^4*c^5*d^3 + 10*a^2*b^3*c^4*d^4 - 10*a^3*b^2*c^
3*d^5 + 5*a^4*b*c^2*d^6 - a^5*c*d^7)*(b*x^4 + a)^(1/4)*((b^7*c^7 - 7*a*b^6*
c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a
^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4))/((b^7*c^7 - 7*a
*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 -
 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*x)) + 4*d*((256*b^7*c^4 - 17
92*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d
^4)/d^8)^(1/4)*arctan((d^2*x*sqrt(((256*b^7*c^4*d^4 - 1792*a*b^6*c^3*d^5 +
4704*a^2*b^5*c^2*d^6 - 5488*a^3*b^4*c*d^7 + 2401*a^4*b^3*d^8)*x^2*sqrt((256
*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/d^8) + (4096*b^10*c^6 - 43008*a*b^9*c^5*d + 188160*a^2*b^8
*c^4*d^2 - 439040*a^3*b^7*c^3*d^3 + 576240*a^4*b^6*c^2*d^4 - 403368*a^5*b^5
*c*d^5 + 117649*a^6*b^4*d^6)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^4 - 1792*a*b
^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^
8)^(1/4) + (64*b^5*c^3*d^2 - 336*a*b^4*c^2*d^3 + 588*a^2*b^3*c*d^4 - 343*a^
3*b^2*d^5)*(b*x^4 + a)^(1/4)*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^
5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4))/((256*b^7*c^
4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)*x)) + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3
*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^
7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*
b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5
+ 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5 - 5*a*b^4*c^4*d + 10
*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a
)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b
^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*
d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^
```

```
2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(
1/4))/x) - d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488
*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(-(d^6*x*((256*b^7*c^4 - 1
792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*
d^4)/d^8)^(3/4) + (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c*d^2 - 343*a
^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x) + d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 47
04*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(
(d^6*x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d + 5
88*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{7}{4}}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{7/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^{\frac{7}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

3.194 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4} \, dx$

Optimal. Leaf size=173

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

```
[Out] 1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctan
((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d+1/2*b^(3/4)*arctanh(
b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctanh((-a*d+b*c)^(1/4)*
x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d
```

Rubi [A] time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {408, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d) +
(b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)
*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{c+dx^4}\,dx = \frac{b\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}}{}$$

Mathematica [C] time = 0.18, size = 161, normalized size = 0.93

$$\frac{5acx\left(a+bx^4\right)^{3/4}F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(3bcF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(3/4)*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + 3*b*c*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

fricas [B] time = 1.06, size = 844, normalized size = 4.88

$$\left(\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{cdx\sqrt{\frac{\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5\right)x^2\sqrt{\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{c^3d^4}}+\left(b^4c^4-\right.}{x^2}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] ((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*arcta
n(-(c*d*x*sqrt(((b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^
```

```
5)*x^2*sqrt((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
+ (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*s
qrt(b*x^4 + a))/x^2)*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(
c^3*d^4))^(1/4) - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*(b*x^4 + a)^(1/4)
*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4))/((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)) + (b^3/d^4)^(1/4)*arc
tan(-((b*x^4 + a)^(1/4)*b^2*d*(b^3/d^4)^(1/4) - d*x*(b^3/d^4)^(1/4)*sqrt((b
^3*d^2*x^2*sqrt(b^3/d^4) + sqrt(b*x^4 + a)*b^4)/x^2))/(b^3*x)) - 1/4*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((c^2*d^
3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) +
 (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(-(c^2*d^3*x*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) - (b*x^4
 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^(1/4)*log((
d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b^3/d^4)^(1/4)*log
(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{4}\right)^{\frac{3}{4}}}{c + dx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

3.195 $\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

```
[Out] 1/2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(
1/4)+1/2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+
b*c)^(1/4)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {377, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
 a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(
2*c^(3/4)*(b*c - a*d)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Mathematica [A] time = 0.05, size = 84, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)+\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] (ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))] + ArcTanh[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

maple **[F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{1/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

3.196 $\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=134

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}\,(bc-ad)}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^4+a)^(1/4)-1/2*d*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)-1/2*d*arctanh((-a*d+b*c)^(1/4)*x/c^
(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 377, 212, 208, 205}

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/4)) - (d*ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx &= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}\,(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{b}\cdots}\right.}{2\sqrt{c}\,(\cdots}\\
&= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.58, size = 256, normalized size = 1.91

$$-\frac{45c^3\left(a+bx^4\right)^2 {}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right) - 45c^3\left(a+bx^4\right)^2 + 36c^2dx^4\left(a+bx^4\right)^2 {}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right) - 36c^2dx^4\left(a\cdots\right.}{9c^3x^3\left(a+bx^4\right)^{9/4}(ad-\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] -1/9*(-45*c^3*(a + b*x^4)^2 - 36*c^2*d*x^4*(a + b*x^4)^2 + 45*c^3*(a + b*x^
4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 36
*c^2*d*x^4*(a + b*x^4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(
c*(a + b*x^4))] + 4*c*(b*c - a*d)^2*x^8*Hypergeometric2F1[2, 9/4, 13/4, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 4*d*(b*c - a*d)^2*x^12*Hypergeometric2F1[
2, 9/4, 13/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^3*(-(b*c) + a*d)*x^3*(
a + b*x^4)^(9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^4+a\right)^{5/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{5}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

3.197 $\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=180

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}\,(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

```
[Out] 1/5*b*x/a/(-a*d+b*c)/(b*x^4+a)^(5/4)+1/5*b*(-9*a*d+4*b*c)*x/a^2/(-a*d+b*c)^
2/(b*x^4+a)^(1/4)+1/2*d^2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4)
)/c^(3/4)/(-a*d+b*c)^(9/4)+1/2*d^2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^
4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(9/4)
```

Rubi [A] time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}\,(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b
*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)
*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4)) + (d^2*ArcTanh[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx = \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} - \frac{\int \frac{-4bc+5ad-4bdx^4}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx}{5a(bc-ad)}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{\int \frac{5a^2d^2}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{5a^2(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x \ldots}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx \ldots}{2\sqrt{c}\,(bc-ad)^2 \ldots}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2 \ldots}{\ldots}$$

Mathematica [C] time = 2.37, size = 621, normalized size = 3.45

$$\frac{80c^2x^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{13}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right) + 80d^2x^{20}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{13}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right) + 160cdx^{16}(bc- \ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b*
x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4)^
3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925*c^
5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*
x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c
- a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c - a*d)^
```

```
3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*
x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric2F1[2, 13/
4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)^3*x^12*Hyp
ergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*HypergeometricP
FQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(325*c^4*(b
*c - a*d)^2*x^7*(a + b*x^4)^(13/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{9}{4}}\left(dx^4 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4 + a\right)^{\frac{9}{4}}\left(d\,x^4 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{9}{4}}\left(dx^4 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^4+a\right)^{9/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{9}{4}}\left(c+dx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

3.198 $\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx$

Optimal. Leaf size=233

$$\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}+\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}\,(bc-ad)^3}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+$$

```
[Out] 1/9*b*x/a/(-a*d+b*c)/(b*x^4+a)^(9/4)+1/45*b*(-17*a*d+8*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^4+a)^(5/4)+1/45*b*(113*a^2*d^2-100*a*b*c*d+32*b^2*c^2)*x/a^3/(-a*d
+b*c)^3/(b*x^4+a)^(1/4)-1/2*d^3*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)
^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)-1/2*d^3*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4
)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)
```

Rubi [A] time = 0.29, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x)/(45*a^2*
(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d
^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4)) - (d^3*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^
3*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b
*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{13/4}\left(c+dx^4\right)}\,dx = \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} - \frac{\int \frac{-8bc+9ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{9a(bc-ad)}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{\int \frac{32b^2c^2-68abcd+45a^2d^2+4bd}{\left(a+bx^4\right)^{5/4}\left(c+dx\right.}}{45a^2(bc-ad}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+1\right.}{45a^3(bc-ad)^3\sqrt[4]{a+}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+1\right.}{45a^3(bc-ad)^3\sqrt[4]{a+}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+1\right.}{45a^3(bc-ad)^3\sqrt[4]{a+}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+1\right.}{45a^3(bc-ad)^3\sqrt[4]{a+}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+1\right.}{45a^3(bc-ad)^3\sqrt[4]{a+}}$$

<u>**Mathematica**</u> **[A]** time = 5.44, size = 231, normalized size = 0.99

$$\frac{bx\left(\left(a+bx^4\right)^2\left(113a^2d^2-100abcd+32b^2c^2\right)+5a^2(bc-ad)^2+a\left(a+bx^4\right)(ad-bc)(17ad-8bc)\right)}{45a^3\left(a+bx^4\right)^{9/4}(bc-ad)^3} - \frac{d^3\left(-\log\left(\sqrt[4]{\right.\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x*(5*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-8*b*c + 17*a*d)*(a + b*x^4)
+ (32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d^2)*(a + b*x^4)^2))/(45*a^3*(b*c - a
*d)^3*(a + b*x^4)^(9/4)) - (d^3*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b
```

```
 + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]
+ Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(4*c^(3/4)*(b*c
- a*d)^(13/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{13}{4}}\left(dx^4 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4 + a\right)^{\frac{13}{4}}\left(d\,x^4 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{13}{4}}\left(dx^4 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{13/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{13}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(13/4)*(c + d*x**4)), x)
```

3.199 $\int \frac{\left(a+bx^4\right)^{9/4}}{c+dx^4}\,dx$

Optimal. Leaf size=316

$$\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}\,(6bc-11ad)}{12d^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(bc-ad)^2}{2}$$

```
[Out] -1/12*b*(-11*a*d+6*b*c)*x*(b*x^4+a)^(1/4)/d^2+1/6*b*x*(b*x^4+a)^(5/4)/d+1/1
2*b^(3/2)*(-11*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*
arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d^2/(b*x^4+a)^(3/4)+1/2*(-a*d
+b*c)^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1
/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2+1/2*(-a*d+b*c)^2*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/
(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d^2
```

Rubi [A] time = 0.34, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {416, 528, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}\,(6bc-11ad)}{12d^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,(bc-ad)^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] -(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/(12*d^2) + (b*x*(a + b*x^4)^(5/4)
)/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*d^2*(a + b*x^4)^(3/4)) + ((b*
c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d^2) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(2*b^(1/4)*c*d^2)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
```

```
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{9/4}}{c+dx^4}\,dx = \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\int \frac{\sqrt[4]{a+bx^4}\left(-a(bc-6ad)-b(6bc-11ad)x^4\right)}{c+dx^4}\,dx}{6d}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\int \frac{a\left(6b^2c^2-13abcd+12a^2d^2\right)+b\left(12b^2c^2-30abcd+23a^2d^2\right)x^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}}{12d^2}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} - \frac{(ab(6bc-11ad))\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{12d^2} + \frac{(bc-ad)^2}{d}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} - \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}}{12d^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{}{(1+}\right.}{12d^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin\right.}{2\sqrt[4]{b}\,cd^2}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\sqrt{a}\,b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\right.}{12d^2\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.70, size = 294, normalized size = 0.93

$$\frac{x\left(\frac{bx^4\left(\frac{bx^4}{a}+1\right)^{3/4}\left(23a^2d^2-30abcd+12b^2c^2\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} - \frac{25a^2c\left(12a^2d^2-13abcd+6b^2c^2\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\right.}\right.}{60d^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] (x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) + (b*(12*b^2*c^2 - 30*a*b
*c*d + 23*a^2*d^2)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((
b*x^4)/a), -((d*x^4)/c)])/c - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2*d^
2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5
*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*Ap
pellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4,
7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*d^2*(a + b*x^4)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{9}{4}}}{dx^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4 + a\right)^{\frac{9}{4}}}{d\,x^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{9}{4}}}{dx^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{9/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(9/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(9/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{4}\right)^{\frac{9}{4}}}{c+dx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(9/4)/(c + d*x**4), x)
```

3.200 $\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx$

Optimal. Leaf size=274

$$-\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\left(a+bx^4\right)^{3/4}}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd}-$$

```
[Out] 1/2*b*x*(b*x^4+a)^(1/4)/d-1/2*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot
(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Ellipt
icF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/d/(b*x^4+a)^(3/4)
-1/2*(-a*d+b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1
/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d-1/2*(-a*d+b*
c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)
*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/d
```

Rubi [A] time = 0.16, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {408, 195, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\left(a+bx^4\right)^{3/4}}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,cd}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*
EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a + b*x^4)^(3/4)) - ((
b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
b^(1/4)*c*d)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx = \frac{b\int \sqrt[4]{a+bx^4}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{d}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} + \frac{(ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{2d} - \frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)\right.}\right.}{d}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2d\left(a+bx^4\right)^{3/4}} - \frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\right.}{2c}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{}}{}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{}}{}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\left(a+bx^4\right)^{3/4}} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi}{2}$$

Mathematica [C] time = 0.32, size = 346, normalized size = 1.26

$$\frac{x\left(\frac{5\left(bx^4\left(a+bx^4\right)\left(c+dx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5ac\left(2a^2d+abdx^4+b^2x^4\left(c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+\right.}{10d\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4
, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(2*a^2*d + a*b*d*x^4 + b^2*x^
4*(c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x
^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)
])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(10*d*(a
 + b*x^4)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4 + a)^{\frac{5}{4}}}{d\,x^4 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{5}{4}}}{dx^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4 + a\right)^{5/4}}{d\,x^4 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(5/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^4\right)^{\frac{5}{4}}}{c + dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

3.201 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}$$

```
[Out] 1/2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),
I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c+1/2*EllipticPi(b^(1/4)*x/(
b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x
^4+a)^(1/2)/b^(1/4)/c
```

Rubi [A] time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {407, 409, 1218}

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*
Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c) + (Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c)
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$

$$= \frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}\,x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c} + \frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\right.}{}$$

$$= \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\right.\right.}{2\sqrt[4]{b}\,c}$$

Mathematica [C] time = 0.17, size = 160, normalized size = 0.96

$$\frac{5acx\sqrt[4]{a+bx^4}\,F_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(bcF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{1}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/(c + d*x^4),x)
[Out] int((a + b*x^4)^(1/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

3.202 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)} dx$

Optimal. Leaf size=259

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+b}}{}$$

```
[Out] -b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/
2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2
)/a^(1/2))),2^(1/2))/(-a*d+b*c)/(b*x^4+a)^(3/4)/a^(1/2)-1/2*d*EllipticPi(b^
(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^
(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)-1/2*d*EllipticPi(b^(1/4)*x/(b*x^
4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a
)^(1/2)/b^(1/4)/c/(-a*d+b*c)
```

Rubi [A] time = 0.15, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {410, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)]*
Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a
+ b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}dx = \frac{b\int \frac{1}{\left(a+bx^4\right)^{3/4}}dx}{bc-ad} - \frac{d\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}dx}{bc-ad}$$

$$= \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}dx}{(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad\right.}\right.}{bc-ad}$$

$$= -\frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}dx,x,\frac{1}{x}\right)}{(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \right.}{2c(bc}$$

$$= -\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{2\sqrt[4]{b}c(}$$

$$= -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\right.}{2\sqrt[4]{b}c(bc-ad)}$$

Mathematica [C] time = 0.04, size = 161, normalized size = 0.62

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*x
^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{3/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{3}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

3.203 $\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=304

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)^2}+-d$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/4)-1/3*b^(3/2)*(-5*a*d+2*b*c)*(1+a/b/x^4)
^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^
2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))
/a^(3/2)/(-a*d+b*c)^2/(b*x^4+a)^(3/4)+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a
)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^
(1/2)/b^(1/4)/c/(-a*d+b*c)^2+1/2*d^2*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(
-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1
/4)/c/(-a*d+b*c)^2
```

Rubi [A] time = 0.24, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)^2}+-d$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a*d)*(1 + a
/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/
2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x
^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)
]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(
1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
```

```
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\int \frac{-2bc+3ad-2bdx^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{d^2\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{\left(d^2\sqrt{\right.}}{}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x\right.}{3a(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}\,c(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}(bc-ad)^2\left(a+bx^4\right)^{3/4}} +$$

Mathematica [C] time = 0.27, size = 332, normalized size = 1.09

$$\frac{x\left(\frac{5\left(bx^4(c+dx^4)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5ac\left(3ad-b\left(3c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}-\frac{2bdx^4\left(\frac{bx^4}{a}+1\right)^{3/4}F_1\left(\frac{5}{4}\right.}{c}\right.}{15a\left(a+bx^4\right)^{3/4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/c + (5*(5*a*c*(3*a*d - b*(3*c + d*x^4))*AppellF1[1/4, 3/
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4*(c + d*x^4)*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 3/4,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])))))/(15*a*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{7}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{7/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{7}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

3.204 $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

Optimal. Leaf size=357

$$\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a}}{}$$

```
[Out] 1/7*b*x/a/(-a*d+b*c)/(b*x^4+a)^(7/4)+1/21*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^4+a)^(3/4)-1/21*b^(3/2)*(47*a^2*d^2-38*a*b*c*d+12*b^2*c^2)*(1+a/b/
x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcco
t(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1
/2))/a^(5/2)/(-a*d+b*c)^3/(b*x^4+a)^(3/4)-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x
^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4
+a)^(1/2)/b^(1/4)/c/(-a*d+b*c)^3-1/2*d^3*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/
b^(1/4)/c/(-a*d+b*c)^3
```

Rubi [A] time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {414, 527, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x)/(21*a^2*
(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*a
^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2
, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a + b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[
(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{11/4}\left(c+dx^4\right)}\,dx = \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} - \frac{\int \frac{-6bc+7ad-6bdx^4}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx}{7a(bc-ad)}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{\int \frac{12b^2c^2-26abcd+21a^2d^2+2\ldots}{\left(a+bx^4\right)^{3/4}\left(c+\ldots\right.}}{21a^2(bc-\ldots}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{d^3\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^3} + \frac{\left(b\left(1\ldots\right.\right.}{\ldots}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{\left(b\left(12b^2c^2-38abcd+\ldots\right.\right.}{21a^2\ldots}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{\left(b\left(12b^2c^2-38abcd+\ldots\right.\right.}{\ldots}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\ldots\right.}{2\sqrt[4]{b}\ldots}$$

$$= \frac{bx}{7a(bc-ad)\left(a+bx^4\right)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{b^{3/2}\left(12b^2c^2-38abcd+\ldots\right.}{21a\ldots}$$

Mathematica [C] time = 0.77, size = 430, normalized size = 1.20

$$\frac{x\left(-\frac{5\left(bx^4\left(c+dx^4\right)\left(16a^2d+ab\left(13dx^4-9c\right)-6b^2cx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5ac\left(21a^3d^2+a^2bd\left(5dx^4-42c\right)+ab^2\left(2\ldots\right.\right.}{\left(a+bx^4\right)\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1\ldots\right.\right.}\right.}{105a^2\left(a+bx^4\right)^{3/4}(bc-\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*(-6*b*c + 13*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c - (5*(5*a*c*(21*a^3*d^2 + 6*b^3*c*x^
4*(3*c + d*x^4) + a^2*b*d*(-42*c + 5*d*x^4) + a*b^2*(21*c^2 - 30*c*d*x^4 -
13*d^2*x^8))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4
*(c + d*x^4)*(16*a^2*d - 6*b^2*c*x^4 + a*b*(-9*c + 13*d*x^4))*(4*a*d*Appell
F1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(105*a^2*(b*c - a*d)^2*(a + b*x^4)^(
3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{11}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{11}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{11}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{11/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(11/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(11/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{11}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(11/4)*(c + d*x**4)), x)
```

3.205 $\int \frac{\left(a+bx^4\right)^{11/4}}{\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=280

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}$$

```
[Out] 1/4*b*(-a*d+2*b*c)*x*(b*x^4+a)^(3/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(7/4)
/c/d/(d*x^4+c)-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4)
)/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/
(b*x^4+a)^(1/4))/c^(7/4)/d^3-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctanh(b^(1/4)*x/
(b*x^4+a)^(1/4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctanh((-a*d+b*c)^
(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3
```

Rubi [A] time = 0.36, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {413, 528, 530, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTan[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4
)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{11/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)}+\frac{\int \frac{\left(a+bx^4\right)^{3/4}\left(a(bc+3ad)+4b(2bc-ad)x^4\right)}{c+dx^4}\,dx}{4cd}$$

$$=\frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2}-\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)}+\frac{\int \frac{-4a\left(2b^2c^2-2abcd-3a^2d^2\right)-4b^2c(8bc-11ad)x^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}}{16cd^2}$$

$$=\frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2}-\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)}-\frac{\left(b^2(8bc-11ad)\right)\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^3}+\frac{((bc}{}$$

$$=\frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2}-\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)}-\frac{\left(b^2(8bc-11ad)\right)\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\right.}{4d^3}$$

$$=\frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2}-\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)}-\frac{\left(b^2(8bc-11ad)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx,\right.}{8d^3}$$

$$=\frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2}-\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)}-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(b}{}$$

Mathematica [C] time = 0.96, size = 560, normalized size = 2.00

$$\frac{1}{80}\left(\frac{15a^3\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{c^{7/4}\sqrt[4]{bc-ad}}+\frac{10a^2b\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((20*x*(a + b*x^4)^(3/4)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^4))))/d^2 - (32*b
^3*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*
x^4)/c)])/(d^2*(a + b*x^4)^(1/4)) + (44*a*b^2*x^5*(1 + (b*x^4)/a)^(1/4)*App
ellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/(c*d*(a + b*x^4)^(1/4)
) + (15*a^3*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] -
Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*
c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(c^(7/4)*(b*c - a*d)^(1/4)) - (10*a*
b^2*c^(1/4)*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] -
Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*
c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(d^2*(b*c - a*d)^(1/4)) + (10*a^2*b*
(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4)
- ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/
4)*x)/(b + a*x^4)^(1/4)]))/(c^(3/4)*d*(b*c - a*d)^(1/4)))/80
```

fricas [B] time = 45.48, size = 3308, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 4646
4*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^
5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*
c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12
))^(1/4)*arctan(-(c^2*d^3*x*sqrt(((4096*b^11*c^14*d^6 - 22528*a*b^10*c^13*d
^7 + 46464*a^2*b^9*c^12*d^8 - 37664*a^3*b^8*c^11*d^9 - 5071*a^4*b^7*c^10*d^
10 + 25641*a^5*b^6*c^9*d^11 - 7931*a^6*b^5*c^8*d^12 - 6259*a^7*b^4*c^7*d^13
 + 2739*a^8*b^3*c^6*d^14 + 891*a^9*b^2*c^5*d^15 - 297*a^10*b*c^4*d^16 - 81*
a^11*c^3*d^17)*x^2*sqrt((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b
^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c
^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8
 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12)) + (2
62144*b^16*c^16 - 2031616*a*b^15*c^15*d + 6451200*a^2*b^14*c^14*d^2 - 10168
```

320*a^3*b^13*c^13*d^3 + 6467520*a^4*b^12*c^12*d^4 + 3123216*a^5*b^11*c^11*d
^5 - 7258119*a^6*b^10*c^10*d^6 + 2307030*a^7*b^9*c^9*d^7 + 2428965*a^8*b^8*
c^8*d^8 - 1607320*a^9*b^7*c^7*d^9 - 387134*a^10*b^6*c^6*d^10 + 436356*a^11*
b^5*c^5*d^11 + 40770*a^12*b^4*c^4*d^12 - 63720*a^13*b^3*c^3*d^13 - 6075*a^1
4*b^2*c^2*d^14 + 4374*a^15*b*c*d^15 + 729*a^16*d^16)*sqrt(b*x^4 + a))/x^2)*
((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*
b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c
^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9
- 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4) + (512*b^8*c^10*d^3 -
1984*a*b^7*c^9*d^4 + 2456*a^2*b^6*c^8*d^5 - 413*a^3*b^5*c^7*d^6 - 1175*a^4
*b^4*c^6*d^7 + 478*a^5*b^3*c^5*d^8 + 234*a^6*b^2*c^4*d^9 - 81*a^7*b*c^3*d^1
0 - 27*a^8*c^2*d^11)*(b*x^4 + a)^(1/4)*((4096*b^11*c^11 - 22528*a*b^10*c^10
*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 +
25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739
*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/
(c^7*d^12))^(1/4))/((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c
^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d
^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 8
91*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)*x)) + 4*(c*d^3*x^4 +
c^2*d^2)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42
592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*arctan((d^3*x*sqrt(((409
6*b^11*c^4*d^6 - 22528*a*b^10*c^3*d^7 + 46464*a^2*b^9*c^2*d^8 - 42592*a^3*b
^8*c*d^9 + 14641*a^4*b^7*d^10)*x^2*sqrt((4096*b^11*c^4 - 22528*a*b^10*c^3*d
+ 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12) +
(262144*b^16*c^6 - 2162688*a*b^15*c^5*d + 7434240*a^2*b^14*c^4*d^2 - 13629
440*a^3*b^13*c^3*d^3 + 14055360*a^4*b^12*c^2*d^4 - 7730448*a^5*b^11*c*d^5 +
1771561*a^6*b^10*d^6)*sqrt(b*x^4 + a))/x^2)*((4096*b^11*c^4 - 22528*a*b^10
*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d
^12)^(1/4) + (512*b^8*c^3*d^3 - 2112*a*b^7*c^2*d^4 + 2904*a^2*b^6*c*d^5 - 1
331*a^3*b^5*d^6)*(b*x^4 + a)^(1/4)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 4
6464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)
)/((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)*x)) + (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11
- 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 50
71*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^
7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d
^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22
528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^
4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4
*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 -
81*a^11*d^11)/(c^7*d^12))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a
^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c
^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/
4))/x) - (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 464
64*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a

```
^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3
*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^1
2))^(1/4)*log((c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2
*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6
*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d
^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3
/4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*
c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6
- 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4 + c^2*d^2
)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*log(-(d^9*x*((4096*b^11*c^4 - 22
528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*
b^7*d^4)/d^12)^(3/4) + (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2
 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b
^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3
+ 14641*a^4*b^7*d^4)/d^12)^(1/4)*log((d^9*x*((4096*b^11*c^4 - 22528*a*b^10*
c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^
12)^(3/4) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3
*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + 4*(b^2*c*d*x^5 + (2*b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*x)*(b*x^4 + a)^(3/4))/(c*d^3*x^4 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{11}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{11}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{11}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{11/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
[Out] Timed out
```

3.206 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=230

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)}{8c^{7/4}d}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^4+a)^(3/4)/c/d/(d*x^4+c)+1/2*b^(7/4)*arctan(b^(1/4)*
x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctan((-a*d+b*c)
^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2+1/2*b^(7/4)*arctanh(b^(1/4)*x
/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctanh((-a*d+b*c)
^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2
```

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {413, 530, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)}{8c^{7/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^2)
+ (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(
3/4)*(4*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/
4))])/(8*c^(7/4)*d^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
```

```
, n}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{a(bc+3ad)+4b^2cx^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)} + \frac{b^2\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d^2} - \frac{((bc-ad)(4bc+3ad))\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2} - \frac{((bc-ad)(4bc+3ad))\,\text{Subs}}{4c}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx, x,\right.}{2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)} + \frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right.}{8c^{7/4}d^2}$$

Mathematica [C] time = 0.64, size = 358, normalized size = 1.56

$$\frac{\frac{15a^2\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{\sqrt[4]{bc-ad}} + \frac{16b^2c^{3/4}x^5\sqrt[4]{\frac{bx^4}{a}+1}\,F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{d\sqrt[4]{a+bx^4}} - \frac{20c^{3/4}x\left(a+bx^4\right)^{3/4}(bc-a}{d\left(c+dx^4\right)}}{80c^{7/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((-20*c^(3/4)*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(d*(c + d*x^4)) + (16*b^2*c^
(3/4)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)])/(d*(a + b*x^4)^(1/4)) + (15*a^2*(2*ArcTan[((b*c - a*d)^(1/4)*x
)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a
*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(b*
c - a*d)^(1/4) + (5*a*b*c*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x
^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[
c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(d*(b*c - a*d)^(1/4)))
/(80*c^(7/4))
```

fricas [B] time = 3.26, size = 1667, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x - 4*(c*d^2*x^4 + c^2*d)*((256*b^7*
c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189
*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*arctan(-(
c^2*d^2*x*sqrt(((256*b^7*c^10*d^4 - 672*a^2*b^5*c^8*d^6 - 112*a^3*b^4*c^7*d
^7 + 609*a^4*b^3*c^6*d^8 + 189*a^5*b^2*c^5*d^9 - 189*a^6*b*c^4*d^10 - 81*a^
7*c^3*d^11)*x^2*sqrt((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d
^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d
^7)/(c^7*d^8)) + (4096*b^10*c^10 + 2048*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2
 - 9472*a^3*b^7*c^7*d^3 + 18928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9
639*a^6*b^4*c^4*d^6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^
9*b*c*d^9 + 729*a^10*d^10)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^7 - 672*a^2*b^
5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5
 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4) - (64*b^5*c^7*d^2 + 16*a*
b^4*c^6*d^3 - 116*a^2*b^3*c^5*d^4 - 45*a^3*b^2*c^4*d^5 + 54*a^4*b*c^3*d^6 +
 27*a^5*c^2*d^7)*(b*x^4 + a)^(1/4)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 11
2*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c
*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4))/((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 -
112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b
*c*d^6 - 81*a^7*d^7)*x)) - 16*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*arctan(-(
(b*x^4 + a)^(1/4)*b^5*d^2*(b^7/d^8)^(1/4) - d^2*x*(b^7/d^8)^(1/4)*sqrt((b^7
*d^4*x^2*sqrt(b^7/d^8) + sqrt(b*x^4 + a)*b^10)/x^2))/(b^7*x)) + (c*d^2*x^4
+ c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^
4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^
8))^(1/4)*log((c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*
d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/
x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d
^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*
a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d -
 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b
*x^4 + a)^(1/4))/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*log((d^6*x*(b^7
/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(
1/4)*log(-(d^6*x*(b^7/d^8)^(3/4) - (b*x^4 + a)^(1/4)*b^5)/x))/(c*d^2*x^4 +
c^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{7}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{7/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.207 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=135

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

```
[Out] 1/4*x*(b*x^4+a)^(3/4)/c/(d*x^4+c)+3/8*a*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(
b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)+3/8*a*arctanh((-a*d+b*c)^(1/4)*x/c
^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)
```

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {378, 377, 212, 208, 205}

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4)) + (3*a*ArcTanh
[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)
^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{\left(c+dx^4\right)^2}\,dx = \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{(3a)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}} + \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}\,x^2}\,dx, \right.}{8c^{3/2}}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

Mathematica [C] time = 0.06, size = 78, normalized size = 0.58

$$\frac{x\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{(ad-bc)x^4}{a\left(dx^4+c\right)}\right)}{c^2\left(\frac{bx^4}{a}+1\right)^{3/4}\sqrt[4]{\frac{dx^4}{c}+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]

[Out] (x*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, ((-(b*c) + a*d)*x^4)/(a*(c + d*x^4))])/(c^2*(1 + (b*x^4)/a)^(3/4)*(1 + (d*x^4)/c)^(1/4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{\left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4 + a\right)^{\frac{3}{4}}}{\left(d\,x^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)

[Out] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{\left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^{\frac{3}{4}}}{\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

3.208 $\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

```
[Out] -1/4*d*x*(b*x^4+a)^(3/4)/c/(-a*d+b*c)/(d*x^4+c)+1/8*(-3*a*d+4*b*c)*arctan((
-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(5/4)+1/8*(-3
*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a
*d+b*c)^(5/4)
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 377, 212, 208, 205}

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*A
rcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
 a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a +
 b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)^2}\,dx = -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)}+\frac{(4bc-$$

$$= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{c}}\right)}{8c^{7/4}(bc-ad)^{5/4}}$$

Mathematica [C] time = 0.14, size = 99, normalized size = 0.61

$$\frac{x\left(\left(c+dx^4\right)(4bc-3ad)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-cd\left(a+bx^4\right)\right)}{4c^2\sqrt[4]{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(-(c*d*(a + b*x^4)) + (4*b*c - 3*a*d)*(c + d*x^4)*Hypergeometric2F1[1/4,
 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]))/(4*c^2*(b*c - a*d)*(a + b*x^4
)^(1/4)*(c + d*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^4+a\right)^{1/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)**2), x)
```

3.209 $\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=205

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}\,(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,\left(c+\right.}$$

```
[Out] 1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d+b*c)/(
b*x^4+a)^(1/4)/(d*x^4+c)-1/8*d*(-3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(
1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)-1/8*d*(-3*a*d+8*b*c)*arctanh
((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)
```

Rubi [A] time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}\,(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,\left(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b
*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c - 3*a*d)*ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
 - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1
/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} + \frac{\int \frac{4bc-3ad-4bdx^4}{(a+bx^4)^{5/4}(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{\int \frac{ad(8bc-3ad)}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx}{4ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad))\int \frac{}{\sqrt[4]{a}}}{4c(bc-a}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad))\,\text{Subs}}{4}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{(d(8bc-3ad))\,\text{Subs}}{8}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{d(8bc-3ad)\tan^{-1}}{8c^{7/4}(bc-a}$$

Mathematica [C] time = 2.27, size = 625, normalized size = 3.05

$$\frac{c\,(a+bx^4)^{3/4}\left(\frac{320d^2x^{20}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{9}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^5(a+bx^4)^3} + \frac{640dx^{16}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{9}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^4(a+bx^4)^3} + \frac{320x^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{9}{4};1,\frac{17}{4};\right.}{c^3(a+bx^4)^3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (c*(a + b*x^4)^(3/4)*(-47385 - (94770*d*x^4)/c - (44460*d^2*x^8)/c^2 + (514
8*(b*c - a*d)*x^4)/(c*(a + b*x^4)) + (14976*d*(b*c - a*d)*x^8)/(c^2*(a + b*
x^4)) + (7488*d^2*(b*c - a*d)*x^12)/(c^3*(a + b*x^4)) + 47385*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + (94770*d*x^4*Hyperge
ometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c + (44460*d^2*
x^8*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^2
- (14625*(b*c - a*d)*x^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(
```

```
c*(a + b*x^4))])/(c*(a + b*x^4)) + (33930*d*(-(b*c) + a*d)*x^8*Hypergeometr
ic2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^2*(a + b*x^4)) +
(16380*d^2*(-(b*c) + a*d)*x^12*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*
x^4)/(c*(a + b*x^4))])/(c^3*(a + b*x^4)) + (320*(b*c - a*d)^3*x^12*Hypergeo
metricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^3*
(a + b*x^4)^3) + (640*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 9/4}, {
1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^4*(a + b*x^4)^3) + (320*d^
2*(b*c - a*d)^3*x^20*HypergeometricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)
*x^4)/(c*(a + b*x^4))])/(c^5*(a + b*x^4)^3)))/(2340*(b*c - a*d)^2*x^7*(c +
d*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{5/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{5}{4}} \left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)
```

3.210 $\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{\quad}{4c\left(a+bx\right.}$$

```
[Out] 1/20*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(5/4)+1/20*b*(-5*a^2*d^2-
56*a*b*c*d+16*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d
+b*c)/(b*x^4+a)^(5/4)/(d*x^4+c)+3/8*d^2*(-a*d+4*b*c)*arctan((-a*d+b*c)^(1/4
)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13/4)+3/8*d^2*(-a*d+4*b*c)
*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13
/4)
```

Rubi [A] time = 0.29, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{\quad}{4c\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(16*b^2
*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)^3*(a + b*x^4)^(1/4)
) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + (3*d^2*(4*b*c -
 a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)
*(b*c - a*d)^(13/4)) + (3*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} - \frac{\int \frac{-16b^2c^2+40abc\ldots}{(a+\ldots}}{20\ldots}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+b\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+b\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+b\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+b\ldots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+b\ldots\right.}$$

Mathematica [C] time = 5.01, size = 1216, normalized size = 4.57

$$-\frac{69120d^3(bc-ad)^4\,{}_3F_2\left(2,2,\frac{13}{4};1,\frac{21}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)x^{28} + 11520d^3(bc-ad)^4\,{}_4F_3\left(2,2,2,\frac{13}{4};1,1,\frac{21}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)x^{28} + 21\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/198900*(285532*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 + 933504*c^4*d*(b*c -
 a*d)^2*x^12*(a + b*x^4)^2 + 891072*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^
2 + 282880*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 + 9793836*c^6*(b*c - a*
```

```
d)*x^4*(a + b*x^4)^3 + 27973296*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 + 25968
384*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3 + 8146944*c^3*d^3*(b*c - a*d)*x^
16*(a + b*x^4)^3 - 23529870*c^7*(a + b*x^4)^4 - 65547495*c^6*d*x^4*(a + b*x
^4)^4 - 60505380*c^5*d^2*x^8*(a + b*x^4)^4 - 18935280*c^4*d^3*x^12*(a + b*x
^4)^4 - 14499810*c^6*(b*c - a*d)*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1
, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 41082795*c^5*d*(b*c - a*d)*x^8*
(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^
4))] - 38069460*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3*Hypergeometric2F1[1/
4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 11934000*c^3*d^3*(b*c - a*d
)*x^16*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
 + b*x^4))] + 23529870*c^7*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 65547495*c^6*d*x^4*(a + b*x^4)^4*Hypergeo
metric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 60505380*c^5*d^
2*x^8*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
+ b*x^4))] + 18935280*c^4*d^3*x^12*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1,
5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 77760*c^3*(b*c - a*d)^4*x^16*Hype
rgeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] +
 224640*c^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4},
 ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 216000*c*d^2*(b*c - a*d)^4*x^24*Hyper
geometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] +
69120*d^3*(b*c - a*d)^4*x^28*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 11520*c^3*(b*c - a*d)^4*x^16*Hypergeometr
icPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 3
4560*c^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/
4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 34560*c*d^2*(b*c - a*d)^4*x^24*Hyp
ergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x
^4))] + 11520*d^3*(b*c - a*d)^4*x^28*HypergeometricPFQ[{2, 2, 2, 13/4}, {1,
 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^8*(-b + (a*d)/c)^3*x^11*(
a + b*x^4)^(13/4)*(c + d*x^4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{4}+a\right)^{\frac{9}{4}} \left(dx^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{9/4}\,{\left(d\,x^4+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{9}{4}} \left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)**2), x)
```

3.211 $\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2} dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{a}\, b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\left(a+bx^4\right)^{3/4}}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin\right.}{8\sqrt[4]{b}\,c^2d^2}$$

```
[Out] 1/4*b*(-a*d+3*b*c)*x*(b*x^4+a)^(1/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(5/4)
/c/d/(d*x^4+c)-1/4*b^(3/2)*(-a*d+3*b*c)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arcc
ot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/d^2/(b*x^4+a)
^(3/4)-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a
*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4
)/c^2/d^2-3/8*(-a*d+b*c)*(a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(
-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1
/4)/c^2/d^2
```

Rubi [A] time = 0.35, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {413, 528, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{\sqrt{a}\, b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\left(a+bx^4\right)^{3/4}}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin\right.}{8\sqrt[4]{b}\,c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4
))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*d^2*(a + b
*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*S
qrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{9/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{\sqrt[4]{a+bx^4}\left(a(bc+3ad)+2b(3bc-ad)x^4\right)}{c+dx^4}\,dx}{4cd}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{-2a\left(3b^2c^2-2abcd-3a^2d^2\right)-4b\left(3b^2c^2-3abcd-a^2d^2\right)}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}}{8cd^2}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} + \frac{(ab(3bc-ad))\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{4cd^2} - \frac{(3(bc-a}{}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} + \frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}}}{4cd^2\left(a+bx^4\right)^{3/4}}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} - \frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int -\right.}{4cd^2\left(a+bx^4\right)^{3/4}}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} - \frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi}{8\sqrt[4]{b}\,c^2d^2}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)} - \frac{\sqrt{a}\,b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot\right.}{4cd^2\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.59, size = 392, normalized size = 1.11

$$\frac{2bx^5\left(\frac{bx^4}{a}+1\right)^{3/4}\left(a^2d^2+3abcd-3b^2c^2\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(x^5\left(a+bx^4\right)\left(a^2d^2-2abcd+b^2c\left(3c+2dx^4\right)\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4}\right.\right.\right.}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2\right.\right.\right.}}{20c^2d^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (2*b*(-3*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^4)/a)^(3/4)*AppellF1[
5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*x*(4*a^3*d^2 +
 a^2*b*d^2*x^4 + b^3*c*x^4*(3*c + 2*d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b
*x^4)/a), -((d*x^4)/c)] + x^5*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c*(3*
c + 2*d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*
x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(
4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appell
F1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d^2*(a + b*x^4
)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{9}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{9}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{9}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{9/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(9/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(9/4)/(c + d*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
[Out] Timed out
```

3.212 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=298

$$\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(3ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d}+$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^4+a)^(1/4)/c/d/(d*x^4+c)+1/4*b^(3/2)*(1+a/b/x^4)^(3/
4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^
(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(
1/2)/c/d/(b*x^4+a)^(3/4)+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(
1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/
2)/b^(1/4)/c^2/d+1/8*(3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a
*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4
)/c^2/d
```

Rubi [A] time = 0.24, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {413, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a}\,b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(3ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d*(c + d*x^4)) + (Sqrt[a]*b^(3/2)*(
1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*
c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^
4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
```

```
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)}+\frac{\int \frac{a(bc+3ad)+2b(bc+ad)x^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)}-\frac{(ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{4cd}-\frac{(-2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)}-\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4cd\left(a+bx^4\right)^{3/4}}-\frac{\left((-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)}+\frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)}{4cd\left(a+bx^4\right)^{3/4}}-\frac{\left((-2bc-3ad)\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)}+\frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\,\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2d}+$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)}+\frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}}{}$$

Mathematica [C] time = 0.49, size = 341, normalized size = 1.14

$$\frac{x\left(\frac{5c\left(x^4(a+bx^4)(ad-bc)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5ac\left(4a^2d+abdx^4-b^2cx^4\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{(c+dx^4)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+2bx^4\left(\frac{bx}{a}\right.\right.}{20c^2d\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(2*b*(b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -(
(b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*(4*a^2*d - b^2*c*x^4 + a*b*d*x^4)*
AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + (-(b*c) + a*d)*x^4
*(a + b*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
+ 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x
^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4
*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF
1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*d*(a + b*x^4)^
(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{5}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{5}{4}}}{\left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4 + a\right)^{5/4}}{\left(d\,x^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(5/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(5/4)/(c + d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^4\right)^{\frac{5}{4}}}{\left(c + dx^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4)**2, x)
```

3.213 $\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2} \, dx$

Optimal. Leaf size=308

$$-\frac{\sqrt{a}\, b^{3/2} x^3 \left(\frac{a}{bx^4}+1\right)^{3/4} F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right)\middle|2\right)}{4c\left(a+bx^4\right)^{3/4}(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\, c^2(bc-ad)}+$$

```
[Out] 1/4*x*(b*x^4+a)^(1/4)/c/(d*x^4+c)-1/4*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*(cos(1/
2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x^2*b^(1/2)/a^(1/2))
)*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)/c/(-a*d+b
*c)/(b*x^4+a)^(3/4)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4)
,-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b
^(1/4)/c^2/(-a*d+b*c)+1/8*(-3*a*d+2*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/
b^(1/4)/c^2/(-a*d+b*c)
```

Rubi [A] time = 0.20, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {412, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{\sqrt{a}\, b^{3/2} x^3 \left(\frac{a}{bx^4}+1\right)^{3/4} F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right)\middle|2\right)}{4c\left(a+bx^4\right)^{3/4}(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}\, c^2(bc-ad)}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*(b*c - a*d)*(
a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*El
lipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^
4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a +
b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSi
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
```

```
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx = \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} - \frac{\int \frac{-3a-2bx^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{4c}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} + \frac{(ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{4c(bc-ad)} + \frac{(2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\Big(}{4c(bc-}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx}\right.}{}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}\,c^2(bc-ad)} + \frac{(2bc-3ad)}{}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} - \frac{\sqrt{a}\,b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\Big(}{8\sqrt[4]{b}\,c^2(b}$$

Mathematica [C] time = 0.31, size = 233, normalized size = 0.76

$$\frac{x\left(\frac{5\left(\frac{a+bx^4}{c}-\frac{15a^2F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}\right)}{c+dx^4}+\frac{2bx^4\left(\frac{bx^4}{a}+1\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}\right)}{20\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((2*b*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c - (15*a^2*AppellF1[1/4, 3/4, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)
/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
)))/(c + d*x^4)))/(20*(a + b*x^4)^(3/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{1/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(1/4)/(c + d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4)**2, x)
```

3.214 $\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=330

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\right.}{8\sqrt[4]{b}\,c^2(bc-ad)^2}$$

```
[Out] -1/4*d*x*(b*x^4+a)^(1/4)/c/(-a*d+b*c)/(d*x^4+c)-1/4*b^(3/2)*(-a*d+4*b*c)*(1
+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*
arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2)))
,2^(1/2))/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)/a^(1/2)-3/8*d*(-a*d+2*b*c)*Ellipti
cPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b*x^
4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2-3/8*d*(-a*d+2*b*c)*Ell
ipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a/(b
*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^2
```

Rubi [A] time = 0.25, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\,\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{bx^4+a}}\right)\right.}{8\sqrt[4]{b}\,c^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(1/4))/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)*(4*b*c -
a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (3*d*(2*b*c - a*d)*Sqrt[
a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^2) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\int \frac{4bc-3ad-2bdx^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{(3d(2bc-ad))\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)^2}+\frac{(b(4bc-ad))\int \frac{1}{\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{\left(3d(2b\right.}{}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx,x,\right.}{4c(bc-ad)^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a+}}\right.\right.}{8\sqrt[4]{b}\,c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)\left(c+dx^4\right)}-\frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\,c(bc-ad)^2\left(a+bx^4\right)^{3/4}}-\frac{3}{}$$

Mathematica [C] time = 0.34, size = 337, normalized size = 1.02

$$\frac{x\left(\frac{2bdx^4\left(\frac{bx^4}{a}+1\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ad-bc}+\frac{c\left(25ac\left(4ad-4bc+bdx^4\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5dx^4\left(a+bx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}{\left(c+dx^4\right)(bc-ad)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}\right)}{20c^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*((2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a
), -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*d*x^4*(a + b*x^4)*
(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appel
lF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c + d*x^
4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*
a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1
[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*(a + b*x^4)^(3/
4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{3/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{3}{4}}\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)**2), x)
```

3.215 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

Optimal. Leaf size=390

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right.}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

```
[Out] 1/12*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)-1/4*d*x/c/(-a*d+b*c
)/(b*x^4+a)^(3/4)/(d*x^4+c)-1/12*b^(3/2)*(3*a^2*d^2-32*a*b*c*d+8*b^2*c^2)*(
1+a/b/x^4)^(3/4)*x^3*(cos(1/2*arccot(x^2*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2
*arccot(x^2*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x^2*b^(1/2)/a^(1/2))
),2^(1/2))/a^(3/2)/c/(-a*d+b*c)^3/(b*x^4+a)^(3/4)+1/8*d^2*(-3*a*d+10*b*c)*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)*(a
/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3+1/8*d^2*(-3*a*d+
10*b*c)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/
2),I)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)/b^(1/4)/c^2/(-a*d+b*c)^3
```

Rubi [A] time = 0.41, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {414, 527, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right.}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (d*x)/(4*c
*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*(8*b^2*c^2 - 32*a*b*
c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*
c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-6bdx^4}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{\int \frac{-8b^2c^2+24abc}{(a+}}{12}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\left(d^2(10bc-3a\right.}{4c(b}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\left(b\left(8b^2c^2-3\right.\right.}{}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{\left(b\left(8b^2c^2-3\right.\right.}{}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{d^2(10bc-3a}{}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{b^{3/2}\left(8b^2c^2-\right.}{}$$

Mathematica [C] time = 0.58, size = 387, normalized size = 0.99

$$\frac{x\left(\frac{c\left(25ac\left(12a^2d^2+3abd\left(dx^4-8c\right)+4b^2c\left(3c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5x^4\left(3a^2d^2+3abd^2x^4+4b^2c\left(c+dx^4\right)\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\right.\right.\right.}{\left(c+dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\right.}{60ac^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c +
d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(4*a*
d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5
/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF
1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4,
3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x^4)^(3/4)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{7}{4}} \left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4 + a\right)^{\frac{7}{4}} \left(d\,x^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{7}{4}}\left(dx^4 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4 + a\right)^{7/4}\,\left(d\,x^4 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
[Out] Timed out
```

3.216 $\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

```
[Out] 1/4*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)+1/4*arctanh(1/2*x*2^(3/4)/(
x^4+1)^(1/4))*2^(1/4)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1+x^4}\left(2+x^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}}$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] (ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(2*2^(3/4))
```

fricas [B] time = 16.99, size = 208, normalized size = 3.92

$$-\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(8^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+8^{\frac{1}{4}}\left(3\,x^4+2\right)\right)}{2\left(x^4+2\right)}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}\left(x^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")
```

```
[Out] -1/16*8^(3/4)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(1/4)*(x^4 + 1
)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*x^4 + 2)))/(x^4
```

+ 2)) + 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x
^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64
*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 -
8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)

maple [C] time = 3.24, size = 211, normalized size = 3.98

$$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{-3x^4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)-2\left(x^4+1\right)^{\frac{1}{4}}x^3\,\mathrm{RootOf}\left(\_Z^4-2\right)^2+2\sqrt{x^4+1}\,x^2\,\mathrm{RootOf}\left(\right.}{x^4+}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+1)^(1/4)/(x^4+2),x)

[Out] -1/8*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(2*(x^4+1)^(1/2)*RootOf(_Z^4-2)^2*Ro
otOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3-3*Root
Of(_Z^2+RootOf(_Z^4-2)^2)*x^4+4*(x^4+1)^(3/4)*x-2*RootOf(_Z^2+RootOf(_Z^4-2
)^2))/(x^4+2))+1/8*RootOf(_Z^4-2)*ln((2*(x^4+1)^(1/2)*RootOf(_Z^4-2)^3*x^2+
2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3+3*RootOf(_Z^4-2)*x^4+4*(x^4+1)^(3/4)*x
+2*RootOf(_Z^4-2))/(x^4+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(x^4+1\right)^{1/4}\,\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)
[Out] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^4+1}\,\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

3.217 $\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

```
[Out] 1/2*arctan(a^(1/4)*x/(b*x^4+a)^(1/4))/a^(5/4)+1/2*arctanh(a^(1/4)*x/(b*x^4+
a)^(1/4))/a^(5/4)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*a^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx = \text{Subst}\left(\int \frac{1}{a-(ab-a(-a+b))x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
[Out] (ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(1
/4)])/(2*a^(5/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
[Out] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(a-b)\,x^4+a\right)\left(b\,x^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
[Out] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
[Out] -integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^4+a\right)^{1/4}\,\left(a-x^4\,\left(a-b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))),x)
[Out] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

```
[Out] -Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(a
 + b*x**4)**(1/4)), x)
```

3.218 $\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \,dx$

Optimal. Leaf size=79

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

```
[Out] x*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^4/a,-d*x^4/c)/((1+b*x
^4/a)^p)/((1+d*x^4/c)^q)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((
d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q\,dx = \left(\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p \left(c+dx^4\right)^q\,dx$$
$$= \left(\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(c+dx^4\right)^q \left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \left(1+\frac{bx^4}{a}\right)^p \left(1+\frac{dx^4}{c}\right)^q\,dx$$
$$= x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(c+dx^4\right)^q \left(1+\frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

Mathematica [B] time = 0.26, size = 172, normalized size = 2.18

$$\frac{5acx\left(a+bx^4\right)^p\left(c+dx^4\right)^q F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{4x^4\left(bcpF_1\left(\frac{5}{4};1-p,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+adqF_1\left(\frac{5}{4};-p,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a
), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)
/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + a*d*q*AppellF1[5/4, -p, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

fricas [F] time = 1.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)^p\left(dx^4+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)^p\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p*(d*x^4+c)^q,x)
[Out] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{4}+a\right)^{p}\left(dx^{4}+c\right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^{4}+a\right)^{p}\,\left(d\,x^{4}+c\right)^{q}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + d*x^4)^q,x)
[Out] int((a + b*x^4)^p*(c + d*x^4)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
[Out] Timed out
```

3.219 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx$

Optimal. Leaf size=176

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right){}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d^2(4q+5)(4q+9)}-\frac{bx\left(c+dx^4\right)^{q+1}}{d^2(4q+}$$

```
[Out] -b*(5*b*c-a*d*(13+4*q))*x*(d*x^4+c)^(1+q)/d^2/(16*q^2+56*q+45)+b*x*(b*x^4+a
)*(d*x^4+c)^(1+q)/d/(9+4*q)+(5*b^2*c^2-2*a*b*c*d*(9+4*q)+a^2*d^2*(16*q^2+56
*q+45))*x*(d*x^4+c)^q*hypergeom([1/4, -q],[5/4],-d*x^4/c)/d^2/(16*q^2+56*q+
45)/((1+d*x^4/c)^q)
```

Rubi [A] time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {416, 388, 246, 245}

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right){}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d^2(4q+5)(4q+9)}-\frac{bx\left(c+dx^4\right)^{q+1}}{d^2(4q+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*q)*(9 + 4*
q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) + ((5*b^2*c^2 -
2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hyperge
ometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^
4)/c)^q)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^q\,dx=\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)}+\frac{\int \left(c+dx^4\right)^q\left(-a(bc-ad(9+4q))-b(5bc-ad(13+4\right.}{d(9+4q)}$$

$$=-\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)}+\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)}+\frac{\left(5b^2c^2-2abcd\right.}{}$$

$$=-\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)}+\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)}+\frac{\left(\left(5b^2c^2-2abc\right.\right.}{}$$

$$=-\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)}+\frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)}+\frac{\left(5b^2c^2-2abcd\right.}{}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.60

$$\frac{1}{45}x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(45a^2\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+bx^4\left(18a\,{}_2F_1\left(\frac{5}{4},-q;\frac{9}{4};-\frac{dx^4}{c}\right)+5bx^4\,{}_2F_1\left(\frac{9}{4},-q;\frac{13}{4};-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(45*a^2*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)] + b*
x^4*(18*a*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)] + 5*b*x^4*Hypergeom
etric2F1[9/4, -q, 13/4, -((d*x^4)/c)])))/(45*(1 + (d*x^4)/c)^q)
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2x^8 + 2\,abx^4 + a^2\right)\left(dx^4 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="fricas")
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(d*x^4 + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^2 \left(dx^4 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \left(b\,x^4 + a\right)^2 \left(d\,x^4 + c\right)^q\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^q,x)
[Out] int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4 + a\right)^2 \left(dx^4 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^4 + a\right)^2 \left(d\,x^4 + c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^q,x)
```

```
[Out] int((a + b*x^4)^2*(c + d*x^4)^q, x)
```

sympy [C] time = 166.03, size = 119, normalized size = 0.68

$$\frac{a^2c^qx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-q\\\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{abc^qx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-q\\\frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{b^2c^qx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},-q\\\frac{13}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**q,x)
```

```
[Out] a**2*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(
4*gamma(5/4)) + a*b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*ex
p_polar(I*pi)/c)/(2*gamma(9/4)) + b**2*c**q*x**9*gamma(9/4)*hyper((9/4, -q)
, (13/4,), d*x**4*exp_polar(I*pi)/c)/(4*gamma(13/4))
```

3.220 $\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx$

Optimal. Leaf size=93

$$\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}-\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}(bc-ad(4q+5))\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d(4q+5)}$$

```
[Out] b*x*(d*x^4+c)^(1+q)/d/(5+4*q)-(b*c-a*d*(5+4*q))*x*(d*x^4+c)^q*hypergeom([1/
4, -q],[5/4],-d*x^4/c)/d/(5+4*q)/((1+d*x^4/c)^q)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a-\frac{bc}{4dq+5d}\right){}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) + ((a - (b*c)/(5*d + 4*d*q))*x*(c +
 d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(1 + (d*x^4)/c)^q
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx=\frac{bx\left(c+dx^4\right)^{1+q}}{d(5+4q)}-\left(-a+\frac{bc}{5d+4dq}\right)\int \left(c+dx^4\right)^q\,dx$$
$$=\frac{bx\left(c+dx^4\right)^{1+q}}{d(5+4q)}-\left(\left(-a+\frac{bc}{5d+4dq}\right)\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \left(1+\frac{dx^4}{c}\right)^q\,dx$$
$$=\frac{bx\left(c+dx^4\right)^{1+q}}{d(5+4q)}+\left(a-\frac{bc}{5d+4dq}\right)x\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)$$

Mathematica [A] time = 0.03, size = 90, normalized size = 0.97

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left((ad(4q+5)-bc)\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+b\left(c+dx^4\right)\left(\frac{dx^4}{c}+1\right)^q\right)}{d(4q+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(b*(c + d*x^4)*(1 + (d*x^4)/c)^q + (-(b*c) + a*d*(5 + 4*q)
)*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]))/(d*(5 + 4*q)*(1 + (d*x^4)
/c)^q)
```

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+a\right)\left(dx^4+c\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*(d*x^4 + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^4+a\right)\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="giac")
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^{4}+a\right)\left(d\,x^{4}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^q,x)
[Out] int((b*x^4+a)*(d*x^4+c)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{4}+a\right)\left(dx^{4}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^{4}+a\right)\,\left(d\,x^{4}+c\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^q,x)
[Out] int((a + b*x^4)*(c + d*x^4)^q, x)
```

sympy [C] time = 61.01, size = 75, normalized size = 0.81

$$\frac{ac^{q}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-q\\\frac{5}{4}\end{matrix}\middle|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{bc^{q}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},-q\\\frac{9}{4}\end{matrix}\middle|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**q,x)
[Out] a*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(4*g
amma(5/4)) + b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*exp_pol
ar(I*pi)/c)/(4*gamma(9/4))
```

3.221 $\int \frac{(c+dx^4)^q}{a+bx^4}\,dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

```
[Out] x*(d*x^4+c)^q*AppellF1(1/4,1,-q,5/4,-b*x^4/a,-d*x^4/c)/a/((1+d*x^4/c)^q)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*
(1 + (d*x^4)/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^4)^q}{a+bx^4}\,dx = \left((c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^4}{c}\right)^q}{a+bx^4}\,dx$$
$$= \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

Mathematica [B] time = 0.22, size = 162, normalized size = 2.84

$$\frac{5acx(c+dx^4)^q F_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(4x^4\left(adqF_1\left(\frac{5}{4};1-q,1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-bcF_1\left(\frac{5}{4};-q,2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)+5acF_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] -
 b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^4+c)^q}{bx^4+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^4+c)^q}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

maple [F]   time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{4}+c\right)^{q}}{b\,x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^q/(b*x^4+a),x)
[Out] int((d*x^4+c)^q/(b*x^4+a),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{4}+c\right)^{q}}{bx^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="maxima")
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^{4}+c\right)^{q}}{b\,x^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^q/(a + b*x^4),x)
[Out] int((c + d*x^4)^q/(a + b*x^4), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a),x)
[Out] Timed out
```

3.222 $\int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx$

Optimal. Leaf size=57

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

```
[Out] x*(d*x^4+c)^q*AppellF1(1/4,2,-q,5/4,-b*x^4/a,-d*x^4/c)/a^2/((1+d*x^4/c)^q)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(c+dx^4)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^
2*(1 + (d*x^4)/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx = \left((c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^4}{c}\right)^q}{(a+bx^4)^2}\,dx$$
$$= \frac{x(c+dx^4)^q\left(1+\frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Mathematica [B] time = 0.24, size = 162, normalized size = 2.84

$$\frac{5acx(c+dx^4)^q F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^2\left(4x^4\left(adqF_1\left(\frac{5}{4};2,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2bcF_1\left(\frac{5}{4};3,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)
])/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 - 2*b*c*AppellF1[5/4, 3, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx^4+c)^q}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^4+c)^q}{(bx^4+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="giac")
```

[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{4}+c\right)^{q}}{\left(b\,x^{4}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^4+c)^q/(b*x^4+a)^2,x)

[Out] int((d*x^4+c)^q/(b*x^4+a)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{4}+c\right)^{q}}{\left(bx^{4}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="maxima")

[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^{4}+c\right)^{q}}{\left(b\,x^{4}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^4)^q/(a + b*x^4)^2,x)

[Out] int((c + d*x^4)^q/(a + b*x^4)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**4+c)**q/(b*x**4+a)**2,x)

[Out] Timed out

3.223 $\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$

Optimal. Leaf size=545

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\,x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\ldots}{\sqrt[5]{c}\sqrt[5]{a+\ldots}}\right.}{5c^{4/5}\sqrt[5]{bc\ldots}}$$

```
[Out] -1/5*ln(c^(1/5)-(-a*d+b*c)^(1/5)*x/(b*x^5+a)^(1/5))/c^(4/5)/(-a*d+b*c)^(1/5
)+1/20*ln((2*(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5
)+2*c^(2/5)*(b*x^5+a)^(2/5)-c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1
/2))/(b*x^5+a)^(2/5))*(-5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/20*ln((2*(-a*
d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)+2*c^(2/5)*(b*x^
5+a)^(2/5)+c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)*5^(1/2))/(b*x^5+a)^(2
/5))*(5^(1/2)+1)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(1/5*(-a*d+b*c)^(1/5)*
x*(50+10*5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^(1/5)+1/5*(25+10*5^(1/2))^(1/2))*
(10-2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)+1/10*arctan(-1/5*(25-10*5^(1/
2))^(1/2)+2*(-a*d+b*c)^(1/5)*x*2^(1/2)/(5+5^(1/2))^(1/2)/c^(1/5)/(b*x^5+a)^
(1/5))*(10+2*5^(1/2))^(1/2)/c^(4/5)/(-a*d+b*c)^(1/5)
```

Rubi [A] time = 1.09, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 202, 634, 618, 204, 628, 31}

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+2x^2(bc-ad)^{2/5}-\sqrt{5}\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[2/(5 + Sqr
t[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(5*c^(4/5)*(b*c -
 a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] + (Sqr
t[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(
5*c^(4/5)*(b*c - a*d)^(1/5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x
^5)^(1/5)]/(5*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a*d
)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*c^(1/
5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a
+ b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 + Sqrt[5])*Log[(2*(b*
```

```
c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt[5
]*c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/
5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^5}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt[5]{c}+\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x+(bc-ad)^{2/5}x^2}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt[5]{c}+}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right.}\right.}{}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x+(bc-ad)^{2/5}x^2}\,dx, x,\right.}{20c^{3/5}}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1-\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{\left(a+bx^5\right)^{2/5}}+\frac{\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}-\frac{\sqrt{5}\,\sqrt[5]{c}\,\sqrt[5]{bc-}}{\sqrt[5]{a+bx^5}}\right.}{20c^{4/5}\sqrt[5]{bc-ad}}$$

$$= \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{5+\sqrt{5}}\left(\left(1+\sqrt{5}\right)\sqrt[5]{c}+\right.}{2\sqrt{10}\sqrt[5]{c}}\right.}{5c^{4/5}\sqrt[5]{bc-ad}}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.09

$$\frac{x\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};\frac{(bc-ad)x^5}{c\left(bx^5+a\right)}\right)}{c\sqrt[5]{a+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] (x*Hypergeometric2F1[1/5, 1, 6/5, ((b*c - a*d)*x^5)/(c*(a + b*x^5))])/(c*(a
 + b*x^5)^(1/5))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^5+a\right)^{\frac{1}{5}}\left(dx^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^5+a\right)^{\frac{1}{5}}\left(d\,x^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

```
[Out] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^5+a\right)^{\frac{1}{5}}\left(dx^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^5+a\right)}^{1/5}\,\left(d\,x^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)),x)
[Out] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
[Out] Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

3.224 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$

Optimal. Leaf size=143

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}$$

```
[Out] c^2*(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-7/5*d*(c+d/x)^2*(a+b
/x)^(1/2)-1/15*d*(-4*a^2*d^2+30*a*b*c*d+114*b^2*c^2+b*d*(2*a*d+33*b*c)/x)*(
a+b/x)^(1/2)/b^2+(c+d/x)^3*x*(a+b/x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 97, 153, 147, 63, 208}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a
*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + Sqrt[a + b/x]*(
c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^3}{x^2}dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(c+dx)^2\left(\frac{1}{2}(bc+6ad)+\frac{7bdx}{2}\right)}{x\sqrt{a+bx}}dx, x, \frac{1}{x}\right)$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{(c+dx)\left(\frac{5}{4}bc(bc+6ad)+\frac{1}{4}bd(33bc+2\right.}{x\sqrt{a+bx}}\right.}{5b}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{}$$

Mathematica [A] time = 0.19, size = 118, normalized size = 0.83

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^2d^3x^2-2abd^2x(15cx+d)-3b^2\left(-5c^3x^3+30c^2dx^2+10cd^2x+2d^3\right)\right)}{15b^2x^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 + 1
0*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.19, size = 306, normalized size = 2.14

$$\left[\frac{15\left(b^3c^3+6\,ab^2c^2d\right)\sqrt{a}\,x^2\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,ab^2c^3x^3-6\,ab^2d^3-2\left(45\,ab^2c^2d+15\,a^2bcd^2\right.\right.}{30\,ab^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2*d
 + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*sqrt
((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a)*x^
2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 -
2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a
^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1
,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[
2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+
%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%
},0,%%%{1,[0,2,2]%%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%
%%}] at parameters values [18,15.451549686,-33]Sign error (%%%{-b,0%%%}+%%%
{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%
%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.06, size = 248, normalized size = 1.73

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(90a\,b^{2}c^{2}d\,x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+15b^{3}c^{3}x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+180\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}b\,c^{2}d\,x^{4}+30\right.}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3*(a+b/x)^(1/2),x)
```

```
[Out] 1/30*((a*x+b)/x)^(1/2)*(180*(a*x^2+b*x)^(1/2)*a^(3/2)*x^4*b*c^2*d+30*(a*x^2
+b*x)^(1/2)*a^(1/2)*x^4*b^2*c^3+90*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1
/2))/a^(1/2))*x^4*a*b^2*c^2*d+15*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^4*b^3*c^3+8*(a*x^2+b*x)^(3/2)*a^(3/2)*x*d^3-180*(a*x^2+b*x)^(
3/2)*a^(1/2)*x^2*b*c^2*d-60*d^2*c*(a*x^2+b*x)^(3/2)*x*b*a^(1/2)-12*(a*x^2+b
*x)^(3/2)*a^(1/2)*b*d^3)/x^3/((a*x+b)*x)^(1/2)/b^2/a^(1/2)
```

maxima [A] time = 1.45, size = 164, normalized size = 1.15

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^3-3\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)c^2d-\frac{2}{15}d^3\left(\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^3 - 3*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c^2*d - 2/15*d^3*(3*(a + b/x)^(5/2)/b^2
 - 5*(a + b/x)^(3/2)*a/b^2) - 2*(a + b/x)^(3/2)*c*d^2/b
```

mupad [B] time = 2.59, size = 173, normalized size = 1.21

$$\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{3\,b^2}-\frac{4\,a\,d^3}{3\,b^2}\right)+\sqrt{a+\frac{b}{x}}\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)+c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(3/2)*((6*a*d^3 - 6*b*c*d^2)/(3*b^2) - (4*a*d^3)/(3*b^2)) + (a +
b/x)^(1/2)*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b
*c)^2)/b^2 + (2*a^2*d^3)/b^2) + c^3*x*(a + b/x)^(1/2) - (2*d^3*(a + b/x)^(5
/2))/(5*b^2) - (c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + b*c)*1i)/a^
(1/2)
```

sympy [A] time = 57.22, size = 454, normalized size = 3.17

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7
/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a*c**2*d*atan(sqrt(a + b
/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 6*c**2*d*sq
rt(a + b/x) + 3*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/
2)/(3*b), True)) + b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.225 $\int \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2 \, dx$

Optimal. Leaf size=99

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c \sqrt{a+\frac{b}{x}} (4 a d+b c)}{a}+\frac{c (4 a d+b c) \tanh ^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2 d^2 \left(a+\frac{b}{x}\right)^{3/2}}{3 b}$$

```
[Out] -2/3*d^2*(a+b/x)^(3/2)/b+c^2*(a+b/x)^(3/2)*x/a+c*(4*a*d+b*c)*arctanh((a+b/x
)^(1/2)/a^(1/2))/a^(1/2)-c*(4*a*d+b*c)*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c \sqrt{a+\frac{b}{x}} (4 a d+b c)}{a}+\frac{c (4 a d+b c) \tanh ^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2 d^2 \left(a+\frac{b}{x}\right)^{3/2}}{3 b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3*b) + (c^2
*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^2}{x^2}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(\frac{1}{2}c(bc+4ad)+ad^2x\right)}{x}dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(c(bc+4ad))\,\text{Subst}\left(\int \right.$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\,\text{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.}{b}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right.}{\sqrt{a}}$$

Mathematica [A] time = 0.16, size = 84, normalized size = 0.85

$$\frac{\sqrt{a+\frac{b}{x}}\left(b\left(3c^2x^2-12cdx-2d^2\right)-2ad^2x\right)}{3bx} + \frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
[Out] (Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
(c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.01, size = 208, normalized size = 2.10

$$\left[\frac{3\left(b^2c^2+4\,abcd\right)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,abc^2x^2-2\,abd^2-2\left(6\,abcd+a^2d^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{6\,abx}, -\frac{3\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqr
t((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan(sq
rt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a
^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1
,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[
2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+
%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%
},0,%%%{1,[0,2,2]%%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%
%%}] at parameters values [18,15.451549686,-33]Sign error (%%%{-b,0%%%}+%%%
{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%
%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 191, normalized size = 1.93

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-12abcd\,x^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-3b^2c^2x^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-24\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}cd\,x^3-6\sqrt{a\,x^2}\right.}{6\sqrt{(ax+b)\,x}\,\sqrt{a}\,b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2*(a+b/x)^(1/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-24*(a*x^2+b*x)^(1/2)*a^(3/2)*x^3*c*d-6*(a*x^2+
b*x)^(1/2)*a^(1/2)*x^3*b*c^2-12*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2)
)/a^(1/2))*x^3*a*b*c*d-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2))*x^3*b^2*c^2+24*(a*x^2+b*x)^(3/2)*a^(1/2)*x*c*d+4*d^2*(a*x^2+b*x)^(3/2)*
a^(1/2))/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.19, size = 126, normalized size = 1.27

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^2-2\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)cd-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}d^2}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^2 - 2*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c*d - 2/3*(a + b/x)^(3/2)*d^2/b
```

mupad [B] time = 1.90, size = 99, normalized size = 1.00

$$\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)\sqrt{a+\frac{b}{x}}+c^2\,x\sqrt{a+\frac{b}{x}}-\frac{2\,d^2\left(a+\frac{b}{x}\right)^{3/2}}{3\,b}-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)(4\,a\,d+b\,c)\,1\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^2,x)
```

```
[Out] ((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b)*(a + b/x)^(1/2) + c^2*x*(a + b/x)^(1/
2) - (2*d^2*(a + b/x)^(3/2))/(3*b) - (c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*
(4*a*d + b*c)*1i)/a^(1/2)
```

sympy [A] time = 36.58, size = 121, normalized size = 1.22

$$-\frac{4acd\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}-4cd\sqrt{a+\frac{b}{x}}+d^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] -4*a*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**2*sqrt(x)*sqrt(
a*x/b + 1) - 4*c*d*sqrt(a + b/x) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/s
qrt(a)
```

3.226 $\int \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a+\frac{b}{x}}\,(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

```
[Out] c*(a+b/x)^(3/2)*x/a+(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)-(2*a
*d+b*c)*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}-\frac{\left(\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a}+\frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}-\frac{1}{2}(bc+2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a}+\frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}-\frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a}+\frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a}+\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.70

$$\sqrt{a+\frac{b}{x}}\,(cx-2d)+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/Sqrt[a]
```

fricas [A] time = 1.50, size = 128, normalized size = 1.73

$$\left[\frac{(bc+2\,ad)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a},-\frac{(bc+2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
+ 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
```

```
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1
,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[
2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+
%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%
},0,%%%{1,[0,2,2]%%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%
%%}] at parameters values [18,15.451549686,-33]Sign error (%%%{-b,0%%%}+%%%
{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%
%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 163, normalized size = 2.20

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(2abd\,x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+b^2c\,x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}d\,x^2+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{(ax+b)\,x}\,\sqrt{a}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*(4*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2*d+2*a^(1/2)*(a*x^2+b
*x)^(1/2)*x^2*b*c+2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x
^2*a*b*d+ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^2*c-4*
a^(1/2)*(a*x^2+b*x)^(3/2)*d)/x/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.37, size = 106, normalized size = 1.43

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c-\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c - (sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a))) + 2*sqrt(a + b/x))*d
```

mupad [B] time = 1.96, size = 92, normalized size = 1.24

$$2\sqrt{a}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,d\,\sqrt{a+\frac{b}{x}}+c\,x\,\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,c\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x),x)
```

```
[Out] 2*a^(1/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*d*(a + b/x)^(1/2) + c*x*(b*x
 + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*c*x*log((b/2 + a*x + a^(1/2)*(b*x + a*x^
2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [A] time = 41.31, size = 87, normalized size = 1.18

$$-\frac{2ad\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c\sqrt{x}\sqrt{\frac{ax}{b}+1}-2d\sqrt{a+\frac{b}{x}}+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)**(1/2),x)
```

```
[Out] -2*a*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b
 + 1) - 2*d*sqrt(a + b/x) + b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.227 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=39

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+x*(a+b/x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 47, 63, 208}

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x - \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\ &= \sqrt{a+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 39, normalized size = 1.00

$$x\sqrt{a+\frac{b}{x}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x],x]
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.08, size = 99, normalized size = 2.54

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}\,b\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,a}, \frac{ax\sqrt{\frac{ax+b}{x}} - \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqrt(
(a*x + b)/x)/a))/a]
```

giac [B] time = 0.20, size = 64, normalized size = 1.64

$$-\frac{b \log \left(\left|-2 \left(\sqrt{a} x-\sqrt{a x^2+b x}\right) \sqrt{a}-b\right|\right) \operatorname{sgn}(x)}{2 \sqrt{a}}+\frac{b \log \left(|b|\right) \operatorname{sgn}(x)}{2 \sqrt{a}}+\sqrt{a x^2+b x} \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

maple [B] time = 0.05, size = 74, normalized size = 1.90

$$\frac{\sqrt{\frac{a x+b}{x}}\, \left(b \ln \left(\frac{2 a x+b+2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}}{2 \sqrt{a}}\right)+2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}\right) x}{2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.20, size = 50, normalized size = 1.28

$$\sqrt{a+\frac{b}{x}}\, x-\frac{b \log \left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt
(a)))/sqrt(a)
```

mupad [B] time = 0.08, size = 58, normalized size = 1.49

$$x\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2),x)
```

```
[Out] x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [A] time = 2.19, size = 42, normalized size = 1.08

$$\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.228 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

```
[Out] (-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^2/a^(1/2)+2*arctan(d^(1/2)*(a
+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)*(-a*d+b*c)^(1/2)/c^2+x*(a+b/x)^(1/2)/
c
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 99, 156, 63, 208, 205}

$$\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(Sqrt[a]*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
```

```
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = -\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-\frac{bdx}{2}}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(d(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(2d(bc-ad))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \ldots\right)}{bc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} + \frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}$$

Mathematica [A] time = 0.24, size = 100, normalized size = 0.96

$$\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + cx\sqrt{a+\frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a
])/c^2
```

fricas [A] time = 0.97, size = 482, normalized size = 4.63

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\,\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\,\sqrt{-bcd+ad^2}\,a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+ad^2}\,x\sqrt{\frac{ax+\ldots}{x}}}{cx+d}\right.}{2\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*
a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2), 1
/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d -
 a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log(2*a*
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) -
 (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(-b*c*d
+ a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x
+ b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*c*d - a*d
^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c
- 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.11, size = 287, normalized size = 2.76

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{3}{2}}d^{2}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)-2\sqrt{a}\,bcd\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{\frac{(ad-b}{c^{2}}}\right.}{2\sqrt{(ax+b)\,x}\,\sqrt{(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*c*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
(1/2))/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2+2*ln((2*(d*(a*d-b*c)/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*d^2-2*ln((2*(d*(
a*d-b*c)/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)
*b*c*d-2*((a*x+b)*x)^(1/2)*c^2*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2))/((a*x+b)*x)
^(1/2)/c^3/a^(1/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x), x)
```

mupad [B] time = 1.63, size = 149, normalized size = 1.43

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}+\frac{\ln\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(a\,d-\frac{b\,c}{2}\right)}{\sqrt{a}\,c^2}-\frac{\ln\left(\sqrt{a+\frac{b}{x}}+\sqrt{a}\right)(2\,a\,d-b\,c)}{2\,\sqrt{a}\,c^2}-\frac{\operatorname{atan}\left(\frac{b^4\,d^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}\,4\mathrm{i}}{4\,a\,b^4\,d^4-4\,b^5\,c\,d^3}\right)\sqrt{a\,d^2-}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x),x)
```

```
[Out] (x*(a + b/x)^(1/2))/c - (atan((b^4*d^3*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2
)*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3))*(a*d^2 - b*c*d)^(1/2)*2i)/c^2 + (log((a
+ b/x)^(1/2) - a^(1/2))*(a*d - (b*c)/2))/(a^(1/2)*c^2) - (log((a + b/x)^(1/
2) + a^(1/2))*(2*a*d - b*c))/(2*a^(1/2)*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x),x)
```

```
[Out] Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

3.229 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal. Leaf size=147

$$\frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}+\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] (-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^3/a^(1/2)+(-4*a*d+3*b*c)*arct
an(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/c^3/(-a*d+b*c)^(1/2)+2*d
*(a+b/x)^(1/2)/c^2/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)) + (Sq
rt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3
*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]
*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-4ad)-\frac{3bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-4ad)(bc-ad)+bd(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} + \frac{(d(3bc-4ad))\,\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{2c^3}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} + \frac{(d(3bc-4ad))\,\text{Subst}\left(\right.}{}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}$$

Mathematica [A] time = 0.39, size = 122, normalized size = 0.83

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\,(cx+2d)}{cx+d} + \frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

fricas [A] time = 1.36, size = 801, normalized size = 5.45

$$\left[-\frac{\left(bcd-4\,ad^2+\left(bc^2-4\,acd\right)x\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+\left(3\,abcd-4\,a^2d^2+\left(3\,abc^2-4\,a^2cd\right)x\right)\sqrt{-\frac{}{bc}}}{2\left(ac^4x+ac^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*
d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt(
(a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x
)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2
 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b*c*d -
 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c -
 a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*
x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d),
 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(
a*c^4*x + a*c^3*d), ((3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sq
rt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/
x)/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))
/(a*c^4*x + a*c^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[-2,0]:[1,0,%%%{-1,[1
]%%%}]%%},[4,6,4,0]%%%}+%%%{%%%{8,[1]%%%},[3,5,4,1]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,5,5,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,4,4,2]%%%}+%%%{%%%{8,[1]%%%},[1,4,5,2]%%%}+%%%{%%{[-2,0]:[1,0,%%
```

```
%{-1,[1]%%%}]%%},[0,4,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,2,0,0]%%%}+%%%{%%{pol
y1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1]%%%}+%%%{%%%{2,[1]%%%
},[2,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.07, size = 943, normalized size = 6.41

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(4a^{\frac{7}{2}}c\,d^{3}x\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)-7a^{\frac{5}{2}}b\,c^{2}d^{2}x\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+3a^{\frac{3}{2}}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2
*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^3+2*a^(5/2)*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^4+4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b
*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^4-2*a^(5/2)*((a*x+b)*x)^(1
/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d-7*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*
d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*c^2*d^2-4*a^(5/2)*((a
*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^2-7*a^(5/2)*ln((-2*a*d*x+b*c*x
-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^3-2*c^4*
((a*x+b)*x)^(3/2)*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)+4*a^(3/2)*((a*x+b)*x)^(1/
2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4+3*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d+4*a^(3/2)*((a*
x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d+3*a^(3/2)*ln((-2*a*d*x+b*c*x-
b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d^2+4*a
^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^
(1/2)*x*c^2*d^2-5*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))
*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d+a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4+4*a^3*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*c*d^3-5*a^2*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b
*c^2*d^2+a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)
/c^2*d)^(1/2)*b^2*c^3*d)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)/(c*x+d)/a^(3/2)/((
a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

mupad [B] time = 2.26, size = 1195, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^2,x)
```

```
[Out] - ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((a
 + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^3*
(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a +
b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(a^(1
/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^
5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^
6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d
- b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/
2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 -
a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*
b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3)
)/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c)
)^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*
a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*
d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/c^6 - ((d*(a*d
- b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^
3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7
*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*
c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c
^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(
1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))
/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*
a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*
(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^
3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d))))*(d*(a*d - b*c))^(1/2)*(4*a*
d - 3*b*c)*1i)/(b*c^4 - a*c^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+\frac{b}{x}}}{(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

3.230 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^4}+\frac{d\sqrt{a+\frac{b}{x}}\,(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\cdots}}{2c^2\left(c+\cdots\right.}$$

```
[Out] (-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^4/a^(1/2)+1/4*(24*a^2*d^2-40*
a*b*c*d+15*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*d^(1/2)/
c^4/(-a*d+b*c)^(3/2)+3/2*d*(a+b/x)^(1/2)/c^2/(c+d/x)^2+1/4*d*(-12*a*d+11*b*
c)*(a+b/x)^(1/2)/c^3/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

Rubi [A] time = 0.34, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{(bc-6ad)\tanh^{-1}\left(\cdots\right.}{\sqrt{a}\,c^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sqrt[a + b/x
])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)^2) + (Sqr
t[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-6ad)-\frac{5bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-(bc-6ad)(bc-ad)+\frac{9}{2}bd(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{(bc-6ad)(bc-ad)^2-\frac{1}{4}bd(11bc-12ad)(bc-a}{x\sqrt{a+bx}\,(c+dx)}\right.}{2c^3(bc-ad)^2}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^4} + \frac{(}{}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right.}{bc^4}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}\left(15b^2c^2-40abcd+24a^2d^2\right)\tan^{-1}\left(\right.}{4c^4(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.82, size = 330, normalized size = 1.55

$$\frac{(cx+d)\left(\frac{1}{2}cd^{5/2}\sqrt{a+\frac{b}{x}}\,(ax+b)\left(12a^2d^2-17abcd+4b^2c^2\right)+(cx+d)\left(-\frac{1}{2}ad^2\left(24a^2d^2-40abcd+15b^2c^2\right)\left(\sqrt{d}\,\sqrt{\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (2*c^3*d^(3/2)*(b*c - a*d)^2*(a + b/x)^(3/2)*x^3 + c^2*d^(5/2)*(2*b*c - 3*a
*d)*(b*c - a*d)*Sqrt[a + b/x]*x*(b + a*x) + (d + c*x)*((c*d^(5/2)*(4*b^2*c^
2 - 17*a*b*c*d + 12*a^2*d^2)*Sqrt[a + b/x]*(b + a*x))/2 + (d + c*x)*(-1/2*(
a*d^2*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*(Sqrt[d]*Sqrt[a + b/x] - Sqrt[
b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])) - d^(3/2)*(b*c
 - 6*a*d)*(b*c - a*d)^2*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]))))/(2*a*c^4*d^(3/2)*(b*c - a*d)^2*(d + c*x)^2)
```

fricas [B] time = 2.25, size = 1749, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)
*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40*a^
2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x
^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
 (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3
*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b
)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d
 - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4 +
(15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d -
40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)
*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^2*c^2*d^2 -
7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18
*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(
a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*
c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2
)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*
c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*
c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x
+ b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^
6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4
```

```
 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d
 - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^2*c^2*d^2
 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c
^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^
5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^
2)*x)]
```

giac [B] time = 0.41, size = 820, normalized size = 3.85

$$-\frac{\left(15\sqrt{a}\,b^2c^2d\arctan\left(\frac{\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right) - 40\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right) + 24\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right) - 2\sqrt{bcd-ad^2}\,b^2c^2\right.}{4\left(\sqrt{bcd-ad^2}\,\sqrt{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3/
2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(sq
rt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) +
14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d^2
*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*a^2*d
^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a^(3/2)
*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^3*sgn(x
))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*sgn(x)/c^3
 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*d*sgn(x) - 32*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^3*a^(5/2)*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^2*a*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d
^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^4*sgn(x) + 7*(sqrt(a
)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sqrt(
a*x^2 + b*x))*a^(3/2)*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
*a^(5/2)*b*d^4*sgn(x) - 9*a*b^3*c*d^3*sgn(x) + 10*a^2*b^2*d^4*sgn(x))/((sqr
t(a)*b*c^5 - a^(3/2)*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn
(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)
```

maple [B] time = 0.07, size = 1972, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(55*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^2*c^4*d^2+24*a^4*ln(1/2*(2*a
*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^3*
d^3-36*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^3-128*a^(7
/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(
c*x+d))*x*b*c^2*d^4+110*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^
(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^3+48*a^4*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^4+46*a^
(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^3-52*a^3*ln(1/2*(2*
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b*c^2*d
^4+32*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/
c^2*d)^(1/2)*b^2*c^3*d^3-22*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/
2)*b^2*c^4*d^2-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d^2+14*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c
^2*d)^(1/2)*x*b*c^6-22*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^
2*b^2*c^6-15*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x
+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^5*d-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1
/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^6+10*a^(3/2)*((a*x+
b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^5*d+12*a^(7/2)*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*x^3*c^5*d-14*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2
*d)^(1/2)*x^3*b*c^6-12*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*
c^5*d-64*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*
x)^(1/2)*c)/(c*x+d))*x^2*b*c^3*d^3-30*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*
d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^3*c^4*d^2+24*a^(9/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*d^6+64*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-
b*c)/c^2*d)^(1/2)*x*b^2*c^4*d^2+78*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2
*d)^(1/2)*x*b*c^4*d^2-104*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/
a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d^3+18*a^(5/2)*((a*x+b)*x)^(1/2)*(
(a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^5*d-52*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*d^2+32*a^2*ln(1/2*(2
*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^
2*c^5*d-44*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d-8*
a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*x*b^3*c^5*d+48*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^5-24*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-
b*c)/c^2*d)^(1/2)*c^2*d^4-64*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^5+55*a^(5/2)*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d^4
+24*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^
2*d)^(1/2)*c*d^5+24*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
```

```
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*c^2*d^4-8*a^(5/2)*((a*x+b)*x)^(3/2)*((a
*d-b*c)/c^2*d)^(1/2)*c^4*d^2-15*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)
/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c^3*d^3)/c^5/((a*x+b)*x)^(1
/2)/(a*d-b*c)^2/(c*x+d)^2/a^(3/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

mupad [B] time = 3.73, size = 1895, normalized size = 8.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^3,x)
```

```
[Out] (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1/2) - a^2*d^2 - b^2*c^2 + 2*a*b*c*
d)*(d*(a*d - b*c)^3)^(1/2)*(3*a^2*d^2 + (15*b^2*c^2)/8 - 5*a*b*c*d))/(b^3*c
^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - ((b*(a + b/x)^(1/2)*(
12*a^2*d^2 + 4*b^2*c^2 - 17*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(5/2)*(12*a*d^
3 - 11*b*c*d^2))/(4*c^3*(a*d - b*c)) - (d*(a + b/x)^(3/2)*(17*b^3*c^2 + 24*
a^2*b*d^2 - 40*a*b^2*c*d))/(4*c^3*(a*d - b*c)))/((a + b/x)^2*(3*a*d^2 - 2*b
*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3
*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1
/2) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(d*(a*d - b*c)^3)^(1/2)*(24*a^2*d^2 +
15*b^2*c^2 - 40*a*b*c*d))/(8*(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a
*b^2*c^6*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^
3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^
2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*
a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c
^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 -
256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2
)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*
c)*1i)/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4
*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*
(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^
```

```
2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2
 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(
1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d -
 b*c)*1i)/(2*a^(1/2)*c^4))/((216*a^4*b^3*d^7 + (165*b^7*c^4*d^3)/8 - (805*a
*b^6*c^3*d^4)/4 - 594*a^3*b^4*c*d^6 + 558*a^2*b^5*c^2*d^5)/(b^2*c^11 + a^2*
c^9*d^2 - 2*a*b*c^10*d) - ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^
4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8
*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a
^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^
2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^
(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d
- b*c))/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^
4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8
*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a
^2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^
2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^
(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d
- b*c))/(2*a^(1/2)*c^4)))*(6*a*d - b*c)*1i)/(a^(1/2)*c^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.231 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=164

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}\,(2ad+bc)+3\sqrt{a}\,c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -9/7*d*(a+b/x)^(3/2)*(c+d/x)^2-1/35*d*(a+b/x)^(3/2)*(2*(-a*d+13*b*c)*(2*a*d
+5*b*c)+3*b*d*(2*a*d+19*b*c)/x)/b^2+(a+b/x)^(3/2)*(c+d/x)^3*x+3*c^2*(2*a*d+
b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*c^2*(2*a*d+b*c)*(a+b/x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}\,(2ad+bc)+3\sqrt{a}\,c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 -
(d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a
*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)^3*x + 3*Sqrt[a]*c^2*(b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^2\left(\frac{3}{2}(bc+2ad)+\frac{9bdx}{2}\right)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)\left(\frac{21}{4}bc(bc+2ad\right.}{x}\right.}{7b}$$
$$= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2} + ($$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+\right.}{35b^2}$$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+\right.}{35b^2}$$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+\right.}{35b^2}$$

Mathematica [A] time = 0.21, size = 159, normalized size = 0.97

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^3d^3x^3 - 2a^2bd^2x^2(21cx+d) + ab^2x\left(35c^3x^3 - 280c^2dx^2 - 84cd^2x - 16d^3\right) - 2b^3\left(35c^3x^3 + 35c^2dx^2 + \right.\right.}{35b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-16
*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2*x
```

```
 + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 1.24, size = 380, normalized size = 2.32

$$\left[\frac{105\left(b^3c^3 + 2\,ab^2c^2d\right)\sqrt{a}\,x^3\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\left(35\,ab^2c^3x^4 - 10\,b^3d^3 - 2\left(35\,b^3c^3 + 140\,ab^2c^2d\right.\right.}{70\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 + 1
40*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*a*b
^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x + b
)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c
^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d +
42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a
*x + b)/x))/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
```

```
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.8Limi
t: Max order reached or unable to make series expansion Error: Bad Argument
 Value
```

maple [B] time = 0.06, size = 353, normalized size = 2.15

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(210a^2b^2c^2d\,x^5\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+105a\,b^3c^3x^5\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+420\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b\,c^2d\,x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x)^3,x)
```

```
[Out] 1/70*((a*x+b)/x)^(1/2)*(420*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2*d+210*(a*x^
2+b*x)^(1/2)*a^(3/2)*x^5*b^2*c^3+8*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^3-420*(a
*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2*d+210*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)
*a^(1/2))/a^(1/2))*x^5*a^2*b^2*c^2*d+105*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2
)*a^(1/2))/a^(1/2))*x^5*a*b^3*c^3-84*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d^2-
140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^2*c^3-12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*
d^3-140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2*c^2*d-84*(a*x^2+b*x)^(3/2)*a^(1/2
)*x*b^2*c*d^2-20*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^3)/x^4/b^2/((a*x+b)*x)^(1/
2)/a^(1/2)
```

maxima [A] time = 1.49, size = 190, normalized size = 1.16

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5\,b}+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c^3-\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/5*(a + b/x)^(5/2)*c*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((
sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^
3 - (3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2
*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c^2*d - 2/35*(5*(a + b/x)^(7/2)/b^2 -
 7*(a + b/x)^(5/2)*a/b^2)*d^3
```

mupad [B] time = 3.88, size = 327, normalized size = 1.99

$$\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{5\,b^2}-\frac{4\,a\,d^3}{5\,b^2}\right)+\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-}{b^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(5/2)*((6*a*d^3 - 6*b*c*d^2)/(5*b^2) - (4*a*d^3)/(5*b^2)) + (a +
b/x)^(1/2)*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (
4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - a^2*((6*a*d^3
- 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2)) + (a + b/x)^(3/2)*((2*a*((6*a*d^3 - 6*b*
c*d^2)/b^2 - (4*a*d^3)/b^2))/3 - (2*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/(3*b
^2)) - (2*d^3*(a + b/x)^(7/2))/(7*b^2) + a*c^3*x*(a + b/x)^(1/2) - 2*c^2*at
an((2*c^2*(a + b/x)^(1/2)*(2*a*d + b*c)*(-(9*a)/4)^(1/2))/(6*a^2*c^2*d + 3*
a*b*c^3))*(2*a*d + b*c)*(-(9*a)/4)^(1/2)
```

sympy [A] time = 123.48, size = 1817, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

```
[Out] -16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d**3*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19/2)*d**3*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(1
1/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(21/2
)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 6*
```

```
a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(23/2
)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a
**(7/2)*b**(9/2)*c*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 6*a**(7/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(11/2)*c*d*
*2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
+ sqrt(a)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**6*d**3*x**(1
1/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**7*d**3*x**(
9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**8*d**3*x**(
7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**7*b*d**3*x**(7/2)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**2*c*d
**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a*
*6*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 12*a**5*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 6*a**2*c**2*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a*sq
rt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**3*atan(sqrt(a + b/x)/sqrt(-a)
)/sqrt(-a) - 6*a*c**2*d*sqrt(a + b/x) + 3*a*c*d**2*Piecewise((-sqrt(a)/x, E
q(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**3*sqrt(a + b/x) + 3*b
*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
)
```

3.232 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx$

Optimal. Leaf size=126

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{3/2}(4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}(4ad+3bc)+\sqrt{a}\,c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

```
[Out] -1/3*c*(4*a*d+3*b*c)*(a+b/x)^(3/2)/a-2/5*d^2*(a+b/x)^(5/2)/b+c^2*(a+b/x)^(5
/2)*x/a+c*(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-c*(4*a*d+3*b
*c)*(a+b/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{3/2}(4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}(4ad+3bc)+\sqrt{a}\,c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^(3/2))/(3
*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a + Sqrt[a]*c
*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2}{x^2}dx, x, \frac{1}{x}\right)$$
$$= \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}c(3bc+4ad)+ad^2x\right)}{x}dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(c(3bc+4ad))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \frac{1}{x}\right)}{2a}$$
$$= -\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(c(3bc+4ad))\,\text{Sub}$$
$$= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a}$$
$$= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a}$$
$$= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a}$$

Mathematica [A] time = 0.26, size = 106, normalized size = 0.84

$$-\frac{c(4ad+3bc)\left(\sqrt{a+\frac{b}{x}}(4ax+b)-3a^{3/2}x\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)}{3ax} + \frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a - (c*(3*b*c + 4*
a*d)*(Sqrt[a + b/x]*(b + 4*a*x) - 3*a^(3/2)*x*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
]))/(3*a*x)
```

fricas [A] time = 1.09, size = 268, normalized size = 2.13

$$\left[\frac{15\left(3\,b^2c^2+4\,abcd\right)\sqrt{a}\,x^2\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,abc^2x^3-6\,b^2d^2-2\left(15\,b^2c^2+40\,abcd+3\,a^2d^2\right)\right.}{30\,bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*
c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x
^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a
*x + b)/x)/a) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d +
3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.43Lim
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

maple [B] time = 0.06, size = 260, normalized size = 2.06

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-60a^2bcd\,x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-45a\,b^2c^2x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-120\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}cd\,x^4-9\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x)^2,x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-120*(a*x^2+b*x)^(1/2)*a^(5/2)*x^4*c*d-90*(a
*x^2+b*x)^(1/2)*a^(3/2)*x^4*b*c^2-60*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^
(1/2))/a^(1/2))*x^4*a^2*b*c*d-45*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^4*a*b^2*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*c*d+60*(a*x^2+b
*x)^(3/2)*a^(1/2)*x^2*b*c^2+12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*d^2+40*(a*x^2+b*
x)^(3/2)*a^(1/2)*x*b*c*d+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b*d^2)/((a*x+b)*x)^(1
/2)/a^(1/2)
```

maxima [A] time = 1.16, size = 152, normalized size = 1.21

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}d^2}{5\,b}+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c^2-\frac{2}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sq
rt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^2
- 2/3*(3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c*d
```

mupad [B] time = 2.58, size = 197, normalized size = 1.56

$$\sqrt{a+\frac{b}{x}}\left(2\,a\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-\frac{2\,(a\,d-b\,c)^2}{b}+\frac{2\,a^2\,d^2}{b}\right)+\left(\frac{4\,a\,d^2-4\,b\,c\,d}{3\,b}-\frac{4\,a\,d^2}{3\,b}\right)\left(a+\frac{b}{x}\right)^{3/2}-\frac{2\,d^2\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(1/2)*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b) + ((4*a*d^2 - 4*b*c*d)/(3*b) - (4*a*d^2)/(3*b))*(a +
```

```
 b/x)^(3/2) - (2*d^2*(a + b/x)^(5/2))/(5*b) + a*c^2*x*(a + b/x)^(1/2) - 2*c
*atan((2*c*(a + b/x)^(1/2)*(4*a*d + 3*b*c)*(-a/4)^(1/2))/(3*a*b*c^2 + 4*a^2
*c*d))*(4*a*d + 3*b*c)*(-a/4)^(1/2)
```

sympy [A] time = 94.07, size = 534, normalized size = 4.24

$$\frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\sqrt{a}\,bc^2\operatorname{asinh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] 4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(9
/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqr
t(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) - 4*a**2*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
+ a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**2*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) - 4*a*c*d*sqrt(a + b/x) + a*d**2*Piecewise((-sqrt(a)/x, Eq
(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**2*sqrt(a + b/x) + 2*b*
c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.233 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=100

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}\,(2ad+3bc)+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

```
[Out] -1/3*(2*a*d+3*b*c)*(a+b/x)^(3/2)/a+c*(a+b/x)^(5/2)*x/a+(2*a*d+3*b*c)*arctan
h((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-(2*a*d+3*b*c)*(a+b/x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}\,(2ad+3bc)+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/2))/(3*a)
+ (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\left(\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(3bc+2ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(a(3bc+2ad))\,\text{Su}$$

$$= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(a(3bc+2ad))\,\text{Sub}}{}$$

$$= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}\,(3bc+2ad)\,\text{tanh}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 0.73

$$\frac{\sqrt{a+\frac{b}{x}}\,(ax(3cx-8d)-2b(3cx+d))}{3x}+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*b
*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 1.03, size = 164, normalized size = 1.64

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x\right)\sqrt{\frac{ax+b}{x}}}{6\,x},-\frac{3\,(3\,bc+2\,ad)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x, -
1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3
*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
```

```
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached
 or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 205, normalized size = 2.05

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(6a^{2}bd\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+9a\,b^{2}c\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+12\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}d\,x^{3}+18\sqrt{a\,x^{2}+\right.}{6\sqrt{(ax+b)\,x}\,\sqrt{a}\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*(12*a^(5/2)*(a*x^2+b*x)^(1/2)*x^3*d+18*a^(3/2)*(a*x^2
+b*x)^(1/2)*x^3*b*c-12*a^(3/2)*(a*x^2+b*x)^(3/2)*x*d+6*ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b*d+9*ln(1/2*(2*a*x+b+2*(a*x^2+b*
x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a*b^2*c-12*a^(1/2)*(a*x^2+b*x)^(3/2)*x*b*c-4
*d*(a*x^2+b*x)^(3/2)*b*a^(1/2))/x^2/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.33, size = 132, normalized size = 1.32

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c-\frac{1}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c - 1/3*(3*a^(3/2)*log((sqrt(a +
b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*(a + b/x)^(3/2) + 6*sqrt(a +
 b/x)*a)*d
```

mupad [B] time = 2.51, size = 81, normalized size = 0.81

$$2\,a^{3/2}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2\,d\left(a+\frac{b}{x}\right)^{3/2}}{3}-2\,a\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,c\,x\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\,\frac{1}{2};\,-\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x),x)
```

```
[Out] 2*a^(3/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - (2*d*(a + b/x)^(3/2))/3 - 2*a*
d*(a + b/x)^(1/2) - (2*c*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a
*x)/b))/((a*x)/b + 1)^(3/2)
```

sympy [A] time = 56.15, size = 163, normalized size = 1.63

$$\sqrt{a}\,bc\,\text{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)-\frac{2a^2d\,\text{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}-\frac{2abc\,\text{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2ad\sqrt{a+\frac{b}{x}}-2bc\sqrt{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a**2*d*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c*atan(sqrt(
a + b/x)/sqrt(-a))/sqrt(-a) - 2*a*d*sqrt(a + b/x) - 2*b*c*sqrt(a + b/x) + b
*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.234 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=54

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] (a+b/x)^(3/2)*x+3*b*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)-3*b*(a+b/x)^(1/2
)
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {242, 47, 50, 63, 208}

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2} x-\frac{1}{2}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}}+\left(a+\frac{b}{x}\right)^{3/2} x-\frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}}+\left(a+\frac{b}{x}\right)^{3/2} x-(3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -3b\sqrt{a+\frac{b}{x}}+\left(a+\frac{b}{x}\right)^{3/2} x+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.85

$$
\sqrt{a+\frac{b}{x}}\,(ax-2b)+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2),x]
```

```
[Out] Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 0.88, size = 100, normalized size = 1.85

$$\left[\frac{3}{2}\sqrt{a}\,b\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},-3\,\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+(ax-2\,b)\sqrt{\frac{ax}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b)
*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a
*x - 2*b)*sqrt((a*x + b)/x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.05, size = 100, normalized size = 1.85

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(3ab\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{a\,x^{2}+bx}\;a^{\frac{3}{2}}x^{2}-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2\sqrt{\left(ax+b\right)x}\;\sqrt{a}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*(6*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2+3*ln(1/2*(2*a*x+b+2*
(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a*b-4*(a*x^2+b*x)^(3/2)*a^(1/2))/x/
((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.20, size = 63, normalized size = 1.17

$$\sqrt{a+\frac{b}{x}}\,ax-\frac{3}{2}\,\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\,\sqrt{a+\frac{b}{x}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b
/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

mupad [B] time = 1.50, size = 34, normalized size = 0.63

$$-\frac{2\,x\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2),x)
```

```
[Out] -(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1)
^(3/2)
```

sympy [B] time = 2.69, size = 92, normalized size = 1.70

$$3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*
x/b + 1))
```

3.235 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}} dx$

Optimal. Leaf size=106

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

```
[Out] (-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^2-2*(-a*d+b*c)^(3/2
)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(1/2)+a*x*(a+b/x)^(1
/2)/c
```

Rubi [A] time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 98, 156, 63, 208, 205}

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-2ad)-\frac{1}{2}b(2bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x\right.}{c^2}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\right.}{bc^2}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

Mathematica [A] time = 0.24, size = 102, normalized size = 0.96

$$\frac{-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}} + \sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + acx\sqrt{a+\frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/Sqrt[d] + Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]])/c^2
```

fricas [A] time = 1.08, size = 519, normalized size = 4.90

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (3\,bc-2\,ad)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right) - 2\,(bc-ad)\sqrt{-\frac{bc-ad}{d}}\,\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-}{cx+d}\right.}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d
*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d
)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-
(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2,
1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-
d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt
(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sqrt((a*x +
 b)/x) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sq
rt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/c^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.06, size = 528, normalized size = 4.98

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{5}{2}}d^3\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)-4a^{\frac{3}{2}}bc\,d^2\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{a}\,b^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2
*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^3-2*a^(3/2)*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*c^2*d-4*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^2+2*a^(1/2)*((a*x+b)*x)^(1/2
)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3-2*a^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*
x)^(1/2)*b*c^3+2*a^(1/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2
```

```
)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^2*c*d^2-3*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a*b*c^2*d+ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2
*c^3-((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a
^(1/2))*b^2*c^3)/((a*x+b)*x)^(1/2)/d/c^3/a^(1/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{c + \frac{d}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x), x)
```

mupad [B] time = 1.68, size = 556, normalized size = 5.25

$$\frac{a\,x\,\sqrt{a+\frac{b}{x}}}{c} - \frac{\sqrt{a}\ \mathrm{atanh}\left(\frac{58\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}} + \frac{46\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{46\,a^3\,b^5\,d^3-58\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+24\,a\,b^7\,c^2\,d} + \frac{1}{12\,a^4\,b^4\,d^4-46\,a^3}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x),x)
```

```
[Out] (a*x*(a + b/x)^(1/2))/c - (a^(1/2)*atanh((58*a^(3/2)*b^6*d^2*(a + b/x)^(1/2
))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c
^2) + (46*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(46*a^3*b^5*d^3 - 58*a^2*b^6*c*d
^2 - (12*a^4*b^4*d^4)/c + 24*a*b^7*c^2*d) + (12*a^(7/2)*b^4*d^4*(a + b/x)^(
1/2))/(12*a^4*b^4*d^4 - 46*a^3*b^5*c*d^3 + 58*a^2*b^6*c^2*d^2 - 24*a*b^7*c^
3*d) - (24*a^(1/2)*b^7*c*d*(a + b/x)^(1/2))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d
- (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c^2))*(2*a*d - 3*b*c))/c^2 + (2*ata
nh((12*a^2*b^4*d^2*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3)^(1/2))/(12*a^4*b^4*d^4 - 40*a^3*b^5*c*d^3 + 44*a^2*b^6*c^2*
d^2 - 16*a*b^7*c^3*d) + (16*a*b^5*d*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(40*a^3*b^5*d^3 - 44*a^2*b^6*c*d^2
- (12*a^4*b^4*d^4)/c + 16*a*b^7*c^2*d))*(d*(a*d - b*c)^3)^(1/2))/(c^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{c x + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x),x)
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

3.236 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] (-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^3-(-4*a*d+b*c)*arct
an(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^3/d^(1/2)-(-2
*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)
```

Rubi [A] time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 151, 156, 63, 208, 205}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}-\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/x]*x)/(c*(
c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-4ad)-\frac{1}{2}b(2bc-3ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3bc-4ad)(bc-ad)+\frac{1}{2}b(bc-2ad)(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - \frac{((bc-4ad)(bc\cdots}{\cdots}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} - \frac{((bc-4a\cdots}{\cdots}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}\,(3bc-4ad)\,\text{ta}\cdots}{c^3}$$

Mathematica [A] time = 0.39, size = 143, normalized size = 0.92

$$\frac{-\frac{\left(4a^2d^2-5abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}\,(acx+2ad-bc)}{cx+d} + \sqrt{a}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*b
*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]
```

```
*Sqrt[b*c - a*d]) + Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/c^3
```

fricas [A] time = 0.95, size = 769, normalized size = 4.93

$$\left[-\frac{\left(3\,bcd-4\,ad^2+\left(3\,bc^2-4\,acd\right)x\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+\left(bcd-4\,ad^2+\left(bc^2-4\,acd\right)x\right)\sqrt{-\frac{bc-ad}{d}}}{2\left(c^4x+c^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a
*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*
sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt
((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a
*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (b*c*d - 4*a*d^2 +
 (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*
sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c
^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), ((b*c*d - 4*a*d^2 + (b
*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((
a*x + b)/x)/(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*
x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[-2,0]:[1,0,%%%{-1,[1
]%%%}]%%},[4,6,4,0]%%%}+%%%{%%%{8,[1]%%%},[3,5,4,1]%%%}+%%%{%%{[-4,0]:[1,0,
```

```
%%%{-1,[1]%%%}]%%},[2,5,5,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,4,4,2]%%%}+%%%{%%%{8,[1]%%%},[1,4,5,2]%%%}+%%%{%%{[-2,0]:[1,0,%%
%{-1,[1]%%%}]%%},[0,4,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,2,0,0]%%%}+%%%{%%{pol
y1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1]%%%}+%%%{%%%{2,[1]%%%
},[2,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.06, size = 834, normalized size = 5.35

$$-\frac{\left(4a^{\frac{7}{2}}c\,d^3x\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)-5a^{\frac{5}{2}}b\,c^2d^2x\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+a^{\frac{3}{2}}b^2c^3dx\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*(4*a^(7/2)*c*d^3*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(
5/2)*c^4*x^2+4*a^(7/2)*d^4*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)
*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*
a^(5/2)*c^3*d*x-5*a^(5/2)*b*c^2*d^2*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c
^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-4*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*a^(5/2)*c^2*d^2-5*a^(5/2)*b*c*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(3/2)*((a*d-
b*c)/c^2*d)^(1/2)*a^(3/2)*c^4+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(3/2)*b*c^4*x+a^(3/2)*b^2*c^3*d*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*
d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d
)^(1/2)*a^(3/2)*b*c^3*d+a^(3/2)*b^2*c^2*d^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+4*((a*d-b*c)/c^2*d)^(1/2)*a
^3*c^2*d^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-3*((a*d-
b*c)/c^2*d)^(1/2)*a^2*b*c^3*d*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)
)/a^(1/2))+4*((a*d-b*c)/c^2*d)^(1/2)*a^3*c*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x
)^(1/2)*a^(1/2))/a^(1/2))-3*((a*d-b*c)/c^2*d)^(1/2)*a^2*b*c^2*d^2*ln(1/2*(2
*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)))*x*((a*x+b)/x)^(1/2)/c^4/((a*d
-b*c)/c^2*d)^(1/2)/a^(3/2)/(c*x+d)/d/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

mupad [B] time = 2.16, size = 448, normalized size = 2.87

$$\frac{\operatorname{atanh}\left(\frac{8\,a^2\,b^5\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{8\,a^3\,b^5\,d^3-10\,a^2\,b^6\,c\,d^2+2\,a\,b^7\,c^2\,d}-\frac{2\,a\,b^6\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{2\,a\,b^7\,c\,d-10\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}\right)\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-b\,c\right)}{c^3\,d}-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{6\,\sqrt{a}\,b}{6\,a\,b^7\,d-\frac{14\,a^2}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x)^2,x)
```

```
[Out] (atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^3
 - 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2 -
b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*d -
 b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(a +
b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (14*a^
(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3*b^5*d
^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2*b^6*c*
d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d)*(a + b
/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d -
 b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.237 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=209

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{3\sqrt{a}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{3\sqrt{a+\frac{b}{x}}\,(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-}{2c^2\left(c+\frac{d}{x}\right.}$$

```
[Out] 3*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))*a^(1/2)/c^4-3/4*(8*a^2*d^2-8*
a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/d^(1/2)
/(-a*d+b*c)^(1/2)-1/2*(-3*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)^2-3/4*(-4*a*d+
b*c)*(a+b/x)^(1/2)/c^3/(c+d/x)+a*x*(a+b/x)^(1/2)/c/(c+d/x)^2
```

Rubi [A] time = 0.34, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 151, 156, 63, 208, 205}

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}-\frac{3\sqrt{a+\frac{b}{x}}\,(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{3\sqrt{a}\,(bc-2ad)\tanh^{-1}\left(\sqrt{\right.}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] -((b*c - 3*a*d)*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*Sqrt[
a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d
]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d]) + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/c^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}a(bc-2ad)-\frac{1}{2}b(2bc-5ad)x}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{3a(bc-2ad)(bc-ad)+\frac{3}{2}b(bc-3ad)(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-3a(bc-2ad)(bc-ad)^2-\frac{3}{4}b(bc-4a}{x\sqrt{a+bx}\,(c+dx)}\right.}{2c^3(bc-ad)^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{2c^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{bc^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3\left(b^2c^2-8abcd+8a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{4c^4\sqrt{d}\,\sqrt{bc-ad}}$$

Mathematica [A] time = 0.60, size = 168, normalized size = 0.80

$$\frac{-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}\left(2a\left(2c^2x^2+9cdx+6d^2\right)-bc(5cx+3d)\right)}{(cx+d)^2} + 12\sqrt{a}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x^
2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqr
t[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) + 12*Sqrt[a]*(b*c -
 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

fricas [B] time = 1.23, size = 1765, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d
^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*
sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2 - 8
*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b
^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d -
(b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)) -
2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2
*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x
+ b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2
- a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(b^2*c^2*
d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2
 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log(
(b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
 d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqr
t((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c
^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^
2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8
*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x +
 b)/x)/(a*d*x + b*d)) - 6*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4
*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2
*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)
/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*
c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a
*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)
```

```
*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d
*x + b*d)) - 12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b
*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^
4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a*b*c^4*d - a^2*c
^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*
c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a
*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x)]
```

giac [B] time = 0.61, size = 727, normalized size = 3.48

$$\frac{\sqrt{ax^2+bx}\,a\,\mathrm{sgn}(x)}{c^3}+\frac{3\left(b^2c^2\mathrm{sgn}(x)-8\,abcd\mathrm{sgn}(x)+8\,a^2d^2\mathrm{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)c+\sqrt{a}\,d}{\sqrt{bcd-ad^2}}\right)}{4\sqrt{bcd-ad^2}\,c^4}-\frac{3\left(abc\mathrm{sgn}(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) + 8
*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sq
rt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2*d*
sgn(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^
4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(3
/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arctan(sqr
t(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 12*
sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*c - 10*sq
rt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4) + 1/4*(5*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*sgn(x) - 24*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 +
b*x))^3*a^(5/2)*c*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c^2*
d*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d^2*sgn(x) + 40*(sq
rt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))*sqrt(a)*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2
)*b^2*c*d^2*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*d^3*sgn(x
) - 5*a*b^3*c*d^2*sgn(x) + 10*a^2*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2
 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)
*c^4)
```

maple [B] time = 0.07, size = 1817, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*(27*a^(5/2)*b^2*c^4*d^2*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)
^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^3*d^3
*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-36*((a*x+b)*x)^(
1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^3*x-96*a^(7/2)*b*c^2*d^4*x*ln((-
2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+5
4*a^(5/2)*b^2*c^3*d^3*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))+48*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^2*d^4*x*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+30*((a*x+b)*x)^(1/2)*((a*d-
b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^3-36*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^
4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+12*((a*d-b*c)/c^2*d
)^(1/2)*a^2*b^2*c^3*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))-6*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^4*d^2+6*((a*x+
b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b*c^6*x-6*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^6*x^2-3*a^(3/2)*b^3*c^5*d*x^2*ln((-2*a*d
*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*
x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b*c^5*d+12*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^5*d*x^3-6*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*a^(5/2)*b*c^6*x^3-12*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(5/2)*c^5*d*x-48*a^(7/2)*b*c^3*d^3*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)
/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-6*a^(3/2)*b^3*c^4*d^2*x*ln((-2*
a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*
a^(9/2)*d^6*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1
/2)*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^(1/2)*a^2*b^2*c^4*d^2*x*ln(1/2*(2*a*x+
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+54*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2
*d)^(1/2)*a^(5/2)*b*c^4*d^2*x-72*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^3*d^3*x*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+18*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^5*d*x^2-36*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*
c^4*d^2*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+12*((a*d-
b*c)/c^2*d)^(1/2)*a^2*b^2*c^5*d*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))-12*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^5
*d*x+48*a^(9/2)*c*d^5*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))-24*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^
(7/2)*c^2*d^4-48*a^(7/2)*b*c*d^5*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)
^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+27*a^(5/2)*b^2*c^2*d^4*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*
c)/c^2*d)^(1/2)*a^4*c*d^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1
/2))+24*a^(9/2)*c^2*d^4*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-8*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)
*a^(5/2)*c^4*d^2-3*a^(3/2)*b^3*c^3*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/
c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d)))*x*((a*x+b)/x)^(1/2)/c^5/d/((a*d
-b*c)/c^2*d)^(1/2)/a^(3/2)/(c*x+d)^2/(a*d-b*c)/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

mupad [B] time = 3.47, size = 1664, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x)^3,x)
[Out] - ((3*(a + b/x)^(1/2)*(3*a*b^3*c^2 + 4*a^3*b*d^2 - 7*a^2*b^2*c*d))/(4*c^3)
- ((a + b/x)^(3/2)*(5*b^3*c^2 + 24*a^2*b*d^2 - 24*a*b^2*c*d))/(4*c^3) + (3*
b*(a + b/x)^(5/2)*(4*a*d^2 - b*c*d))/(4*c^3))/((a + b/x)^2*(3*a*d^2 - 2*b*c
*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3*d
^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (3*a^(1/2)*atanh((27*a^(1/2)*b^7*d*(a + b/x
)^(1/2))/(8*((27*a*b^7*d)/8 - (27*a^2*b^6*d^2)/(4*c))) + (27*a^(3/2)*b^6*d^
2*(a + b/x)^(1/2))/(4*((27*a^2*b^6*d^2)/4 - (27*a*b^7*c*d)/8)))*(2*a*d - b*
c))/c^4 - (atan((((((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a
*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(
a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*
c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d
^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2
*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*
d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8*(a*c^4*d^2 - b*c^5*d)) + ((((a + b/x)^(1/
2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4
 + 864*a^2*b^4*c^2*d^3))/(8*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d
^2 - 12*a^2*b^3*c^8*d^3)/c^9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a +
 b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^
6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2
- b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8
*(a*c^4*d^2 - b*c^5*d)))/((216*a^5*b^3*d^5 - 378*a^4*b^4*c*d^4 - (189*a^2*b
^6*c^3*d^2)/4 + 216*a^3*b^5*c^2*d^3 + (27*a*b^7*c^4*d)/8)/c^9 - (3*(((a + b
/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^
3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(a*d - b*c))^(1/2)*((9*a*b^
4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^
```

```
3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))
/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c
^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)
)/(8*(a*c^4*d^2 - b*c^5*d)) + (3*(((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*
b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8
*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^
9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))
^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(
8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c)
)^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d))))*(d*(
a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(4*(a*c^4*d^2 - b*c
^5*d))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.238 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=198

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}-\frac{1}{3}c^2\left(a+\frac{b}{x}\right.$$

```
[Out] -1/3*c^2*(6*a*d+5*b*c)*(a+b/x)^(3/2)-11/9*d*(a+b/x)^(5/2)*(c+d/x)^2-1/315*d
*(a+b/x)^(5/2)*(-20*a^2*d^2+270*a*b*c*d+938*b^2*c^2+5*b*d*(10*a*d+89*b*c)/x
)/b^2+(a+b/x)^(5/2)*(c+d/x)^3*x+a^(3/2)*c^2*(6*a*d+5*b*c)*arctanh((a+b/x)^(
1/2)/a^(1/2))-a*c^2*(6*a*d+5*b*c)*(a+b/x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}+a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{1}{3}c^2\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a + b/x)^(3/
2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a + b/x)^(5/2)*(2*(469*b
^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(89*b*c + 10*a*d))/x))/(315*b^2
) + (a + b/x)^(5/2)*(c + d/x)^3*x + a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2\left(\frac{1}{2}(5bc+6ad)+\frac{11bdx}{2}\right)}{x}\,dx, x,$$

$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)\left(\frac{9}{4}bc(5bc+\right.}{x}\right.}{9b}$$

$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(469b^2c^2+135abcd-10a^2d^2\right)+\frac{5bd(89bc+}{x}\right.}{315b^2}$$

$$= -\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(469b^2c^2+1\right.\right.}{}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d}{}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d}{}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d}{}$$

Mathematica [A] time = 0.25, size = 201, normalized size = 1.02

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{a+\frac{b}{x}}\left(20a^4d^3x^4-10a^3bd^2x^3(27cx+d)-3a^2b^2x^2\left(-105c^3x^3+966c^2dx^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2*
x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3 +
135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d^2*
x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c + 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

fricas [A] time = 0.85, size = 494, normalized size = 2.49

$$\left[\frac{315 \left(5 \, a b^{3} c^{3} + 6 \, a^{2} b^{2} c^{2} d\right) \sqrt{a} \, x^{4} \log\left(2 \, a x + 2 \, \sqrt{a} \, x \sqrt{\frac{a x + b}{x}} + b\right) + 2 \, \left(315 \, a^{2} b^{2} c^{3} x^{5} - 70 \, b^{4} d^{3} - 2 \, \left(735 \, a b^{3} c^{3} + 144\right.\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*
a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105
*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b
^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*
b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 + 6*a^2*
b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*a^2*b^2
*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c
*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d
^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)
*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
```

```
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 1.79Lim
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

maple [B] time = 0.07, size = 457, normalized size = 2.31

$$\sqrt{\frac{ax+b}{x}}\left(1890a^3b^2c^2d\,x^6\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\;\sqrt{a}}{2\sqrt{a}}\right)+1575a^2b^3c^3x^6\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\;\sqrt{a}}{2\sqrt{a}}\right)+3780\sqrt{a\,x^2+bx}\;a^{\frac{7}{2}}b\,c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^3,x)
```

```
[Out] 1/630*((a*x+b)/x)^(1/2)*(3780*(a*x^2+b*x)^(1/2)*a^(7/2)*x^6*b*c^2*d+3150*(a
*x^2+b*x)^(1/2)*a^(5/2)*x^6*b^2*c^3+40*(a*x^2+b*x)^(3/2)*a^(7/2)*x^3*d^3-37
80*(a*x^2+b*x)^(3/2)*a^(5/2)*x^4*b*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*
b*c*d^2-2520*(a*x^2+b*x)^(3/2)*a^(3/2)*x^4*b^2*c^3+1890*ln(1/2*(2*a*x+b+2*(
a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^6*a^3*b^2*c^2*d+1575*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^6*a^2*b^3*c^3-60*(a*x^2+b*x)^(3/2)*a
^(5/2)*x^2*b*d^3-2016*(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b^2*c^2*d-1080*(a*x^2+b
*x)^(3/2)*a^(3/2)*x^2*b^2*c*d^2-420*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^3*c^3-2
40*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b^2*d^3-756*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^
3*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(1/2)*x*b^3*c*d^2-140*(a*x^2+b*x)^(3/2)*a^(
1/2)*b^3*d^3)/x^5/b^2/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.21, size = 219, normalized size = 1.11

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^2}{7\,b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^2x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c^3-\frac{1}{5}\left(15\,a^{\frac{5}{2}}\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/7*(a + b/x)^(7/2)*c*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*lo
g((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*
b - 24*sqrt(a + b/x)*a*b)*c^3 - 1/5*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a
))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a +
30*sqrt(a + b/x)*a^2)*c^2*d - 2/63*(7*(a + b/x)^(9/2)/b^2 - 9*(a + b/x)^(7/
2)*a/b^2)*d^3
```

mupad [B] time = 6.05, size = 487, normalized size = 2.46

$$\left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{7\,b^2}-\frac{4\,a\,d^3}{7\,b^2}\right)-\sqrt{a+\frac{b}{x}}\,\left(a^2\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(7/2)*((6*a*d^3 - 6*b*c*d^2)/(7*b^2) - (4*a*d^3)/(7*b^2)) - (a +
b/x)^(1/2)*(a^2*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*
d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - 2*a*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*
((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*
a^2*d^3)/b^2) - a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))) + (a + b/
x)^(3/2)*((2*(a*d - b*c)^3)/(3*b^2) + (2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2
- (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2))/3 - (a^2*((6
*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3) + (a + b/x)^(5/2)*((2*a*((6*a*
d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/5 - (6*d*(a*d - b*c)^2)/(5*b^2) + (2
*a^2*d^3)/(5*b^2)) - (2*d^3*(a + b/x)^(9/2))/(9*b^2) + a^2*c^3*x*(a + b/x)^
(1/2) - a^(3/2)*c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + 5*b*c)*1i
```

sympy [A] time = 158.70, size = 5513, normalized size = 27.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

```
[Out] 32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(
21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6
300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(1
1/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(29/
2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2
)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x
**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2)
+ 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*sqrt(a*x
/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
```

725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(315*a**(21/2
)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x
(17/2) + 6300*a(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
+ 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(2
1/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1
890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(1
5/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**2
0*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2)*b**(11/2)*d**3*x*
*6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 378*a**(
19/2)*b**(37/2)*d**3*x**5*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +
1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(
15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**
20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 48*a**(19/2)*b**(13/2)*c*d**2
*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a*
*(19/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b
10*x(7/2)) - 1134*a**(17/2)*b**(39/2)*d**3*x**4*sqrt(a*x/b + 1)/(315*a*
*(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b
17*x(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(
13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 120
*a**(17/2)*b**(15/2)*c*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7
/2)*b**10*x**(7/2)) - 60*a**(17/2)*b**(15/2)*d**3*x**4*sqrt(a*x/b + 1)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 1494*a**(15/2)*b**(41/2)*d**3*x
3*sqrt(a*x/b + 1)/(315*a(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x
(19/2) + 4725*a(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2)
+ 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**
(9/2)*b**21*x**(9/2)) - 90*a**(15/2)*b**(17/2)*c*d**2*x**4*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a**(15/2)*b**(17/2)*d**3
*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**
(15/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
(5/2)*b4*x**(5/2)) - 1098*a**(13/2)*b**(43/2)*d**3*x**2*sqrt(a*x/b + 1)
/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2
)) - 120*a**(13/2)*b**(19/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**
7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 1
05*a**(7/2)*b**10*x**(7/2)) - 200*a**(13/2)*b**(19/2)*d**3*x**2*sqrt(a*x/b

```
+ 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 24*a**(13/2)*b**(5/2)*
c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) + 2*a**(13/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 430*a**(11/2)*b**(45/2)*d**3*x*sqr
t(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2
) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*
a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b
**21*x**(9/2)) - 300*a**(11/2)*b**(21/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(105*a
**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*
x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**(11/2)*b**(21/2)*d**3*x*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(11/2)*
b**(7/2)*c**2*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) + 12*a**(11/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(11/2)*b**(7/2)*d
**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 70*a**(9/2)*b**(47/2)*d**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21
/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 630
0*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/
2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 288*a**(9/2)*b**(23/2)*
c*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 60
*a**(9/2)*b**(23/2)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 6*a**(9/2)*b**(9/2)*c**2*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*
b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 48*a**(9/2)*b**(9/2)*c*d**2*x*
sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6
*a**(9/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 90*a**(7/2)*b**(25/2)*c*d**2*sqrt(a*x/b + 1)/(105*a*
*(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x
**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**(7/2)*b**(11/2)*c**2*d*x*sqr
t(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 36*a
**(7/2)*b**(11/2)*c*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(13/2)*c**2*d*sqrt(a*x/b + 1)/(15*a*
*(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c**3*asinh(s
qrt(a)*sqrt(x)/sqrt(b)) - 32*a**15*b**13*d**3*x**(21/2)/(315*a**(21/2)*b**1
5*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/
2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890
*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 192*a**14*b**14
*d**3*x**(19/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(1
9/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 47
25*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2
)*b**21*x**(9/2)) - 480*a**13*b**15*d**3*x**(17/2)/(315*a**(21/2)*b**15*x**
(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) +
```

```
6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(
11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 640*a**12*b**16*d**3
*x**(15/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2)
+ 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a*
*(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**
21*x**(9/2)) - 480*a**11*b**17*d**3*x**(13/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 32*a**11*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**10*b**18*d**3*x*
*(11/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 48*a**10*b**6*c*d**2*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) +
315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b*
*10*x**(7/2)) + 96*a**10*b**6*d**3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2)
+ 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*
b**10*x**(7/2)) - 32*a**9*b**19*d**3*x**(9/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 144*a**9*b**7*c*d**2*x**(
11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 96*a**9*b**7*d**3*x**
(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 144*a**8*b**8*c*d**2*
x**(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 32*a**8*b**8*d**3*
x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**8*b*d**3*x**(
7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 48*a**7*b**9
*c*d**2*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**7*b*
*2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 24*a**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2
)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**3*c**2*d*atan(sqrt(a +
b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 4*a**
2*b*c**3*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 6*a**2*c**2*d*sqrt(a + b/x
) + 3*a**2*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3
*b), True)) - 4*a*b*c**3*sqrt(a + b/x) + 6*a*b*c**2*d*Piecewise((-sqrt(a)/x
, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*Piecewise((-sqr
t(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.239 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 \, dx$

Optimal. Leaf size=152

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+}$$

```
[Out] -1/3*c*(4*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*c*(4*a*d+5*b*c)*(a+b/x)^(5/2)/a-2/7*
d^2*(a+b/x)^(7/2)/b+c^2*(a+b/x)^(7/2)*x/a+a^(3/2)*c*(4*a*d+5*b*c)*arctanh((
a+b/x)^(1/2)/a^(1/2))-a*c*(4*a*d+5*b*c)*(a+b/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^(3/2))/
3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*(a + b/x)^(7/2))/(7*
b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^2}{x^2}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(\frac{1}{2}c(5bc+4ad)+ad^2x\right)}{x}dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{(c(5bc+4ad))\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(c(5bc+4ad))\,\text{Sub}$$

$$= -\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)}{a}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2a}{}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2a}{}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2a}{}$$

Mathematica [A] time = 0.16, size = 121, normalized size = 0.80

$$-\frac{c(4ad+5bc)\left(\sqrt{a+\frac{b}{x}}\left(23a^2x^2+11abx+3b^2\right)-15a^{5/2}x^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)}{15ax^2}+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(7/2))/(7*b) + (c^2*(a + b/x)^(7/2)*x)/a - (c*(5*b*c + 4*
a*d)*(Sqrt[a + b/x]*(3*b^2 + 11*a*b*x + 23*a^2*x^2) - 15*a^(5/2)*x^2*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]]))/(15*a*x^2)
```

fricas [A] time = 1.02, size = 350, normalized size = 2.30

$$\left[\frac{105\left(5\,ab^{2}c^{2}+4\,a^{2}bcd\right)\sqrt{a}\,x^{3}\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(105\,a^{2}bc^{2}x^{4}-30\,b^{3}d^{2}-2\left(245\,ab^{2}c^{2}+322\,a^{2}b\right.\right.}{210\,bx^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*
c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*
a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3
), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 32
2*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^
2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.81Lim
```

```
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

maple [B] time = 0.06, size = 336, normalized size = 2.21

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-420a^{3}bcd\,x^{5}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-525a^{2}b^{2}c^{2}x^{5}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-840\sqrt{a\,x^{2}+bx}\;a^{\frac{7}{2}}cd\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^2,x)
```

```
[Out] -1/210*((a*x+b)/x)^(1/2)/x^4/b*(-840*(a*x^2+b*x)^(1/2)*a^(7/2)*x^5*c*d-1050
*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2-420*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2
)*a^(1/2))/a^(1/2))*x^5*a^3*b*c*d-525*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a
^(1/2))/a^(1/2))*x^5*a^2*b^2*c^2+840*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*c*d+840*
(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^2+44
8*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d+140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2
*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d^2+168*(a*x^2+b*x)^(3/2)*a^(1/2)*x*
b^2*c*d+60*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^2)/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.25, size = 181, normalized size = 1.19

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^{2}}{7\,b}+\frac{1}{6}\left(6\,\sqrt{a+\frac{b}{x}}\,a^{2}x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\,\sqrt{a+\frac{b}{x}}\,ab\right)c^{2}-\frac{2}{15}\left(15\,a^{\frac{5}{2}}\,\mathrm{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log(
(sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b
- 24*sqrt(a + b/x)*a*b)*c^2 - 2/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a)
)/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 3
0*sqrt(a + b/x)*a^2)*c*d
```

mupad [B] time = 3.79, size = 271, normalized size = 1.78

$$\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,a\,\left(\frac{4\,a\,d^{2}-4\,b\,c\,d}{b}-\frac{4\,a\,d^{2}}{b}\right)}{3}-\frac{2\,{\left(a\,d-b\,c\right)}^{2}}{3\,b}+\frac{2\,a^{2}\,d^{2}}{3\,b}\right)+\left(\frac{4\,a\,d^{2}-4\,b\,c\,d}{5\,b}-\frac{4\,a\,d^{2}}{5\,b}\right)\left(a+\frac{b}{x}\right)^{5/2}-\sqrt{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d -
b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^2)
/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b - (4*
a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b)) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a + b/
x)^(1/2) - a^(3/2)*c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(4*a*d + 5*b*c)*1i
```

sympy [A] time = 112.03, size = 1841, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)/(15*a**
(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(23/2)*d*
*2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**
(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(
9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(
5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(25/2)*d**2
*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 16*a**(7/2
)*b**(11/2)*c*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 12*a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b*
*3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2
)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
```

```
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**7*d**2*x**(11/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**2*d**2*x**(7/2)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*c*d*x**(7
/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d
**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a*
*5*b**4*c*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)
) - 4*a**3*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**2*sq
rt(x)*sqrt(a*x/b + 1) - 4*a**2*b*c**2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
 - 4*a**2*c*d*sqrt(a + b/x) + a**2*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) - 4*a*b*c**2*sqrt(a + b/x) + 4*a*b*c*d*Pi
ecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c
**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.240 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=125

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}\,(2ad+5bc)+\frac{cx\left(a+\right.}{a}$$

```
[Out] -1/3*(2*a*d+5*b*c)*(a+b/x)^(3/2)-1/5*(2*a*d+5*b*c)*(a+b/x)^(5/2)/a+c*(a+b/x
)^(7/2)*x/a+a^(3/2)*(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))-a*(2*a*d+5
*b*c)*(a+b/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}\,(2ad+5bc)+\frac{cx\left(a+\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(3/2))/3 -
((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^(7/2)*x)/a + a^(3/2)
*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)}{x^2}dx, x, \frac{1}{x}\right)$$
$$= \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\left(\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(5bc+2ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \right.$$
$$= -\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(a(5bc+2ad)$$
$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\right.}{a}$$
$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\right.}{a}$$
$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\right.}{a}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.75

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{a+\frac{b}{x}}\left(a^2x^2(15cx-46d)-2abx(35cx+11d)-2b^2(5cx+3d)\right)}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*(
11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

fricas [A] time = 0.96, size = 222, normalized size = 1.78

$$\left[\frac{15\left(5\,abc+2\,a^2d\right)\sqrt{a}\,x^2\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,a^2cx^3-6\,b^2d-2\left(35\,abc+23\,a^2d\right)x^2-2\left(5\,b^2c\right.\right.}{30\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2
*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*a^2
*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 - 6*b
^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x +
b)/x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached
 or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 253, normalized size = 2.02

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-30a^3bd\,x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-75a^2b^2c\,x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-60\sqrt{a\,x^2+bx}\,a^{\frac{7}{2}}d\,x^4-150\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x),x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-60*(a*x^2+b*x)^(1/2)*a^(7/2)*x^4*d-150*(a*x
^2+b*x)^(1/2)*a^(5/2)*x^4*b*c-30*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^4*a^3*b*d-75*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(
1/2))*x^4*a^2*b^2*c+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d+120*(a*x^2+b*x)^(3/2
)*a^(3/2)*x^2*b*c+32*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d+20*(a*x^2+b*x)^(3/2)*a
^(1/2)*x*b^2*c+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d)/((a*x+b)*x)^(1/2)/a^(1/2
)
```

maxima [A] time = 1.51, size = 161, normalized size = 1.29

$$\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^2x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c-\frac{1}{15}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sq
rt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b - 24*sqrt(a + b/x)*a*b)*c - 1
/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 30*sqrt(a + b/x)*a^2)*d
```

mupad [B] time = 3.48, size = 99, normalized size = 0.79

$$-\frac{2\,d\left(a+\frac{b}{x}\right)^{5/2}}{5}-2\,a^2\,d\sqrt{a+\frac{b}{x}}-\frac{2\,a\,d\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\,c\,x\left(a+\frac{b}{x}\right)^{5/2}{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\,-\frac{1}{2};\,-\frac{a\,x}{b}\right)}{3\left(\frac{a\,x}{b}+1\right)^{5/2}}-a^{5/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x),x)
```

```
[Out] - (2*d*(a + b/x)^(5/2))/5 - 2*a^2*d*(a + b/x)^(1/2) - a^(5/2)*d*atan(((a +
b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*d*(a + b/x)^(3/2))/3 - (2*c*x*(a + b/x)^(
5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^(5/2))
```

sympy [A] time = 82.77, size = 520, normalized size = 4.16

$$\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^{\frac{3}{2}}bc\,\mathrm{asinh}\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2)*d
*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**
6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5
/2)) - 2*a**3*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c*sqrt
(x)*sqrt(a*x/b + 1) - 4*a**2*b*c*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 2*
a**2*d*sqrt(a + b/x) - 4*a*b*c*sqrt(a + b/x) + 2*a*b*d*Piecewise((-sqrt(a)/
x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c*Piecewise((-sqrt(
a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.241 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=71

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

```
[Out] -5/3*b*(a+b/x)^(3/2)+(a+b/x)^(5/2)*x+5*a^(3/2)*b*arctanh((a+b/x)^(1/2)/a^(1
/2))-5*a*b*(a+b/x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {242, 47, 50, 63, 208}

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2} dx, x, \frac{1}{x}\right) \\
&= \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \frac{1}{x}\right) \\
&= -\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x}\right) \\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right) \\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right) \\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.90

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{a+\frac{b}{x}}\left(3a^2x^2-14abx-2b^2\right)}{3x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2),x]

[Out] (Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

fricas [A] time = 0.94, size = 139, normalized size = 1.96

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^{2}x^{2}-14\,abx-2\,b^{2}\right)\sqrt{\frac{ax+b}{x}}}{6\,x},\,-\frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-(3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2),x, algorithm="fricas")

[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parameters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.06, size = 120, normalized size = 1.69

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(15a^{2}b\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+30\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}x^{3}-24\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\right)}{6\sqrt{(ax+b)\,x}\,\sqrt{a}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*(30*a^(5/2)*(a*x^2+b*x)^(1/2)*x^3-24*a^(3/2)*(a*x^2+b
*x)^(3/2)*x+15*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^
2*b-4*b*(a*x^2+b*x)^(3/2)*a^(1/2))/x^2/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.25, size = 78, normalized size = 1.10

$$\sqrt{a+\frac{b}{x}}\,a^{2}x-\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-4\sqrt{a+\frac{b}{x}}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

mupad [B] time = 1.63, size = 34, normalized size = 0.48

$$-\frac{2\,x\left(a+\frac{b}{x}\right)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};\,-\frac{1}{2};\,-\frac{a\,x}{b}\right)}{3\left(\frac{a\,x}{b}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2),x)
```

```
[Out] -(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
+ 1)^(5/2))
```

sympy [A] time = 4.36, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}\,b^{2}\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/
2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b
**2*sqrt(1 + b/(a*x))/(3*x)
```

3.242 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=134

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}\,(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

```
[Out] a*(a+b/x)^(3/2)*x/c+2*(-a*d+b*c)^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b
*c)^(1/2))/c^2/d^(3/2)+a^(3/2)*(-2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2)
)/c^2-b*(a*d+2*b*c)*(a+b/x)^(1/2)/c/d
```

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 154, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}\,(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/c + (2*(b*
c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2
)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-2ad)-\frac{1}{2}b(2bc+ad)x\right)}{x(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}a^2d(5bc-2ad)+\frac{1}{4}b\left(2b^2c^2-6abcd+a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{cd}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(bc-a}{}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \cdots$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}} + \frac{a^{3/2}(5bc-2ad)\tan}{c^2}$$

Mathematica [A] time = 0.27, size = 116, normalized size = 0.87

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{c\sqrt{a+\frac{b}{x}}\left(a^2dx-2b^2c\right)}{d} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] ((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(Sq
rt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) + a^(3/2)*(5*b*c - 2*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

fricas [A] time = 1.31, size = 659, normalized size = 4.92

$$\left[-\frac{\left(5\,abcd-2\,a^{2}d^{2}\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-\frac{bc-ad}{d}}\,\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+b}{cx+d}\right.}{2\,c^{2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*
d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x +
d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*d -
2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((
a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d*x - 2*b^2*c^2)*s
qrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*
c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) +
(5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
 b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -(2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt
((a*x + b)/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-
a)*sqrt((a*x + b)/x)/a) - (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.06, size = 859, normalized size = 6.41

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{7}{2}}d^{4}x^{2}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)-6a^{\frac{5}{2}}bc\,d^{3}x^{2}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+6a^{\frac{3}{2}}b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^3*c*d^3-5*ln(1/2*(2*a*x+b+2*((a*x+b)*x
)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^2*b*c^2*d^2+4*ln(1/
2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^
2*a*b^2*c^3*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b
*c)/c^2*d)^(1/2)*x^2*b^3*c^4-8*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^(1/2)*a^
(3/2)*x^2*b*c^3*d+2*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^(1/2)*a^(1/2)*x^2*b
^2*c^4-4*((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^2*a*b^2*c^3*d+((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^
2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^3*c^4-2*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2
)*((a*x+b)*x)^(1/2)*x^2*c^2*d^2+4*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*((a*x+b)*
x)^(1/2)*x^2*b*c^3*d-2*((a*d-b*c)/c^2*d)^(1/2)*a^(1/2)*((a*x+b)*x)^(1/2)*x^
2*b^2*c^4+2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1
/2)*c)/(c*x+d))*a^(7/2)*x^2*d^4-6*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d
)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*a^(5/2)*x^2*b*c*d^3+6*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*a^(3/2)*x^2
*b^2*c^2*d^2-2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)
^(1/2)*c)/(c*x+d))*a^(1/2)*x^2*b^3*c^3*d+4*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b
*x)^(3/2)*a^(1/2)*b*c^3*d)/((a*x+b)*x)^(1/2)/d^2/c^3/a^(1/2)/((a*d-b*c)/c^2
*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{c + \frac{d}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x), x)
```

mupad [B] time = 2.16, size = 1427, normalized size = 10.65

$$\frac{a^2\, b\, d\, \sqrt{a + \frac{b}{x}}}{c\, \left(d\, \left(a + \frac{b}{x}\right) - a\, d\right)} - \frac{2\, b^2\, \sqrt{a + \frac{b}{x}}}{d} + \frac{\mathrm{atan}\left(\frac{a^3\, b^5\, \sqrt{a+\frac{b}{x}}\, \sqrt{a^5\, d^8-5\, a^4\, b\, c\, d^7+10\, a^3\, b^2\, c^2\, d^6-10\, a^2\, b^3\, c^3\, d^5+5\, a\, b^4\, c^4\, d^4-b^5\, c^5\, d^3}\, 160\mathrm{i}}{448\, a^3\, b^8\, c^3\, d-340\, a^6\, b^5\, d^4-128\, a^2\, b^9\, c^4+740\, a^5\, b^6\, c\, d^3+\frac{16\, a\, b^{10}\, c^5}{d}-796\, a^4\, b^7\, c^2\, d^2+\frac{60\, a^7\, b^4\, d^5}{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x),x)
```

```
[Out] (atan((a^3*b^5*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 1
0*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*160i)/(448*a^
3*b^8*c^3*d - 340*a^6*b^5*d^4 - 128*a^2*b^9*c^4 + 740*a^5*b^6*c*d^3 + (16*a
*b^10*c^5)/d - 796*a^4*b^7*c^2*d^2 + (60*a^7*b^4*d^5)/c) - (a^2*b^6*(a + b/
x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10
*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*80i)/(16*a*b^10*c^4 + 740*a^5*b^6*d
^4 - 128*a^2*b^9*c^3*d - 796*a^4*b^7*c*d^3 + 448*a^3*b^8*c^2*d^2 - (340*a^6
*b^5*d^5)/c + (60*a^7*b^4*d^6)/c^2) - (a^4*b^4*(a + b/x)^(1/2)*(a^5*d^8 - b
^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*
a^4*b*c*d^7)^(1/2)*60i)/(448*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 796*a^4*b^7*c^3
*d - 340*a^6*b^5*c*d^3 + (16*a*b^10*c^6)/d^2 + 740*a^5*b^6*c^2*d^2 - (128*a
^2*b^9*c^5)/d) + (a*b^7*c*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*
c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*16
i)/(740*a^5*b^6*d^5 - 796*a^4*b^7*c*d^4 - 128*a^2*b^9*c^3*d^2 + 448*a^3*b^8
*c^2*d^3 - (340*a^6*b^5*d^6)/c + (60*a^7*b^4*d^7)/c^2 + 16*a*b^10*c^4*d))*(
d^3*(a*d - b*c)^5)^(1/2)*2i)/(c^2*d^3) - (2*b^2*(a + b/x)^(1/2))/d + (atan(
(b^9*c^3*(a + b/x)^(1/2)*(a^3)^(1/2)*40i)/(40*a^2*b^9*c^3 - 790*a^5*b^6*d^3
 - 256*a^3*b^8*c^2*d + 696*a^4*b^7*c*d^2 + (370*a^6*b^5*d^4)/c - (60*a^7*b^
4*d^5)/c^2) + (a*b^8*c^2*(a + b/x)^(1/2)*(a^3)^(1/2)*256i)/(256*a^3*b^8*c^2
 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4
*d^4)/c^2 - 696*a^4*b^7*c*d) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*790
i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d
^3)/c + (60*a^7*b^4*d^4)/c^2 - 696*a^4*b^7*c*d) - (a^4*b^5*d^3*(a + b/x)^(1
/2)*(a^3)^(1/2)*370i)/(256*a^3*b^8*c^3 - 370*a^6*b^5*d^3 - 696*a^4*b^7*c^2*
d + 790*a^5*b^6*c*d^2 - (40*a^2*b^9*c^4)/d + (60*a^7*b^4*d^4)/c) + (a^5*b^4
*d^4*(a + b/x)^(1/2)*(a^3)^(1/2)*60i)/(256*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 6
96*a^4*b^7*c^3*d - 370*a^6*b^5*c*d^3 + 790*a^5*b^6*c^2*d^2 - (40*a^2*b^9*c^
5)/d) - (a^2*b^7*c*d*(a + b/x)^(1/2)*(a^3)^(1/2)*696i)/(256*a^3*b^8*c^2 + 7
90*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4*d^4
)/c^2 - 696*a^4*b^7*c*d))*(2*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^2 + (a^2*b*d*(a
 + b/x)^(1/2))/(c*(d*(a + b/x) - a*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Timed out
```

3.243 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\right.}{c\left(c+\right.}$$

```
[Out] a*(a+b/x)^(3/2)*x/c/(c+d/x)-(-a*d+b*c)^(3/2)*(4*a*d+b*c)*arctan(d^(1/2)*(a+
b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)+a^(3/2)*(-4*a*d+5*b*c)*arctanh((a+
b/x)^(1/2)/a^(1/2))/c^3+(-2*a*d+b*c)*(-a*d+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)
```

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 149, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\right.}{c\left(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^
(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + (a^(3/2)*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-4ad)-\frac{1}{2}b(2bc-ad)x\right)}{x(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a^2d(5bc-4ad)+\frac{1}{2}b\left(b^2c^2+2abcd-2a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)}\,dx,\right.}{c^2d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - $$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right.}{bc^3}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}}{}$$

Mathematica [A] time = 0.43, size = 145, normalized size = 0.87

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}\left(a^2d(cx+2d)-2abcd+b^2c^2\right)}{d(cx+d)} - \frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x)
) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
```

```
c - a*d]])/d^(3/2) + a^(3/2)*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/c^3
```

fricas [A] time = 1.23, size = 1001, normalized size = 6.03

$$\left[-\frac{\left(5\,abcd^2-4\,a^2d^3+\left(5\,abc^2d-4\,a^2cd^2\right)x\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+\left(b^2c^2d+3\,abcd^2-4\,a^2d^3+\left(b^2c\right.\right.}{2\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 - 4
*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*lo
g((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c
*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt
((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a
*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x)
 + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*
c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), 1/2*(2*(b^2
*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*
log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c^2*d*x^2 + (b^2*c^
3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), (
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b
*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c^2*d*x^2 + (b^2*c^3 - 2*a
*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[-2,0]:[1,0,%%%{-1,[1
]%%%}]%%},[4,6,4,0]%%%}+%%%{%%%{8,[1]%%%},[3,5,4,1]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,5,5,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,4,4,2]%%%}+%%%{%%%{8,[1]%%%},[1,4,5,2]%%%}+%%%{%%{[-2,0]:[1,0,%%
%{-1,[1]%%%}]%%},[0,4,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,2,0,0]%%%}+%%%{%%{pol
y1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1]%%%}+%%%{%%%{2,[1]%%%
},[2,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.06, size = 1323, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*(2*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4*d-5*a^3*l
n(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2
)*x*b*c^3*d^2-5*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*(
(a*d-b*c)/c^2*d)^(1/2)*b*c^2*d^3+a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d-a*((a*d-b*c)/c^2*d)^(1/2)*l
n(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*b^3*c^4*d-2*a^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^(1/2)*x*b^2*c^5+a^(3/2)*ln((-2*a*d*x+b*c*
x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^3*c^4*d+a
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1
/2)*x*b^3*c^5-a*((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)
*a^(1/2))/a^(1/2))*x*b^3*c^5-2*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^
(1/2)*b^2*c^4*d-2*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c
^5-2*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^2-7*a^(7/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*x*b*c^2*d^3+2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*
((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^2+4*a^4*ln(1/2*(2*a*x+b+2*((a*x+b
)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^3+4*a^(5/2)*((
a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^2+2*a^(7/2)*((a*x+b)*x)^(1/
2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^4*d+4*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a
*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^5-2*a^(5/2)*((a*x+b)*x
)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*c^4*d+4*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((
a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^4-4*a^(7/2)*((a*x
+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^3-7*a^(7/2)*ln((-2*a*d*x+b*c*x-b
*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^4+2*a^(5/2
)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*
x+d))*b^2*c^2*d^3+4*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*((a*d-b*c)/c^2*d)^(1/2)*c*d^4+2*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2
```

```
*d)^(1/2)*b*c^5+a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c^3*d^2)*x*((a*x+b)/x)^(1/2)/c^4/((a*d-b*c)
/c^2*d)^(1/2)/a^(3/2)/(c*x+d)/d^2/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^2, x)
```

mupad [B] time = 2.31, size = 1153, normalized size = 6.95

$$\frac{\frac{\sqrt{a+\frac{b}{x}}\,\left(2\,a^3\,b\,d^2-3\,a^2\,b^2\,c\,d+a\,b^3\,c^2\right)}{c^2\,d}-\frac{b\,\left(a+\frac{b}{x}\right)^{3/2}\,\left(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{c^2\,d}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,\left(a+\frac{b}{x}\right)^2-a^2\,d+a\,b\,c}-\frac{\mathrm{atanh}\left(\frac{10\,b^9\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{10\,a^2\,b^9+\frac{32\,a^3\,b^8\,d}{c}-\frac{132\,a^4\,b^7\,d^2}{c^2}+\frac{130\,a^5\,b^6\,d^3}{c^3}-\frac{40\,a^6\,b^5\,d^4}{c^4}}+\frac{}{32}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x)^2,x)
```

```
[Out] (((a + b/x)^(1/2)*(a*b^3*c^2 + 2*a^3*b*d^2 - 3*a^2*b^2*c*d))/(c^2*d) - (b*(
a + b/x)^(3/2)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c^2*d))/((a + b/x)*(2*a*
d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((10*b^9*(a + b/x)^(1/2)*
(a^3)^(1/2))/(10*a^2*b^9 + (32*a^3*b^8*d)/c - (132*a^4*b^7*d^2)/c^2 + (130*
a^5*b^6*d^3)/c^3 - (40*a^6*b^5*d^4)/c^4) + (32*a*b^8*(a + b/x)^(1/2)*(a^3)^
(1/2))/(32*a^3*b^8 + (10*a^2*b^9*c)/d - (132*a^4*b^7*d)/c + (130*a^5*b^6*d^
2)/c^2 - (40*a^6*b^5*d^3)/c^3) - (132*a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/2)
)/(32*a^3*b^8*c - 132*a^4*b^7*d + (10*a^2*b^9*c^2)/d + (130*a^5*b^6*d^2)/c
- (40*a^6*b^5*d^3)/c^2) + (130*a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2))/(32
*a^3*b^8*c^2 + 130*a^5*b^6*d^2 + (10*a^2*b^9*c^3)/d - (40*a^6*b^5*d^3)/c -
132*a^4*b^7*c*d) - (40*a^4*b^5*d^3*(a + b/x)^(1/2)*(a^3)^(1/2))/(32*a^3*b^8
*c^3 - 40*a^6*b^5*d^3 - 132*a^4*b^7*c^2*d + 130*a^5*b^6*c*d^2 + (10*a^2*b^9
*c^4)/d))*(4*a*d - 5*b*c)*(a^3)^(1/2))/c^3 + (atanh((30*a^3*b^6*(a + b/x)^(
1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(14*a
^2*b^9*c^3 + 110*a^5*b^6*d^3 - 4*a^3*b^8*c^2*d - 82*a^4*b^7*c*d^2 + (2*a*b^
10*c^4)/d - (40*a^6*b^5*d^4)/c) + (18*a^2*b^7*(a + b/x)^(1/2)*(a^3*d^6 - b^
3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(2*a*b^10*c^3 - 82*a^4*
```

```
b^7*d^3 + 14*a^2*b^9*c^2*d - 4*a^3*b^8*c*d^2 + (110*a^5*b^6*d^4)/c - (40*a^
6*b^5*d^5)/c^2) + (40*a^4*b^5*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*
b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(4*a^3*b^8*c^3 + 40*a^6*b^5*d^3 + 82*a^
4*b^7*c^2*d - 110*a^5*b^6*c*d^2 - (2*a*b^10*c^5)/d^2 - (14*a^2*b^9*c^4)/d)
- (2*a*b^8*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2
*b*c*d^5)^(1/2))/(4*a^3*b^8*d^3 - 14*a^2*b^9*c*d^2 + (82*a^4*b^7*d^4)/c - (
110*a^5*b^6*d^5)/c^2 + (40*a^6*b^5*d^6)/c^3 - 2*a*b^10*c^2*d))*(d^3*(a*d -
b*c)^3)^(1/2)*(4*a*d + b*c))/(c^3*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.244 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal. Leaf size=237

$$\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abc\right.}{4c^3d\left(c+\frac{d}{x}\right)}$$

```
[Out] a*(a+b/x)^(3/2)*x/c/(c+d/x)^2+a^(3/2)*(-6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/c^4-1/4*(-24*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/
2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c^4/d^(3/2)+1/2*(-3*a*d+b*c)*(-a*d+b*
c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)^2-1/4*(-12*a^2*d^2+7*a*b*c*d+b^2*c^2)*(a+b/x
)^(1/2)/c^3/d/(c+d/x)
```

Rubi [A] time = 0.37, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 98, 149, 151, 156, 63, 208, 205}

$$-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}+\frac{a^{3/2}(5bc-6ad)\tanh}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2
 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x
)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2
*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a
^(3/2)*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-6ad)-\frac{1}{2}b(2bc-3ad)x\right)}{x(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b\left(b^2c^2+6abcd-9a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\left(a^2(5bc-\right.}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\left(a^2(5bc-\right.}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}}{}$$

Mathematica [A] time = 0.85, size = 191, normalized size = 0.81

$$\frac{-4a^{3/2}(6ad-5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}\left(2a^2d\left(2c^2x^2+9cdx+6d^2\right)-abcd(11cx+7d)+b^2c^2(cx-d)\right)}{d(cx+d)^2} - \frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)}{d^{3/2}}}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d*
(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2
 + 8*a*b*c*d - 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
/d^(3/2) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^
4)
```

fricas [**A**] time = 1.16, size = 1445, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(
5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^
3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)
*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c
^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*
sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 +
 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(
-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b*c*d^3 - 6*
a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^
3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*a^2*c^3*d
*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c
^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4
*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2
 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
b)/x)/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d
 - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*
d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqr
t((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*
a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(-d*
sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 4*(5*a*b*c*d^3 - 6*a^2
*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*
x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*a^2*c^3*d*x^3 + (b^2*
c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*
a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)]
```

giac [B] time = 0.49, size = 945, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))*
log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4) + 1/4
*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^3*d^
3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*
d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan(sqrt(
a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d
 - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*
a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d - a*d^2)*
a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - sqrt(b
*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*sqrt(b*c*d
- a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) - 1/4*((sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a)*x - sqrt(a*
x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(7/2)*
c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^3*d*sgn(x) - 3*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) + 48*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*c^3*d*sgn(x
) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*d^2*sgn(x) + 52*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt
(a*x^2 + b*x))*a^(7/2)*b*d^4*sgn(x) - a*b^4*c^2*d^2*sgn(x) + 11*a^2*b^3*c*d
^3*sgn(x) - 10*a^3*b^2*d^4*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c +
2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)*c^4*d)
```

maple [B] time = 0.07, size = 1638, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*(7*a^(5/2)*b^2*c^4*d^2*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^
(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^3*d^3*
x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-36*((a*x+b)*x)^(1
/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^3*x-64*a^(7/2)*b*c^2*d^4*x*ln((-2
*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+14
*a^(5/2)*b^2*c^3*d^3*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
```

```
*x+b)*x)^(1/2)*c)/(c*x+d))+48*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^2*d^4*x*ln(1/2*
(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+14*((a*x+b)*x)^(1/2)*((a*d-b
*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^3-20*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^4
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+2*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^4*d^2-2*((a*x+b)*x)^(3/2)*((a*d-b*c)/
c^2*d)^(1/2)*a^(3/2)*b*c^6*x+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^
(3/2)*b^2*c^6*x^2+a^(3/2)*b^3*c^5*d*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)
/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-6*((a*x+b)*x)^(3/2)*((a*d-b*c)/
c^2*d)^(1/2)*a^(3/2)*b*c^5*d+12*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(7/2)*c^5*d*x^3+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^6*
x^3-12*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*c^5*d*x-32*a^(7/2)
*b*c^3*d^3*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)
^(1/2)*c)/(c*x+d))+2*a^(3/2)*b^3*c^4*d^2*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b
*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*a^(9/2)*d^6*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+30*((a*x+
b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^4*d^2*x-40*((a*d-b*c)/c^2*d
)^(1/2)*a^3*b*c^3*d^3*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))+18*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^5*d*x^2-20*((a*
d-b*c)/c^2*d)^(1/2)*a^3*b*c^4*d^2*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a
^(1/2))/a^(1/2))+4*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^
5*d*x+48*a^(9/2)*c*d^5*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))-24*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(7/2)*c^2*d^4-32*a^(7/2)*b*c*d^5*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d
)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+7*a^(5/2)*b^2*c^2*d^4*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*
c)/c^2*d)^(1/2)*a^4*c*d^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1
/2))+24*a^(9/2)*c^2*d^4*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-8*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)
*a^(5/2)*c^4*d^2+a^(3/2)*b^3*c^3*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d)))*x*((a*x+b)/x)^(1/2)/c^5/((a*d-b*c
)/c^2*d)^(1/2)/a^(3/2)/(c*x+d)^2/d^2/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

mupad [B] time = 3.44, size = 1476, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x)^3,x)
```

```
[Out] (atan((b^9*(a + b/x)^(1/2)*(a^3)^(1/2)*5i)/(8*((5*a^2*b^9)/8 + (8*a^3*b^8*d
)/c - (159*a^4*b^7*d^2)/(8*c^2) + (45*a^5*b^6*d^3)/(4*c^3))) + (a*b^8*(a +
b/x)^(1/2)*(a^3)^(1/2)*8i)/(8*a^3*b^8 + (5*a^2*b^9*c)/(8*d) - (159*a^4*b^7*
d)/(8*c) + (45*a^5*b^6*d^2)/(4*c^2)) - (a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/
2)*159i)/(8*(8*a^3*b^8*c - (159*a^4*b^7*d)/8 + (5*a^2*b^9*c^2)/(8*d) + (45*
a^5*b^6*d^2)/(4*c))) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*45i)/(4*(8*
a^3*b^8*c^2 + (45*a^5*b^6*d^2)/4 + (5*a^2*b^9*c^3)/(8*d) - (159*a^4*b^7*c*d
)/8)))*(6*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^4 - (((a + b/x)^(3/2)*(b^4*c^3 - 2
4*a^3*b*d^3 + 32*a^2*b^2*c*d^2 - 9*a*b^3*c^2*d))/(4*c^3*d) - (b*(a + b/x)^(
5/2)*(b^2*c^2 - 12*a^2*d^2 + 7*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(1/2)*(12*a
^4*d^3 - a*b^3*c^3 + 14*a^2*b^2*c^2*d - 25*a^3*b*c*d^2))/(4*c^3*d))/((a + b
/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d
^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) + (log(- (5*a^2*b^9*c^6
 + 1728*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^
4*d^2 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d) - ((((a + b
/x)^(1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4
*d^2 - 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6
*d) - (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16
*c^9*d) - ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(8*c^10*d^4))*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*
c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*c))
^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3) - (log(((((a + b/x)^(
1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4*d^2
- 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6*d) +
 (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16*c^9*
d) + ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(64*c^10*d^4))*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3))*(d^3*(a*d - b*c))^
(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3) - (5*a^2*b^9*c^6 + 17
28*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^4*d^2
 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d))*(d^3*(a*d - b*c
))^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.245 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=126

$$-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

```
[Out] -c^2*(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-1/3*d*(-4*a^2*d^2+
18*a*b*c*d+6*b^2*c^2+b*d*(2*a*d+3*b*c)/x)*(a+b/x)^(1/2)/a/b^2+c*(c+d/x)^2*x
*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 147, 63, 208}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] -(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*
a*d))/x))/(3*a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(bc-6ad)-\frac{1}{2}d(3bc+2ad)x\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\left(c^2(bc-6ad)\right)\text{S}}{}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\left(c^2(bc-6ad)\right)\text{S}}{}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc-6ad)\tan}{a^{3/2}}$$

Mathematica [A]   time = 0.15, size = 95, normalized size = 0.75

$$\frac{c^2(6ad-bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a+\frac{b}{x}}\left(4a^2d^3x-2abd^2(9cx+d)+3b^2c^3x^2\right)}{3ab^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a*
b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

fricas [A]   time = 0.98, size = 233, normalized size = 1.85

$$\left[-\frac{3\left(b^3c^3-6\,ab^2c^2d\right)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,ab^2c^3x^2-2\,a^2bd^3-2\left(9\,a^2bcd^2-2\,a^3d^3\right)x\right)\sqrt{\frac{ax}{x}}}{6\,a^2b^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((
a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*
a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d
)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b^2*c^3*x^2 - 2*a^
2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x)]
```

giac [A] time = 0.20, size = 158, normalized size = 1.25

$$-\frac{\frac{3 b^2 c^3 \sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a} - \frac{3\left(b^2c^3-6\,abc^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{2\left(9\,b^3cd^2\sqrt{\frac{ax+b}{x}} - 3\,ab^2d^3\sqrt{\frac{ax+b}{x}} + \frac{(ax+b)b^2d^3\sqrt{\frac{ax+b}{x}}}{x}\right)}{b^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - 3*(b^2*c^3 - 6*a*
b*c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a) + 2*(9*b^3*c*d^2*s
qrt((a*x + b)/x) - 3*a*b^2*d^3*sqrt((a*x + b)/x) + (a*x + b)*b^2*d^3*sqrt((
a*x + b)/x)/x)/b^3)/b
```

maple [B] time = 0.06, size = 535, normalized size = 4.25

$$\sqrt{\frac{ax+b}{x}}\,\left(3a^3b\,d^3x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right) - 3a^3b\,d^3x^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right) - 9a^2b^2c\,d^2x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*x^3*a^3*b*d^3-9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*x^3*a^2*b^2*c*d^2+9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)*x^3*a*b^3*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x
^3*b^4*c^3-6*(a*x^2+b*x)^(1/2)*a^(7/2)*x^3*d^3+18*(a*x^2+b*x)^(1/2)*a^(5/2)
*x^3*b*c*d^2+18*(a*x^2+b*x)^(1/2)*a^(3/2)*x^3*b^2*c^2*d-3*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b*d^3+9*ln(1/2*(2*a*x+b+2*(a*x
^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b^2*c*d^2+9*ln(1/2*(2*a*x+b+2*(a*x^
2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a*b^3*c^2*d-6*a^(7/2)*((a*x+b)*x)^(1/2)*
x^3*d^3+18*a^(5/2)*((a*x+b)*x)^(1/2)*x^3*b*c*d^2-18*a^(3/2)*((a*x+b)*x)^(1/
2)*x^3*b^2*c^2*d+6*a^(1/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^3+12*(a*x^2+b*x)^(3/
```

```
2)*a^(5/2)*x*d^3-36*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*c*d^2-4*d^3*(a*x^2+b*x)^(
3/2)*b*a^(3/2))/((a*x+b)*x)^(1/2)/b^3/a^(3/2)
```

maxima [A] time = 1.33, size = 166, normalized size = 1.32

$$\frac{1}{2}\,c^{3}\left(\frac{2\,\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^{2}}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{2}{3}\,d^{3}\left(\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{b^{2}}-\frac{3\,\sqrt{a+\frac{b}{x}}\,a}{b^{2}}\right)-\frac{3\,c^{2}d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{6\,\sqrt{a+\frac{b}{x}}\,cd^{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2/3*d^3*((a + b/x)^(3/2)/b^2 -
3*sqrt(a + b/x)*a/b^2) - 3*c^2*d*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/sqrt(a) - 6*sqrt(a + b/x)*c*d^2/b
```

mupad [B] time = 1.73, size = 107, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3\,b^2}+\frac{c^3\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d-b\,c\right)\,1{}\mathrm{i}}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (2*d^3*(a + b
/x)^(3/2))/(3*b^2) + (c^3*x*(a + b/x)^(1/2))/a - (c^2*atan(((a + b/x)^(1/2)
*1i)/a^(1/2))*(6*a*d - b*c)*1i)/a^(3/2)
```

sympy [A] time = 90.00, size = 386, normalized size = 3.06

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^{3}x^{2}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^{3}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^{3}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{4a^{4}bd^{3}x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}-\frac{4a^{3}b^{2}d^{3}x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^{3}x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^{4}x^{\frac{3}{2}}}+3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3
*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*a*
*(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d**3
*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*
a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**
(3/2)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b
, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)/a - 6*c**2*d*atan(1/(sqrt(-
1/a)*sqrt(a + b/x)))/(a*sqrt(-1/a)) - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))
/a**(3/2)
```

3.246 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=73

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

```
[Out] -c*(-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)-2*d^2*(a+b/x)^(1/2)/
b+c^2*x*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 89, 80, 63, 208}

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(bc-4ad)+ad^2x}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 66, normalized size = 0.90

$$\frac{c(4ad-bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}\left(bc^2x-2ad^2\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.93, size = 158, normalized size = 2.16

$$\left[-\frac{\left(b^2c^2-4\,abcd\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(abc^2x-2\,a^2d^2\right)\sqrt{\frac{ax+b}{x}}}{2\,a^2b},\frac{\left(b^2c^2-4\,abcd\right)\sqrt{-a}\,\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^2
- 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x - 2
*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

giac [A] time = 0.18, size = 99, normalized size = 1.36

$$-\frac{\frac{b^2c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}+2\,d^2\sqrt{\frac{ax+b}{x}}-\frac{\left(b^2c^2-4\,abcd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) + 2*d^2*sqrt((a*x + b)/x)
 - (b^2*c^2 - 4*a*b*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a))/b
```

maple [B] time = 0.06, size = 348, normalized size = 4.77

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(a^{2}b\,d^{2}x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-a^{2}b\,d^{2}x^{2}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-2a\,b^{2}cd\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1
/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*
x^2*a*b^2*c*d+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^3
*c^2-2*(a*x^2+b*x)^(1/2)*a^(5/2)*x^2*d^2-4*(a*x^2+b*x)^(1/2)*a^(3/2)*x^2*b*
c*d-ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b*d^2-2*l
n(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a*b^2*c*d-2*a^(5/2
)*((a*x+b)*x)^(1/2)*x^2*d^2+4*a^(3/2)*((a*x+b)*x)^(1/2)*x^2*b*c*d-2*a^(1/2)
*((a*x+b)*x)^(1/2)*x^2*b^2*c^2+4*(a*x^2+b*x)^(3/2)*a^(3/2)*d^2)/((a*x+b)*x)
^(1/2)/b^2/a^(3/2)
```

maxima [B] time = 1.24, size = 129, normalized size = 1.77

$$\frac{1}{2}\,c^{2}\left(\frac{2\,\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^{2}}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{2\,cd\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\,\sqrt{a+\frac{b}{x}}\,d^{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2*c*d*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) - 2*sqrt(a + b/x)*d^2/b
```

mupad [B] time = 1.62, size = 63, normalized size = 0.86

$$\frac{c^{2}\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{2\,d^{2}\,\sqrt{a+\frac{b}{x}}}{b}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-b\,c)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(1/2),x)
```

```
[Out] (c^2*x*(a + b/x)^(1/2))/a - (2*d^2*(a + b/x)^(1/2))/b + (c*atanh((a + b/x)^
(1/2)/a^(1/2))*(4*a*d - b*c))/a^(3/2)
```

sympy [A] time = 83.77, size = 114, normalized size = 1.56

$$d^2\left(\begin{cases}-\frac{1}{\sqrt{a}\,x} & \text{for } b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}\,c^2\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{4cd\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sq
rt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - 4*c*d*atan(1/(sqrt(-1/a)*sqrt(a + b/
x)))/(a*sqrt(-1/a)) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

3.247 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal. Leaf size=51

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+c*x*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {375, 78, 63, 208}

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3
/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{c+dx}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(-\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\ &= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(2\left(-\frac{bc}{2}+ad\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\ &= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.04

$$\frac{2\left(ad-\frac{bc}{2}\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a + (2*(-1/2*(b*c) + a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
])/a^(3/2)
```

fricas [A] time = 0.98, size = 115, normalized size = 2.25

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(bc-2\,ad)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2},\frac{acx\sqrt{\frac{ax+b}{x}}+(bc-2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/a^2]
```

giac [A] time = 0.20, size = 78, normalized size = 1.53

$$-\frac{\frac{b^2c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}-\frac{\left(b^2c-2\,abd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - (b^2*c - 2*a*b*d)*arctan(
sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a))/b
```

maple [B] time = 0.06, size = 173, normalized size = 3.39

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(abd\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+abd\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-b^2c\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}$$
$$2\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*a^(3/2)*(a*x^2+b*x)^(1/2)*d-2*a^(3/2)*((a*x+b)*x
)^(1/2)*d+2*a^(1/2)*((a*x+b)*x)^(1/2)*b*c+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*a*b*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(
1/2))*b^2*c+ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*a*b*d)/((
a*x+b)*x)^(1/2)/b/a^(3/2)
```

maxima [B] time = 1.30, size = 109, normalized size = 2.14

$$\frac{1}{2}\,c\left(\frac{2\,\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqrt(
a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - d*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.98, size = 88, normalized size = 1.73

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\,c\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(1/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2) + (2*c*x*((3*b^(1/2)*(b + a*x)
^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2
)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/2))
```

sympy [A] time = 60.25, size = 82, normalized size = 1.61

$$\frac{\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{2d\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - 2*d*atan(1/(sqrt(-1/a)*sqrt(a + b/x))
)/(a*sqrt(-1/a)) - b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

3.248 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=43

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)+x*(a+b/x)^(1/2)/a
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\sqrt{a+\frac{b}{x}}\,x}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\\ &= \frac{\sqrt{a+\frac{b}{x}}\,x}{a}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a}\\ &= \frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x],x]
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.93, size = 98, normalized size = 2.28

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}\,b\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}}+\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a^2]
```

giac [B] time = 0.24, size = 71, normalized size = 1.65

$$-\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\,a^{\frac{3}{2}}}+\frac{b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx}\right)\sqrt{a}\,-b\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}+\frac{\sqrt{ax^2+bx}}{a\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) - b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

maple [A] time = 0.05, size = 71, normalized size = 1.65

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(-b\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{(ax+b)\,x}\,\sqrt{a}\right)x}{2\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*((a*x+b)*x)^(1/2)*a^(1/2)-b*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(3/2)
```

maxima [A] time = 1.21, size = 67, normalized size = 1.56

$$\frac{\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.44, size = 66, normalized size = 1.53

$$\frac{2x\left(\frac{3\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\operatorname{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\sqrt{\frac{a\,x}{b}+1}}{3\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(1/2),x)
```

```
[Out] (2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*
1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/
2))
```

sympy [A] time = 3.09, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/
2)
```

3.249 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=108

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

```
[Out] -(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^2-2*d^(3/2)*arctan(d^
(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(1/2)+x*(a+b/x)^(1/2)/
a/c
```

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 103, 156, 63, 208, 205}

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/(a^(3/2)*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+2ad)+\frac{bdx}{2}}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2} + \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{abc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

Mathematica [A] time = 0.25, size = 104, normalized size = 0.96

$$\frac{-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}}{a}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
 - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2))/c^2
```

fricas [A] time = 0.99, size = 542, normalized size = 5.02

$$\left[\frac{2\,a^2 d\sqrt{-\frac{d}{bc-ad}}\,\log\left(-\frac{2\,(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\,\sqrt{\frac{ax+b}{x}}-bd+(bc-2\,ad)x}{cx+d}\right)+2\,acx\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}\right.}{2\,a^2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d
))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((a*
x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d
/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + a*c*x
*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/
x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt((a*x + b
)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
)/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*
c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b)/x) - (b*c
 + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2)]
```

giac [A] time = 0.18, size = 134, normalized size = 1.24

$$-b^2\left(\frac{2\,d^2\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,b^2c^2}+\frac{\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)abc}-\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ab^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b^2*(2*d^2*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a
*d^2)*b^2*c^2) + sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a*b*c) - (b*c + 2*a*d
)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b^2*c^2))
```

maple [B] time = 0.06, size = 228, normalized size = 2.11

$$-\frac{\left(2a^{\frac{3}{2}}d^2\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{\frac{(ad-bc)d}{c^2}}\,acd\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+\sqrt{\frac{(ad-bc)d}{c^2}}\,b\,c^2\ln\left(\frac{2ax+b}{}\right.\right.}{2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*(2*a^(3/2)*d^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+
b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(1/2)
*c^2+2*((a*d-b*c)/c^2*d)^(1/2)*a*c*d*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
```

```
(1/2))/a^(1/2))+((a*d-b*c)/c^2*d)^(1/2)*b*c^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2)))*x*((a*x+b)/x)^(1/2)/((a*d-b*c)/c^2*d)^(1/2)/c^3/a
^(3/2)/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

mupad [B] time = 1.98, size = 1183, normalized size = 10.95

$$\frac{x\sqrt{a+\frac{b}{x}}}{a\,c}-\frac{\mathrm{atanh}\left(\frac{12\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{12\,b^4\,d^4}{a}+\frac{10\,b^5\,c\,d^3}{a^2}+\frac{2\,b^6\,c^2\,d^2}{a^3}\right)}+\frac{10\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{10\,b^5\,d^3}{a}+\frac{12\,b^4\,d^4}{c}+\frac{2\,b^6\,c\,d^2}{a^2}\right)}+\frac{2\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{2\,b^6\,d^2}{a}+\frac{10\,b^5\,d^3}{c}+\frac{12\,a\,b^4\,d^4}{c^2}\right)}\right)\,(2\,a\,d+b}{c^2\,\sqrt{a^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)),x)
```

```
[Out] (x*(a + b/x)^(1/2))/(a*c) - (atan((((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*
d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2
)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(
1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 +
 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d) -
 (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^
5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^
2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a +
b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4
 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d))/((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b
```

```
^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/
x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c
*d^3)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^
2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*
d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b^4*c^5*d^2 + 2*
a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a
 + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4
- b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b
^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a
*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2)*2i)/(b*c^3 - a*c^2*d) - (atanh((12*b^4*d^
4*(a + b/x)^(1/2))/((a^3)^(1/2)*((12*b^4*d^4)/a + (10*b^5*c*d^3)/a^2 + (2*b
^6*c^2*d^2)/a^3)) + (10*b^5*d^3*(a + b/x)^(1/2))/((a^3)^(1/2)*((10*b^5*d^3)
/a + (12*b^4*d^4)/c + (2*b^6*c*d^2)/a^2)) + (2*b^6*d^2*(a + b/x)^(1/2))/((a
^3)^(1/2)*((2*b^6*d^2)/a + (10*b^5*d^3)/c + (12*a*b^4*d^4)/c^2)))*(2*a*d +
b*c))/(c^2*(a^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

3.250 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

```
[Out] -d^(3/2)*(-4*a*d+5*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/
(-a*d+b*c)^(3/2)-(4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^3+d*(
-2*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d
/x)
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 151, 156, 63, 208, 205}

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b
/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+4ad)+\frac{3bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(bc+4ad)-\frac{1}{2}bd(bc-2ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac^2(bc-ad)}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)} + \frac{(b\cdots}{\cdots}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3(bc-ad)} \cdots$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh\cdots}{a^{3/2}c\cdots}$$

Mathematica [A] time = 0.78, size = 150, normalized size = 0.87

$$\frac{\frac{ad^{3/2}(4ad-5bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{cx\sqrt{a+\frac{b}{x}}\,(bc(cx+d)-ad(cx+2d))}{(cx+d)(bc-ad)} - \frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b*c*(d + c*x) - a*d*(2*d + c*x)))/((b*c - a*d)*(d + c*
x)) + (a*d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
Sqrt[a])/(a*c^3)
```

fricas [A] time = 1.07, size = 1163, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2
*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^2*
b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))
*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c
- 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*
c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*
c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^
2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a
*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3
 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*
sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x),
 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-(
2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d
)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*
x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*
x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/
(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a
*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*
d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - ((a*b*c
^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*
b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x)]
```

giac [A] time = 0.22, size = 300, normalized size = 1.74

$$-b^3\left(\frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^4-ab^3c^3d\right)\sqrt{bcd-ad^2}}+\frac{b^2c^2\sqrt{\frac{ax+b}{x}}-2\,abcd\sqrt{\frac{ax+b}{x}}+2\,a^2d^2\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)bcd\sqrt{\frac{ax+b}{x}}}{x}-\frac{2\,(ax+b)a}{}}{\left(ab^3c^3-a^2b^2c^2d\right)\left(abc-a^2d-\frac{(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}-\frac{(ax+b)^2d}{x^2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b^3*((5*b*c*d^2 - 4*a*d^3)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))
/((b^4*c^4 - a*b^3*c^3*d)*sqrt(b*c*d - a*d^2)) + (b^2*c^2*sqrt((a*x + b)/x)
```

```
 - 2*a*b*c*d*sqrt((a*x + b)/x) + 2*a^2*d^2*sqrt((a*x + b)/x) + (a*x + b)*b*
c*d*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a*d^2*sqrt((a*x + b)/x)/x)/((a*b^3*c^
3 - a^2*b^2*c^2*d)*(a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (
a*x + b)^2*d/x^2)) - (b*c + 4*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt
(-a)*a*b^3*c^3))
```

maple [B] time = 0.07, size = 1135, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(9/2)*c*d^4*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b
*c)/c^2*d)^(1/2)*a^(7/2)*c^4*d*x^2+4*a^(9/2)*d^5*ln((-2*a*d*x+b*c*x-b*d+2*(
(a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^2*x-9*a^(7/2)*b*c^2*d^3*x*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-4*((a*x+b
)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^2*d^3-9*a^(7/2)*b*c*d^4*ln((-2
*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*
((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*c^4*d+6*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^4*d*x+5*a^(5/2)*b^2*c^3*d^2*x*ln((-2*a
*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+6*((
a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^2+5*a^(5/2)*b^2*c^2
*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/
(c*x+d))-2*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^5-2*a^
(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d+4*((a*d-b*c)/c^2*
d)^(1/2)*a^4*c^2*d^3*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)-7*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^3*d^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(
1/2)*a^(1/2))/a^(1/2))+2*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4*d+((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c
^5*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+4*((a*d-b*c)/c^2
*d)^(1/2)*a^4*c*d^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-7
*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))+2*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d^2+((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c^4*d*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)))/c^4/((a*x+b)*x)^(1/2
)/(a*d-b*c)^2/(c*x+d)/a^(5/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

mupad [B] time = 3.54, size = 3813, normalized size = 22.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)
```

```
[Out] (((a + b/x)^(1/2)*(b^3*c^2 + 2*a^2*b*d^2 - 2*a*b^2*c*d))/(c^2*(a^2*d - a*b*
c)) + (d*(a + b/x)^(3/2)*(b^2*c - 2*a*b*d))/(c^2*(a^2*d - a*b*c)))/((a + b/
x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atan(((((2*(a + b/x)^(
1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 2
6*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^
6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^
2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(
4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6
*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*
d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/2)) + (((2
*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b
^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)
 - (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*
c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*
a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8
*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^
5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(
1/2)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*d^5 - 48*a^2*b^4*c
*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - (((2*(a + b/x)^(1/2)*(
32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*
b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^6*c^9*
d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*
c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*
b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))
/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*
c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)) + (((2*(a + b/x
)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6
+ 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - (((4*a
*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/
(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*a*d + b*c
)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*
c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4
```

```
*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2))))*(4*a
*d + b*c)*1i)/(c^3*(a^3)^(1/2)) - (atan((((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d
- 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^
4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*
a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2
 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8
 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2
)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*
d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3
*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*
c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3
+ 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5
*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 -
 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3
*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 +
4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 +
a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(
4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4
 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^
6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3
*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3
*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a
*b^5*c^2*d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*
d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4
*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^
2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^
(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^
7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((
d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2
 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*
c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2
 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*
c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + (
(d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d
^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5)
)/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*
(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4
+ 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a
*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*
a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 +
a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a
*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)
)))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))*(d^3*(a
*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*1i)/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4
*d^2 - 3*a*b^2*c^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

```
[Out] -1/4*d^(3/2)*(24*a^2*d^2-56*a*b*c*d+35*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2
)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(5/2)-(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(3/2)/c^4+1/2*d*(-3*a*d+2*b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c
+d/x)^2+1/4*d*(-4*a*d+b*c)*(-3*a*d+4*b*c)*(a+b/x)^(1/2)/a/c^3/(-a*d+b*c)^2/
(c+d/x)+x*(a+b/x)^(1/2)/a/c/(c+d/x)^2
```

Rubi [A] time = 0.40, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 151, 156, 63, 208, 205}

$$-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b
*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)
) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d
 + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c
 - a*d)^(5/2)) - ((b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^
4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+6ad)+\frac{5bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-(bc-ad)(bc+6ad)-\frac{3}{2}bd(2bc-3ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{(bc-\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\,\text{Su}\cdots}{\cdots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\,\text{Su}\cdots}{\cdots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/2}\left(35b^2c^2-\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.75, size = 216, normalized size = 0.86

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\left(2a^2d^2\left(2c^2x^2+9cdx+6d^2\right)-abcd\left(8c^2x^2+29cdx+19d^2\right)+4b^2c^2(cx+d)^2\right)}{(cx+d)^2(bc-ad)^2} - \frac{ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{4(6ad+bc)\tan\cdots}{\sqrt{\cdots}}}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x + 2
*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/((b*c - a*d)^2*(d + c
*x)^2) - (a*d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(4*a*c^4)
```

fricas [B] time = 1.94, size = 2307, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*
c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d
+ 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 +
 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 +
2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c -
a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
(b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2
)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*
c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c
^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4
*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), 1/8*(
8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*
b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*s
qrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (
35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c
^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^
2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^
2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*
b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 +
 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^
2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2
+ 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c
*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt
((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*
c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2
*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4
```

```
)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*(a*b^2*c^5
 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 1
8*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)
*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2
*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^
6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^
4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*
a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*a
rctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) -
 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a
*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*
sqrt((a*x + b)/x)/a) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (
8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 -
 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 -
2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)
*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x)]
```

giac [A] time = 0.29, size = 352, normalized size = 1.41

$$-\frac{1}{4}\,b^4\left(\frac{\left(35\,b^2c^2d^2-56\,abcd^3+24\,a^2d^4\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^6c^6-2\,ab^5c^5d+a^2b^4c^4d^2\right)\sqrt{bcd-ad^2}}+\frac{13\,b^2c^2d^2\sqrt{\frac{ax+b}{x}}-21\,abcd^3\sqrt{\frac{ax+b}{x}}+8\,a^2d^4\sqrt{\frac{ax+b}{x}}+}{\left(b^5c^5-2\,ab^4c^4d+a^2b^3c^3d^2\right)\left(bc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^4*((35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 24*a^2*d^4)*arctan(d*sqrt((a*x +
 b)/x)/sqrt(b*c*d - a*d^2))/((b^6*c^6 - 2*a*b^5*c^5*d + a^2*b^4*c^4*d^2)*sq
rt(b*c*d - a*d^2)) + (13*b^2*c^2*d^2*sqrt((a*x + b)/x) - 21*a*b*c*d^3*sqrt(
(a*x + b)/x) + 8*a^2*d^4*sqrt((a*x + b)/x) + 11*(a*x + b)*b*c*d^3*sqrt((a*x
 + b)/x)/x - 8*(a*x + b)*a*d^4*sqrt((a*x + b)/x)/x)/((b^5*c^5 - 2*a*b^4*c^4
*d + a^2*b^3*c^3*d^2)*(b*c - a*d + (a*x + b)*d/x)^2) + 4*sqrt((a*x + b)/x)/
((a - (a*x + b)/x)*a*b^3*c^3) - 4*(b*c + 6*a*d)*arctan(sqrt((a*x + b)/x)/sq
rt(-a))/(sqrt(-a)*a*b^4*c^4))
```

maple [B] time = 0.07, size = 2269, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(60*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d^4-12*a^2*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d^3+12*a^
(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^5*d^2-12*a^(7/2)*((a*
x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^5*d^2-80*a^(9/2)*ln((-2*a*d*x+b*c
*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*c^3*d^
4+91*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(
1/2)*c)/(c*x+d))*x^2*b^2*c^4*d^3-35*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^5*d^2+24*a^5*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x
^2*c^3*d^4+18*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^5*d^2-3
6*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^4-160*a^(9/2)*l
n((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d
))*x*b*c^2*d^5-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^7+8*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*b^3*c^5*d^2-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(
1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^5*d^2+182*a^(7/2)*ln((-2*a*d*x+b*c*x-b*
d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^4-70*
a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*
c)/(c*x+d))*x*b^3*c^4*d^3+48*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^5+62*a^(7/2)*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^4-46*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c
^2*d)^(1/2)*b^2*c^4*d^3-68*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))
/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b*c^2*d^5+8*a^(3/2)*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^7+24*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^7-92*a^(5/2)*((a*x+b)*x)
^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d^2-136*a^4*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d^4+120*a^3
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1
/2)*x*b^2*c^4*d^3-24*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^5*d^2+60*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^5*d^2-12*a^2*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/
2)*x^2*b^3*c^6*d+102*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*
c^4*d^3+22*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^6*d+18*a
^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^5*d^2-46*a^(5/2)*(
(a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^6*d-68*a^4*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*
d^3-22*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^6*d-8*a*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*x*b^4*c^6*d+16*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^6
*d-24*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^5-80*a^(9/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*b*c*d^6+91*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d^5-35*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*
```

```
((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c^3*d^4+24*a^5*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*c*d^6+24*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b
)*x)^(1/2)*c)/(c*x+d))*x^2*c^2*d^5-8*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c
^2*d)^(1/2)*c^4*d^3+48*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^
(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^6)/c^5/((a*x+b)*x)^(1/2)/(a*d-b*c
)^3/(c*x+d)^2/a^(5/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

mupad [B] time = 5.48, size = 2890, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^3),x)
```

```
[Out] (log((d^3*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2) - a^3*d^4 + b^3*c^3*d - 3*a*
b^2*c^2*d^2 + 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(3*a^2*d^2 + (35*b^2
*c^2)/8 - 7*a*b*c*d))/(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3
*c^7*d^2 - 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d) - ((b*(a + b/x)^(5/2)*(12*a^
2*d^4 + 4*b^2*c^2*d^2 - 19*a*b*c*d^3))/(4*a*c^3*(a*d - b*c)^2) - ((a + b/x)
^(1/2)*(4*b^4*c^3 - 12*a^3*b*d^3 + 25*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(4*a
*c^3*(a*d - b*c)) + (d*(a + b/x)^(3/2)*(8*b^4*c^3 - 24*a^3*b*d^3 + 56*a^2*b
^2*c*d^2 - 37*a*b^3*c^2*d))/(4*c^3*(a^2*d - a*b*c)*(a*d - b*c)))/((a + b/x)
^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*
(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((d^3*(a*d - b*c)^5)
^(1/2)*(a + b/x)^(1/2) + a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*
d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*b*c*d))/(8*(
b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3*c^7*d^2 - 10*a^3*b^2*c
^6*d^3 - 5*a*b^4*c^8*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16
*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^
5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^
6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^
8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4
```

```
*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)
*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^
11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7)
)/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5
*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*
d + b*c)*1i)/(2*c^4*(a^3)^(1/2)) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 1
6*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d
^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c
^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^
5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 -
4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2
)*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c
^11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7
))/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^
5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a
*d + b*c)*1i)/(2*c^4*(a^3)^(1/2)))/((216*a^5*b^3*d^9 + (35*b^8*c^5*d^4)/2 -
 (49*a*b^7*c^4*d^5)/8 - 810*a^4*b^4*c*d^8 - (1877*a^2*b^6*c^3*d^6)/4 + 1044
*a^3*b^5*c^2*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*
c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*
b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5
 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6
*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^8
*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
- 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*
a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)*
(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^1
1*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))
/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*
b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d
 + b*c))/(2*c^4*(a^3)^(1/2)) - ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*b^
8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5 -
 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6*d
^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b^8*c
^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5 -
49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^
3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2)*(6
*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^11*
d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))/(
16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*
c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d +
 b*c))/(2*c^4*(a^3)^(1/2))))*(6*a*d + b*c)*1i)/(c^4*(a^3)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] Timed out
```

3.252 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] -3*c^2*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+((-2*a*d+b*c)*(2
*a^2*d^2-2*a*b*c*d+3*b^2*c^2)-a*b*d^2*(2*a*d+b*c)/x)/a^2/b^2/(a+b/x)^(1/2)+
c*(c+d/x)^2*x/a/(a+b/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 146, 63, 208}

$$\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d)
)/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) - (3*c^2
*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\mathrm{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\mathrm{Subst}\left(\int \frac{(c+dx)\left(\frac{3}{2}c(bc-2ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\mathrm{Subst}\left(\int\right.}{2a^2}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\mathrm{Subst}\left(\int\right.}{a^2b}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a\cdots}}{\sqrt{\cdots}}\right.}{a^{5/2}}$$

Mathematica [C] time = 0.07, size = 92, normalized size = 0.70

$$\frac{a\left(-4a^2d^3x-2abd^2(d-3cx)+b^2c^3x^2\right)+3b^2c^2x(bc-2ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)}{a^2b^2x\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (a*(-4*a^2*d^3*x + b^2*c^3*x^2 - 2*a*b*d^2*(d - 3*c*x)) + 3*b^2*c^2*(b*c -
2*a*d)*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2*b^2*Sqrt[a + b/
x]*x)
```

fricas [A] time = 0.98, size = 336, normalized size = 2.55

$$\left[-\frac{3\left(b^4c^3-2\,ab^3c^2d+\left(ab^3c^3-2\,a^2b^2c^2d\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^2b^2c^3x^2-2\,a^3bd^3+\left(3\,ab^3\cdots\right.\right.}{2\left(a^4b^2x+a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(a
)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2*a
^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*s
qrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b
^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a)
+ (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b
*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]
```

giac [A] time = 0.21, size = 222, normalized size = 1.68

$$-\frac{\frac{2\,d^3\sqrt{\frac{ax+b}{x}}}{b}-\frac{3\left(b^2c^3-2\,abc^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}-\frac{2\,ab^3c^3-6\,a^2b^2c^2d+6\,a^3bcd^2-2\,a^4d^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{6\,(ax+b)ab^2c^2d}{x}-\frac{6\,(ax+b)a^2bcd^2}{x}+\frac{2\,(ax+b)a^3d^3}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*d^3*sqrt((a*x + b)/x)/b - 3*(b^2*c^3 - 2*a*b*c^2*d)*arctan(sqrt((a*x +
b)/x)/sqrt(-a))/(sqrt(-a)*a^2) - (2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c
*d^2 - 2*a^4*d^3 - 3*(a*x + b)*b^3*c^3/x + 6*(a*x + b)*a*b^2*c^2*d/x - 6*(a
*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x)/((a*sqrt((a*x + b)/x) - (a*x
 + b)*sqrt((a*x + b)/x)/x)*a^2*b))/b
```

maple [B] time = 0.07, size = 969, normalized size = 7.34

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(3a^4b^2c\,d^2x^4\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-3a^4b^2c\,d^2x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-6a^3b^3c^2d\,x^4\ln\left(\frac{2ax+b+2\sqrt{(ax+}}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x/a^(5/2)*(3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*x^2*b^6*c^3+4*(a*x^2+b*x)^(3/2)*a^(9/2)*x^2*d^3-4*a^(9/2)*((
a*x+b)*x)^(3/2)*x^2*d^3+4*(a*x^2+b*x)^(3/2)*a^(5/2)*b^2*d^3+6*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^3*c*d^2-12*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b^4*c^2*d-6*ln(1/2*(2*a*x+
```

```
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^3*b^3*c^2*d-3*ln(1/2*(2*a*x+b
+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^4*b^2*c*d^2+3*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^4*b^2*c*d^2-6*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^3*c*d^2+12*a^(3/2)*((a*x+b)*
x)^(1/2)*x^2*b^4*c^2*d-6*(a*x^2+b*x)^(1/2)*a^(9/2)*x^4*b*c*d^2-6*a^(9/2)*((
a*x+b)*x)^(1/2)*x^4*b*c*d^2+12*a^(7/2)*((a*x+b)*x)^(1/2)*x^4*b^2*c^2*d-12*(
a*x^2+b*x)^(1/2)*a^(7/2)*x^3*b^2*c*d^2+12*a^(7/2)*((a*x+b)*x)^(3/2)*x^2*b*c
*d^2-12*a^(5/2)*((a*x+b)*x)^(3/2)*x^2*b^2*c^2*d-12*a^(7/2)*((a*x+b)*x)^(1/2
)*x^3*b^2*c*d^2+24*a^(5/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^2*d-6*(a*x^2+b*x)^(1
/2)*a^(5/2)*x^2*b^3*c*d^2-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*x^2*a*b^5*c^2*d-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2))*x^2*a^2*b^4*c*d^2+3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*x^2*a^2*b^4*c*d^2-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c*d^2+8*(a*x^2+b*x
)^(3/2)*a^(7/2)*x*b*d^3-12*a^(3/2)*((a*x+b)*x)^(1/2)*x^3*b^4*c^3+4*a^(3/2)*
((a*x+b)*x)^(3/2)*x^2*b^3*c^3-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^4*b^3*c^3+3*ln(
1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^2*b^4*c^3+6*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a*b^5*c^3-6*a^(1/2)*((a
*x+b)*x)^(1/2)*x^2*b^5*c^3)/((a*x+b)*x)^(1/2)/b^3/(a*x+b)^2
```

maxima [A] time = 1.26, size = 200, normalized size = 1.52

$$\frac{1}{2}c^3\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-3c^2d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)-2d^3\left(\frac{\sqrt{a+\frac{b}{x}}}{b^2}+\frac{}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 3*c^2*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
+ 2/(sqrt(a + b/x)*a)) - 2*d^3*(sqrt(a + b/x)/b^2 + a/(sqrt(a + b/x)*b^2))
+ 6*c*d^2/(sqrt(a + b/x)*b)
```

mupad [B] time = 1.91, size = 172, normalized size = 1.30

$$\frac{\frac{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\left(2a^3d^3-6a^2bcd^2+6ab^2c^2d-3b^3c^3\right)}{a^2}}{b^2\left(a+\frac{b}{x}\right)^{3/2}-ab^2\sqrt{a+\frac{b}{x}}}-\frac{2d^3\sqrt{a+\frac{b}{x}}}{b^2}+\frac{3c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad-b}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(3/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/a - ((a + b/x)*(2*
a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2))/a^2)/(b^2*(a + b/x)^(
3/2) - a*b^2*(a + b/x)^(1/2)) - (2*d^3*(a + b/x)^(1/2))/b^2 + (3*c^2*atanh(
(a + b/x)^(1/2)/a^(1/2))*(2*a*d - b*c))/a^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

3.253 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}} \, dx$

Optimal. Leaf size=94

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] -c*(-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(2*a^2*d^2+b*c*(-4
*a*d+3*b*c))/a^2/b/(a+b/x)^(1/2)+c^2*x/a/(a+b/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 89, 78, 63, 208}

$$\frac{\frac{c(3bc-4ad)}{a^2}+\frac{2d^2}{b}}{\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] ((2*d^2)/b + (c*(3*b*c - 4*a*d))/a^2)/Sqrt[a + b/x] + (c^2*x)/(a*Sqrt[a + b
/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(3bc-4ad)+ad^2x}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.86

$$\frac{c(4ad-3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2a^2d^2+abc(cx-4d)+3b^2c^2}{a^2b\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x))/(a^2*b*Sqrt[a + b/x]) + (c*(-3
*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.89, size = 272, normalized size = 2.89

$$\left[-\frac{\left(3b^3c^2-4ab^2cd+\left(3ab^2c^2-4a^2bcd\right)x\right)\sqrt{a}\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^2bc^2x^2+\left(3ab^2c^2-4a^2bcd\right.\right.}{2\left(a^4bx+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*lo
g(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^2*
c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2), (
(3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d
+ 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

giac [A] time = 0.23, size = 160, normalized size = 1.70

$$\frac{\frac{\left(3\,b^{2}c^{2}-4\,abcd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{2\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}-\frac{3\,(ax+b)b^{2}c^{2}}{x}+\frac{4\,(ax+b)abcd}{x}-\frac{2\,(ax+b)a^{2}d^{2}}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] ((3*b^2*c^2 - 4*a*b*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2)
+ (2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 - 3*(a*x + b)*b^2*c^2/x + 4*(a*x +
 b)*a*b*c*d/x - 2*(a*x + b)*a^2*d^2/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sq
rt((a*x + b)/x)/x)*a^2))/b
```

maple [B] time = 0.06, size = 789, normalized size = 8.39

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(-a^{4}b\,d^{2}x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+a^{4}b\,d^{2}x^{2}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4a^{3}b^{2}cd\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2))*a^2*b^3*d^2+2*a^(9/2)*((a*x+b)*x)^(1/2)*x^2*d^2+2*a^(9/2)*(a*x^2+b*x)^(
1/2)*x^2*d^2-4*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c^2+2*a^(5/2)*((a*x+b)*x)^(1/2
)*b^2*d^2+2*a^(5/2)*(a*x^2+b*x)^(1/2)*b^2*d^2-ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2))*a^2*b^3*d^2+6*a^(1/2)*((a*x+b)*x)^(1/2)*b^4*c^2-16
*a^(5/2)*((a*x+b)*x)^(1/2)*x*b^2*c*d+8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))*x*a^2*b^3*c*d+4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/
```

```
2))/a^(1/2))*x^2*a^3*b^2*c*d-8*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*c*d-3*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^5*c^2-4*a^(7/2)*((a*x+b)*
x)^(3/2)*d^2-2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^3*
b^2*d^2-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*b^4*c^2
+2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^3*b^2*d^2+4*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^4*c*d+4*a^(7/2)*((a
*x+b)*x)^(1/2)*x*b*d^2+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c^2+4*a^(7/2)*(a*
x^2+b*x)^(1/2)*x*b*d^2-8*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*c*d+6*a^(5/2)*((a*x+
b)*x)^(1/2)*x^2*b^2*c^2+8*a^(5/2)*((a*x+b)*x)^(3/2)*b*c*d-ln(1/2*(2*a*x+b+2
*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^4*b*d^2-3*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b^3*c^2+ln(1/2*(2*a*x+b+2*(a*x^2+b*
x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^4*b*d^2)/a^(5/2)/((a*x+b)*x)^(1/2)/b^2/(a*
x+b)^2
```

maxima [A] time = 1.16, size = 164, normalized size = 1.74

$$\frac{1}{2}\,c^2\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-2\,cd\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)+\frac{2\,d^2}{\sqrt{a+\frac{b}{x}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 2*c*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
2/(sqrt(a + b/x)*a)) + 2*d^2/(sqrt(a + b/x)*b)
```

mupad [B] time = 1.83, size = 120, normalized size = 1.28

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(4\,a\,d-3\,b\,c\right)}{a^{5/2}}-\frac{\frac{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\,\left(2\,a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2\right)}{a^2}}{b\,{\left(a+\frac{b}{x}\right)}^{3/2}-a\,b\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(3/2),x)
```

```
[Out] (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*a*d - 3*b*c))/a^(5/2) - ((2*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/a - ((a + b/x)*(2*a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d))/
a^2)/(b*(a + b/x)^(3/2) - a*b*(a + b/x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

3.254 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] -(-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+(-2*a*d+3*b*c)/a^2/(
a+b/x)^(1/2)+c*x/a/(a+b/x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 51, 63, 208}

$$\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{\left(-\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.63

$$\frac{(3bc-2ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx}{a^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (a*c*x + (3*b*c - 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2
*Sqrt[a + b/x])
```

fricas [A] time = 1.05, size = 210, normalized size = 2.76

$$\left[-\frac{\left(3\,b^2c-2\,abd+\left(3\,abc-2\,a^2d\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^2cx^2+\left(3\,abc-2\,a^2d\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4x+a^3b\right)}, -\frac{(3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sqrt
((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x
)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c*x^2 + (3*a*b*c - 2
*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

giac [A] time = 0.20, size = 127, normalized size = 1.67

$$\frac{\frac{\left(3\,b^2c-2\,abd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2\,ab^2c-2\,a^2bd-\frac{3\,(ax+b)b^2c}{x}+\frac{2\,(ax+b)abd}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] ((3*b^2*c - 2*a*b*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2
*a*b^2*c - 2*a^2*b*d - 3*(a*x + b)*b^2*c/x + 2*(a*x + b)*a*b*d/x)/((a*sqrt(
(a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2))/b
```

maple [B] time = 0.06, size = 387, normalized size = 5.09

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(2a^3bd\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-3a^2b^2c\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+4a^2b^2dx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*x^2*a^3*b*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*x^2*a^2*b^2*c-4*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*d+6*a^(5/2)*((a*x+b)*x
)^(1/2)*x^2*b*c+4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a
^2*b^2*d-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*b^3*c+
4*a^(5/2)*((a*x+b)*x)^(3/2)*d-4*a^(3/2)*((a*x+b)*x)^(3/2)*b*c-8*a^(5/2)*((a
*x+b)*x)^(1/2)*x*b*d+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^2*c+2*ln(1/2*(2*a*x+b
+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^3*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)
```

```
*x)^(1/2)*a^(1/2))/a^(1/2))*b^4*c-4*a^(3/2)*((a*x+b)*x)^(1/2)*b^2*d+6*a^(1/
2)*((a*x+b)*x)^(1/2)*b^3*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^2
```

maxima [B] time = 1.37, size = 144, normalized size = 1.89

$$\frac{1}{2}c\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3)
+ 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) - d
*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(sqr
t(a + b/x)*a))
```

mupad [B] time = 2.44, size = 71, normalized size = 0.93

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\,d}{a\,\sqrt{a+\frac{b}{x}}}+\frac{2\,c\,x\,{\left(\frac{a\,x}{b}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,{\left(a+\frac{b}{x}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(3/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2*d)/(a*(a + b/x)^(1/2)) +
(2*c*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/
x)^(3/2))
```

sympy [B] time = 81.34, size = 224, normalized size = 2.95

$$c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right)+d\left(-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+\right.}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/
b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqrt(
1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x +
a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
 - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/
(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

3.255 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

[Out] -3*b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)+3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b/x)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + b/x]) + (3*Sqrt[a + b/x]*x)/a^2 - (3*b*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2}\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.60

$$
\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{a+\frac{b}{x}}{a}\right)}{a^2\sqrt{a+\frac{b}{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/x)/a])/(a^2*Sqrt[a + b/x])
```

fricas [A] time = 0.77, size = 156, normalized size = 2.60

$$\left[\frac{3\left(abx+b^{2}\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(a^{2}x^{2}+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^{4}x+a^{3}b\right)},\frac{3\left(abx+b^{2}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^{4}x+a^{3}b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b
) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x + b
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*sqr
t((a*x + b)/x))/(a^4*x + a^3*b)]
```

giac [A] time = 0.16, size = 86, normalized size = 1.43

$$b\left(\frac{3\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{2\,a-\frac{3\,(ax+b)}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*(3*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2*a - 3*(a*x + b)
/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2))
```

maple [B] time = 0.06, size = 198, normalized size = 3.30

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(-3a^{2}b\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-6a\,b^{2}x\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{(ax+b)\,x}\,a^{\frac{5}{2}}x^{2}-3b^{3}\ln\left(\frac{2ax+b+2\right.\right.}{2\sqrt{(ax+b)\,x}\,\left(ax+b\right)^{2}a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2-4*a^(3/2)*((a*x+b)
*x)^(3/2)+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^
(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*x*a*b^2+6*a^(1/2)*((a*x+b)*x)^(1/2)*b^2-3*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^3)/a^(5/2)/((a*x+b)*x)^(1/2)/(a*x+b)^
2
```

maxima [A] time = 1.18, size = 85, normalized size = 1.42

$$\frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] (3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

mupad [B] time = 1.87, size = 34, normalized size = 0.57

$$\frac{2\,x\left(\frac{a\,x}{b}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(3/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x)
^(3/2))
```

sympy [A] time = 4.79, size = 71, normalized size = 1.18

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b +
 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

3.256 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=147

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}\,(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

```
[Out] 2*d^(5/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(3/
2)-(2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^2+b*(-a*d+3*b*c)/
a^2/c/(-a*d+b*c)/(a+b/x)^(1/2)+x/a/c/(a+b/x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 152, 156, 63, 208, 205}

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}\,(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt[a + b/x])
 + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c -
a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^2
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+2ad)+\frac{3bdx}{2}}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{4}(bc-ad)(3bc+2ad)+\frac{1}{4}bd(3bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{a^2c(bc-ad)}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)} + \frac{(3bc+2ad)}{}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2(bc-ad)} + \frac{(3bc}{}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}} - \frac{(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{a^{5/2}c^2}$$

Mathematica [C] time = 0.07, size = 106, normalized size = 0.72

$$\frac{(ad-bc)\left((2ad+3bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx\right)-2a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (-2*a^2*d^2*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)] +
 (-(b*c) + a*d)*(a*c*x + (3*b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ b/(a*x)]))/(a^2*c^2*(-(b*c) + a*d)*Sqrt[a + b/x])
```

fricas [B] time = 1.17, size = 1075, normalized size = 7.31

$$\left[\frac{\left(3\,b^3c^2 - ab^2cd - 2\,a^2bd^2 + \left(3\,ab^2c^2 - a^2bcd - 2\,a^3d^2\right)x\right)\sqrt{a}\,\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) - 2\left(a^4d^2x + a^3bd^2\right)\sqrt{\cdots}}{2\left(a^3b^2c^3 - a^4bc^2d + \left(a^4b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a
^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^4*
d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b*c
^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^
2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2*c*d -
 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (a^4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*l
og(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c -
2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*
d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*
d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*
d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^3*c^2 - a*
b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2 - a^3*c*d)*x^2
 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2
*d + (a^4*b*c^3 - a^5*c^2*d)*x), (2*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) + (3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((a^2*b*c^2 - a
^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3
- a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x)]
```

giac [A] time = 0.21, size = 200, normalized size = 1.36

$$\left(\frac{2\,d^3\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^3 - ab^2c^2d\right)\sqrt{bcd-ad^2}} + \frac{2\,abc - \frac{3\,(ax+b)bc}{x} + \frac{(ax+b)ad}{x}}{\left(a^2b^2c^2 - a^3bcd\right)\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)} + \frac{(3\,bc + 2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2b^2c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] (2*d^3*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^3*c^3 - a*b^2*c^
2*d)*sqrt(b*c*d - a*d^2)) + (2*a*b*c - 3*(a*x + b)*b*c/x + (a*x + b)*a*d/x)
/((a^2*b^2*c^2 - a^3*b*c*d)*(a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)
/x)/x)) + (3*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*
b^2*c^2))*b^2
```

maple [B] time = 0.07, size = 962, normalized size = 6.54

$$-\frac{\left(2a^{\frac{9}{2}}d^3x^2\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{\frac{(ad-bc)d}{c^2}}\,a^4c\,d^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+\sqrt{\frac{(ad-bc)d}{c^2}}\,a^3b\,c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] -1/2*(2*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x
)^(1/2)*c)/(c*x+d))*x^2*d^3-2*a^(7/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(
1/2)*x^2*c^2*d+4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*d^3+6*a^(5/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*
x+b)*x)^(1/2)*x^2*b*c^3-4*a^(5/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)
*x*b*c^2*d+2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x
+b)*x)^(1/2)*c)/(c*x+d))*b^2*d^3-4*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)
*x)^(3/2)*b*c^3+12*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*
c^3+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*
d)^(1/2)*x^2*a^4*c*d^2+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^3*b*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(
1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^2*b^2*c^3-2*a^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^2*d+4*ln(1/2*(2*a*x+b+2*((a*x
+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^3*b*c*d^2+2*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x
*a^2*b^2*c^2*d-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*
d-b*c)/c^2*d)^(1/2)*x*a*b^3*c^3+6*a^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*
x)^(1/2)*b^3*c^3+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*a^2*b^2*c*d^2+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3)*x*((a*x+b)
/x)^(1/2)/a^(5/2)/(a*x+b)^2/((a*d-b*c)/c^2*d)^(1/2)/c^3/(a*d-b*c)/((a*x+b)*
x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

mupad [B] time = 2.68, size = 3000, normalized size = 20.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)),x)
```

```
[Out] (atan((((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 6
6*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c
^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) +
 ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2 - 13
2*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a^14*b
^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^
2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 +
200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*
d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5*(a*d - b
*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 6
8*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c
^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)
^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^10*d^4 -
 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 + ((d^5*(a
*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*
d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6
- 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))/(c^2*(a
*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^5 - 96*a^7*b^7*c^6
*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10*b^4*c^3*d^9 + 8*a^
11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c
^10*d^3 - 66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62
*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*
c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^
12*d^2 - 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6
 + 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*
b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*
c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8
))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3) + ((d^5*
(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9
*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^
11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d -
 b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^
```

```
10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 +
((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b
^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*
c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))
/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))*(d^5*(a*d - b*c)^3)^(1/2)*2i)/
(c^2*(a*d - b*c)^3) - (atanh((54*a^5*b^11*c^10*d^2*(a + b/x)^(1/2))/((a^5)^
(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 +
124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b
^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (216*a^6*b^10*c^9*d^3*(a + b/x)^(1/2))
/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^
8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 1
10*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (234*a^7*b^9*c^8*d^4*(a + b/x)
^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5
*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*
d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (124*a^8*b^8*c^7*d^5*(a
 + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 +
234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b
^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (366*a^9*b^7*c^6
*d^6*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9
*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 12
0*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (120*a^10
*b^6*c^5*d^7*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*
b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*
d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (
110*a^11*b^5*c^4*d^8*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 -
216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*
b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d
^9)) - (60*a^12*b^4*c^3*d^9*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10
*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 3
66*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^
4*c^3*d^9)))*(2*a*d + 3*b*c))/(c^2*(a^5)^(1/2)) - ((2*b^2)/(a^2*d - a*b*c)
+ (b*(a + b/x)*(a*d - 3*b*c))/(a^2*c*(a*d - b*c)))/(a*(a + b/x)^(1/2) - (a
+ b/x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

3.257 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=224

$$-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

```
[Out] d^(5/2)*(-4*a*d+7*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/(
-a*d+b*c)^(5/2)-(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^3+b*
(2*a^2*d^2-2*a*b*c*d+3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(1/2)+d*(-2*a*
d+b*c)/a/c^2/(-a*d+b*c)/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)/(a+b/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[a + b/x
]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + x/(a*c
*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

```
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+4ad)+\frac{5bdx}{2}}{x(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(3bc+4ad)-\frac{3}{2}bd(bc-}{x(a+bx)^{3/2}(c+dx)}\right.}{ac^2(bc-ad)}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{2\,\text{S}}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\left(d^3\right.}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\left(d^3\right.}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{d^{5/2}}{}$$

Mathematica [C] time = 0.14, size = 164, normalized size = 0.73

$$\frac{(bc-ad)\left((cx+d)\left(-4a^2d^2+abcd+3b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx(bc(cx+d)-ad(cx+2d))\right)+a^2d^2(cx+}{a^2c^3\sqrt{a+\frac{b}{x}}\,(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (a^2*d^2*(7*b*c - 4*a*d)*(d + c*x)*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a +
b/x))/(-(b*c) + a*d)] + (b*c - a*d)*(a*c*x*(b*c*(d + c*x) - a*d*(2*d + c*x)
) + (3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*(d + c*x)*Hypergeometric2F1[-1/2, 1,
1/2, 1 + b/(a*x)]))/(a^2*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(d + c*x))
```

fricas [B] time = 2.00, size = 2321, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a
*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^
4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^4*b*
d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*
d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
 - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^
2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a
^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^
3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b
*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 -
 a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3*b^2*c*d^3 -
 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3
*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt
(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c
^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5
*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^
2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a
*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*
d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d
 - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
```

```
^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
), 1/2*(2*(3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 +
(3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^
4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(
-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 +
 (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 -
 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*
c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c
*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*
b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), ((7*a^3*b^2*c*d^3 - 4*a^4*b*d^
4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^
3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a
*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*
a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*
d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3
*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((
a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3
*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3
*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x)]
```

giac [B] time = 0.26, size = 424, normalized size = 1.89

$$b^3\left(\frac{\left(7\,bcd^3-4\,ad^4\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^5c^5-2\,ab^4c^4d+a^2b^3c^3d^2\right)\sqrt{bcd-ad^2}}+\frac{2\,ab^3c^3-2\,a^2b^2c^2d-\frac{3\,(ax+b)b^3c^3}{x}+\frac{7\,(ax+b)ab^2c^2d}{x}-\frac{3\,(ax+b)a^2bcd^2}{x}+\frac{2\,(a}{}}{\left(a^2b^4c^4-2\,a^3b^3c^3d+a^4b^2c^2d^2\right)\left(abc\sqrt{\frac{ax+b}{x}}-a^2d\sqrt{\frac{ax+b}{x}}-\frac{(a}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] b^3*((7*b*c*d^3 - 4*a*d^4)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/
((b^5*c^5 - 2*a*b^4*c^4*d + a^2*b^3*c^3*d^2)*sqrt(b*c*d - a*d^2)) + (2*a*b^
3*c^3 - 2*a^2*b^2*c^2*d - 3*(a*x + b)*b^3*c^3/x + 7*(a*x + b)*a*b^2*c^2*d/x
 - 3*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x - 3*(a*x + b)^2*b^2*c^
2*d/x^2 + 2*(a*x + b)^2*a*b*c*d^2/x^2 - 2*(a*x + b)^2*a^2*d^3/x^2)/((a^2*b^
4*c^4 - 2*a^3*b^3*c^3*d + a^4*b^2*c^2*d^2)*(a*b*c*sqrt((a*x + b)/x) - a^2*d
*sqrt((a*x + b)/x) - (a*x + b)*b*c*sqrt((a*x + b)/x)/x + 2*(a*x + b)*a*d*sq
rt((a*x + b)/x)/x - (a*x + b)^2*d*sqrt((a*x + b)/x)/x^2)) + (3*b*c + 4*a*d)
*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b^3*c^3))
```

maple [B] time = 0.07, size = 3119, normalized size = 13.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(15/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*d^6+4*a^(11/2)*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*d^6-6*a
^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^
(1/2)*x^2*b^5*c^6+2*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^2*c
^4*d^2-4*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^5*d-3*a*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*x*b^6*c^6+6*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^5*d-3*
a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*b^6*c^5*d+2*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*c^4
*d^2-2*a^(11/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^4*d^2-2*a^(
13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^3*d^3+6*a^(7/2)*((a*x
+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^6-11*a^(13/2)*ln((-2*a*d*x+b
*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*b*c^2*
d^4+7*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)
^(1/2)*c)/(c*x+d))*x^3*b^2*c^3*d^3+4*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2
)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^2*d^4-3*a^3*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^4*c^
6-4*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^6-4*a^(13/2)*
((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^2*d^4+12*a^(5/2)*((a*x+b)*x
)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^6-3*a^(13/2)*ln((-2*a*d*x+b*c*x-b
*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*c*d^5-18*a
^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*
c)/(c*x+d))*x*b^2*c*d^5+3*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d
)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^3*c^2*d^4+7*a^(7/2)*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^4*c^3
*d^3-4*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^2*d^4+8*a^(7
/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^3*d^3-10*a^(5/2)*((a*x+
b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^4*d^2+4*a^5*ln(1/2*(2*a*x+b+2*((a
*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c*d^5-9*a^4*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*b^3*c^2*d^4+3*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d^3+5*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)
*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4*d^2-15*a^(11/2)*ln((-2*a
*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*
b^2*c^2*d^4+14*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^3*d^3+4*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)
```

```
*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*c*d^5+6*a^(3/2)*((a
*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^6+13*a^3*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^4*d^2-8*
a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^2*d^4+14*a^(9/2)*(
(a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^3*d^3-12*a^(7/2)*((a*x+b)*
x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^4*d^2+2*a^(5/2)*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^5*d+8*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c*d^5-14*a^5*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^2*
d^4+11*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)
/c^2*d)^(1/2)*x^2*b^3*c^4*d^2+7*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^5*d-a^6*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^2*d^4-1
5*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*
d)^(1/2)*x^2*b^2*c^3*d^3+8*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2
)*x^2*b^2*c^4*d^2-14*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*
b^3*c^5*d-10*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^5*
d-9*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^
2*d)^(1/2)*x^3*b*c^3*d^3+3*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))
/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4*d^2+5*a^4*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^5*d-4*
a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4*d^2+4*a^(7/2)*((a
*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d+4*a^(11/2)*((a*x+b)*x)^(
1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^3*d^3+12*a^(11/2)*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^4*d^2-a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)
*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^5*d-3*a^4*ln(1/2*(2*a*x+
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^3*d
^3+4*a^(15/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^
(1/2)*c)/(c*x+d))*x^3*c*d^5+8*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/
c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*d^6-11*a^(9/2)*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c*d^5
+7*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/
2)*c)/(c*x+d))*b^4*c^2*d^4)/a^(7/2)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)^3/(c*x+
d)/((a*d-b*c)/c^2*d)^(1/2)/(a*x+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

mupad [B] time = 6.20, size = 4274, normalized size = 19.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(a^2*d - a*b*c) + (b*(a + b/x)^2*(2*a^2*d^3 + 3*b^2*c^2*d - 2*a*b*
c*d^2))/(c^2*(a^2*d - a*b*c)^2) - (b*(a + b/x)*(2*a*d - b*c)*(a^2*d^2 + 3*b
^2*c^2 - a*b*c*d))/(c^2*(a^2*d - a*b*c)^2))/(d*(a + b/x)^(5/2) + (a + b/x)^
(1/2)*(a^2*d - a*b*c) - (a + b/x)^(3/2)*(2*a*d - b*c)) + (atan((a^13*b^11*c
^11*d^3*(a + b/x)^(1/2)*35i - a^12*b^12*c^12*d^2*(a + b/x)^(1/2)*441i - a^1
0*b^14*c^14*(a + b/x)^(1/2)*27i + a^14*b^10*c^10*d^4*(a + b/x)^(1/2)*1694i
- a^15*b^9*c^9*d^5*(a + b/x)^(1/2)*3073i + a^16*b^8*c^8*d^6*(a + b/x)^(1/2)
*1316i + a^17*b^7*c^7*d^7*(a + b/x)^(1/2)*2561i - a^18*b^6*c^6*d^8*(a + b/x
)^(1/2)*4375i + a^19*b^5*c^5*d^9*(a + b/x)^(1/2)*2996i - a^20*b^4*c^4*d^10*
(a + b/x)^(1/2)*1015i + a^21*b^3*c^3*d^11*(a + b/x)^(1/2)*140i + a^11*b^13*
c^13*d*(a + b/x)^(1/2)*189i)/(a^5*(a^5)^(1/2)*(a^5*(a^5*(2561*b^7*c^7*d^7 -
 4375*a*b^6*c^6*d^8 + 2996*a^2*b^5*c^5*d^9 - 1015*a^3*b^4*c^4*d^10 + 140*a^
4*b^3*c^3*d^11) - 441*b^12*c^12*d^2 + 35*a*b^11*c^11*d^3 + 1694*a^2*b^10*c^
10*d^4 - 3073*a^3*b^9*c^9*d^5 + 1316*a^4*b^8*c^8*d^6) - 27*a^3*b^14*c^14 +
189*a^4*b^13*c^13*d)))*(4*a*d + 3*b*c)*1i)/(c^3*(a^5)^(1/2)) - (atan((((d^5
*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^
3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 -
 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 -
3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12
+ 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) -
((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*
b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^1
2*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15
*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b
^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^
(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a
^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376
*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280
*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*
a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b
^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d
^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^
7*d)))*1i)/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2
 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) + ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d
 - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 +
362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 10
04*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 59
```

60*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 5
76*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d - b*c)^5)^(1/2)*(
4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^1
2*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*
b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6
*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^
10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^1
0*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^1
3*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^
16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^
19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5
*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*
d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^
2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))*1i)/(2*(b^5*c^8 - a^5
*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*
b^4*c^7*d)))/(((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(
18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*
a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a
^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768
*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^
18*b^2*c^6*d^15) - ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*
c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*
c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*
c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^
12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^
5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^
12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*
b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*
b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*
b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b
*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(
b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c
^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10
*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) - ((d^5*(a*d - b*c)
^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*
b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^
10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^
7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*
b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d -
b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^
3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*
d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^
9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12
+ 8*a^19*b^3*c^10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d
- 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^2

```
1*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^
18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^
15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^1
2*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 -
 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b
*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(
b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c
^5*d^3 - 5*a*b^4*c^7*d)) - 126*a^6*b^13*c^14*d^5 + 744*a^7*b^12*c^13*d^6 -
1742*a^8*b^11*c^12*d^7 + 1756*a^9*b^10*c^11*d^8 + 322*a^10*b^9*c^10*d^9 - 3
248*a^11*b^8*c^9*d^10 + 4606*a^12*b^7*c^8*d^11 - 3668*a^13*b^6*c^7*d^12 + 1
804*a^14*b^5*c^6*d^13 - 512*a^15*b^4*c^5*d^14 + 64*a^16*b^3*c^4*d^15))*(d^5
*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*1i)/(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*
c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.258 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

```
[Out] 3/4*d^(5/2)*(8*a^2*d^2-24*a*b*c*d+21*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/
(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(7/2)-3*(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(5/2)/c^4+3/4*b*(-a*d+2*b*c)*(4*a^2*d^2-a*b*c*d+2*b^2*c^2)/a^2/c
^3/(-a*d+b*c)^3/(a+b/x)^(1/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(c+d/x)
^2/(a+b/x)^(1/2)+1/4*d*(12*a^2*d^2-21*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2
/(c+d/x)/(a+b/x)^(1/2)+x/a/c/(c+d/x)^2/(a+b/x)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d
)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a
*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(
5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{\frac{3}{2}(bc+2ad)+\frac{7bdx}{2}}{x(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-3(bc-ad)(bc+2ad)-\frac{5}{2}b}{x(a+bx)^{3/2}(c+}\right.}{2ac^2(bc-a}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\right.}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-}$$

Mathematica [C] time = 0.32, size = 239, normalized size = 0.75

$$\frac{(cx+d)\left(2(cx+d)\left(\frac{3}{4}a^2d^2\left(8a^2d^2-24abcd+21b^2c^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)+3(2ad+bc)(bc-ad)^3{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2}\right.\right.\right.}{2a^2c^4\sqrt{a+\frac{b}{x}}(cx+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (-(a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2) + 2*a*c^3*(b*c - a*d)^3*x^3
+ (d + c*x)*(-1/2*(a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^
2)*x) + 2*(d + c*x)*((3*a^2*d^2*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*Hyper
geometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)])/4 + 3*(b*c - a*d)
^3*(b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])))/(2*a^2*c^4
*(b*c - a*d)^3*Sqrt[a + b/x]*(d + c*x)^2)
```

fricas [B] time = 5.03, size = 4093, normalized size = 12.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5
 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c
^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 -
a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4
*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)
*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 3*(21*a^3*b^3*
c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c
^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40
*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d
^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)
*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d
)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x), 1/8*(24*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^
2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*
b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d
 - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2
 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a
^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) -
3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^
```

2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 6*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x

```
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x)]
```

giac [A] time = 0.31, size = 516, normalized size = 1.61

$$\frac{1}{4}\,b^4\left(\frac{3\left(21\,b^2c^2d^3-24\,abcd^4+8\,a^2d^5\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^7c^7-3\,ab^6c^6d+3\,a^2b^5c^5d^2-a^3b^4c^4d^3\right)\sqrt{bcd-ad^2}}+\frac{4\left(2\,ab^3c^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{3\,(ax+b)ab^2c^2d}{x}-\frac{3\,(ax+b)a^2b}{x}\right.}{\left(a^2b^6c^6-3\,a^3b^5c^5d+3\,a^4b^4c^4d^2-a^5b^3c^3d^3\right)\left(a\sqrt{\frac{ax}{}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] 1/4*b^4*(3*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5)*arctan(d*sqrt((a*x +
 b)/x)/sqrt(b*c*d - a*d^2))/((b^7*c^7 - 3*a*b^6*c^6*d + 3*a^2*b^5*c^5*d^2 -
 a^3*b^4*c^4*d^3)*sqrt(b*c*d - a*d^2)) + 4*(2*a*b^3*c^3 - 3*(a*x + b)*b^3*c
^3/x + 3*(a*x + b)*a*b^2*c^2*d/x - 3*(a*x + b)*a^2*b*c*d^2/x + (a*x + b)*a^
3*d^3/x)/((a^2*b^6*c^6 - 3*a^3*b^5*c^5*d + 3*a^4*b^4*c^4*d^2 - a^5*b^3*c^3*
d^3)*(a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)) + (17*b^2*c^2*d
^3*sqrt((a*x + b)/x) - 25*a*b*c*d^4*sqrt((a*x + b)/x) + 8*a^2*d^5*sqrt((a*x
 + b)/x) + 15*(a*x + b)*b*c*d^4*sqrt((a*x + b)/x)/x - 8*(a*x + b)*a*d^5*sqr
t((a*x + b)/x)/x)/((b^6*c^6 - 3*a*b^5*c^5*d + 3*a^2*b^4*c^4*d^2 - a^3*b^3*c
^3*d^3)*(b*c - a*d + (a*x + b)*d/x)^2) + 12*(b*c + 2*a*d)*arctan(sqrt((a*x
+ b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b^4*c^4))
```

maple [B] time = 0.08, size = 5158, normalized size = 16.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

mupad [B] time = 9.49, size = 8936, normalized size = 27.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^3),x)
```

```
[Out] ((2*b^4)/(a^2*d - a*b*c) + (b*(a + b/x)*(12*a^4*d^4 + 12*b^4*c^4 + 24*a^2*b
^2*c^2*d^2 - 40*a*b^3*c^3*d - 33*a^3*b*c*d^3))/(4*a*c^3*(a^2*d - a*b*c)*(a*
d - b*c)) + (3*b*(a + b/x)^3*(4*a^3*d^5 - 4*b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 -
 9*a^2*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2) - (b*(a + b/x)^2*(
24*a^4*d^5 + 24*b^4*c^4*d - 56*a*b^3*c^3*d^2 + 65*a^2*b^2*c^2*d^3 - 72*a^3*
b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2))/((a + b/x)^(3/2)*(3*a^2*
d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(5/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
 b/x)^(7/2) - (a + b/x)^(1/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan(
((((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 90
3168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*
d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^
13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^1
7*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 3288
09600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5
*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147
456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c
^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081
344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^2
6*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272
*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^2
1*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 148316
16*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*
d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(d^5*(a*d
- b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192
*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^
4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^1
```

5*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))/(290304*a^6*b^18*c^21*d^5 - 2654208*a^7*b^17*c^20*d^6 + 10675584*a^8*b^16*c^19*d^7 - 23497344*a^9*b^15*c^18*d^8 + 23604480*a^10*b^14*c^17*d^9 + 24731136*a^11*b^13*c^16*d^10 - 148172544*a^12*b^12*c^15*d^11 + 320101632*a^13*b^11*c^14*d^12 - 452086272*a^14*b^10*c^13*d^13 + 459302400*a^15*b^9*c^12*d^14 -

343108224*a^16*b^8*c^11*d^15 + 187373952*a^17*b^7*c^10*d^16 - 72873216*a^1
8*b^6*c^9*d^17 + 19132416*a^19*b^5*c^8*d^18 - 3041280*a^20*b^4*c^7*d^19 + 2
21184*a^21*b^3*c^6*d^20 - (3*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 20
2752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d
^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*
b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^1
1 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 48290457
6*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^
13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 21381
12*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^
(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 1720
32*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*
d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a
^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^
10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*
a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^
15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^
3*c^14*d^18 - (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*
b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3
+ 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b
^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 -
134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^
19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^
13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^2
4*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8
*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^
4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8
*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^
4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^
5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11
- a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 +
35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (3*((a + b/x)
^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^1
7*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 600768
0*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19
*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521
961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b
^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 -
14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2
*c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*
c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^1
7*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 1411
2768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c
^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 440
64768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*

c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 18124 8*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1 /2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c ^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680* a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28* d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 15814656 0*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c ^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314 880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18* d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5* d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5 *b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5* d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5 *b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21* b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^ 2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^ 5 - 7*a*b^6*c^10*d))))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(4*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c ^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a *b^6*c^10*d)) + (atan((((2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26 *d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^ 16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276 416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11 *c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^ 19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^1 8 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2*a*d + b* c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17* c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 141127 68*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^2 3*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064 768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^ 18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248* a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b ^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 4 6964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b ^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^ 22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)))*3i)/(2*c^4*(a^5)^(1/2)) + ((2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^ 17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 60076 80*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^1

9*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 52
1961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*
b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17
- 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^
2*c^9*d^20) + (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c
^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472
*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24
*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616
*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19
*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*
a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 +
(3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^
17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 186
36800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12
*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 -
146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21
*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 +
1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^
17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)))*3i)/(2*c^4*(a^5)^(1/2)
))/(290304*a^6*b^18*c^21*d^5 - 2654208*a^7*b^17*c^20*d^6 + 10675584*a^8*b^1
6*c^19*d^7 - 23497344*a^9*b^15*c^18*d^8 + 23604480*a^10*b^14*c^17*d^9 + 247
31136*a^11*b^13*c^16*d^10 - 148172544*a^12*b^12*c^15*d^11 + 320101632*a^13*
b^11*c^14*d^12 - 452086272*a^14*b^10*c^13*d^13 + 459302400*a^15*b^9*c^12*d^
14 - 343108224*a^16*b^8*c^11*d^15 + 187373952*a^17*b^7*c^10*d^16 - 72873216
*a^18*b^6*c^9*d^17 + 19132416*a^19*b^5*c^8*d^18 - 3041280*a^20*b^4*c^7*d^19
+ 221184*a^21*b^3*c^6*d^20 - (3*(2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*
b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 17510
40*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d
^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*
a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^
16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164
257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4
*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2
*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a
^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6
- 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15
*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^1
1 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^
20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16
+ 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^(1/2)
*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 11059
20*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^2
9*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 1347174
40*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*
c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26

```
091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c
^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a
^5)^(1/2))))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)) + (3*(2*a*d + b*c)*(
(a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 90316
8*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7
 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*
b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d
^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 3288096
00*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^
12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456
*a^23*b^2*c^9*d^20) + (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^
9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 +
 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b
^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 +
45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*
b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 -
1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^1
4*d^18 + (3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264
*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d
^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a
^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25
*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383
232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^2
0*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^2
6*b^2*c^17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^
(1/2))))*(2*a*d + b*c)*3i)/(c^4*(a^5)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.259 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{(bc-ad)\left(-4a^3d^2x-2a^2bd(5cx+3d)+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] c*(c+d/x)^2*x/a/(a+b/x)^(3/2)+1/3*(-a*d+b*c)*(15*b^3*c^2-4*a^3*d^2*x-2*a^2*
b*d*(5*c*x+3*d)+a*b^2*c*(20*c*x-3*d))/a^3/b^2/(a+b/x)^(3/2)/x-c^2*(-6*a*d+5
*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 145, 63, 208}

$$\frac{(bc-ad)\left(-2a^2bd(5cx+3d)-4a^3d^2x+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^
2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b
/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(5bc-6ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc-\right.}{}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc-\right.}{}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc-}{}$$

Mathematica [A] time = 0.43, size = 145, normalized size = 1.01

$$\frac{\frac{4a^5d^3x}{b^2} + \frac{6a^4d^2(cx+d)}{b} + 3a^3c^2x(cx-8d) + 2a^2bc^2(10cx-9d) + 15ab^2c^3 + 3ac^2\sqrt{\frac{b}{ax}+1}\,(ax+b)(6ad-5bc)\tanh^{-1}}{3a^4\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (15*a*b^2*c^3 + (4*a^5*d^3*x)/b^2 + 3*a^3*c^2*x*(-8*d + c*x) + (6*a^4*d^2*(
d + c*x))/b + 2*a^2*b*c^2*(-9*d + 10*c*x) + 3*a*c^2*(-5*b*c + 6*a*d)*Sqrt[1
 + b/(a*x)]*(b + a*x)*ArcTanh[Sqrt[1 + b/(a*x)]])/(3*a^4*Sqrt[a + b/x]*(b +
 a*x))
```

fricas [A] time = 0.98, size = 483, normalized size = 3.38

$$\left[-\frac{3\left(5\,b^5c^3-6\,ab^4c^2d+\left(5\,a^2b^3c^3-6\,a^3b^2c^2d\right)x^2+2\left(5\,ab^4c^3-6\,a^2b^3c^2d\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-}{6\left(a^6b^2x^2+2\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d
+ 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4),
 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
+ 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
 b)/x)/a) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2*c^2*d + 3*a
^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d + 2*a^4*b*d^
3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4)]
```

giac [A] time = 0.26, size = 203, normalized size = 1.42

$$-\frac{\frac{3\,b^2c^3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\left(5\,b^2c^3-6\,abc^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3+\frac{6\,(ax+b)b^3c^3}{x}-\frac{9\,(ax+b)ab^2c^2d}{x}+\frac{3\,(ax+b)a^3d^3}{x}\right)x}{(ax+b)a^3b\sqrt{\frac{ax+b}{x}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c^3 -
6*a*b*c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^3*c
^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 + 6*(a*x + b)*b^3*c^3/x - 9*
(a*x + b)*a*b^2*c^2*d/x + 3*(a*x + b)*a^3*d^3/x)*x/((a*x + b)*a^3*b*sqrt((a
*x + b)/x)))/b
```

maple [B] time = 0.06, size = 1150, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*a^3*b^4*d^3-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a
^(1/2))*a^3*b^4*d^3-30*a^(1/2)*((a*x+b)*x)^(1/2)*b^6*c^3-6*(a*x^2+b*x)^(1/2
)*a^(13/2)*x^3*d^3-6*a^(13/2)*((a*x+b)*x)^(1/2)*x^3*d^3+12*a^(11/2)*((a*x+b
)*x)^(3/2)*x*d^3+16*a^(9/2)*((a*x+b)*x)^(3/2)*b*d^3+20*a^(3/2)*((a*x+b)*x)^
(3/2)*b^4*c^3-6*(a*x^2+b*x)^(1/2)*a^(7/2)*b^3*d^3-6*a^(7/2)*((a*x+b)*x)^(1/
2)*b^3*d^3+36*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*b^2*c^2*d-36*a^(7/2)*((a*x+b)*x
)^(3/2)*x*b^2*c^2*d+108*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c^2*d-54*ln(1/2*(
2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^2*b^5*c^2*d-54*ln(1/2*(2*
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^3*b^4*c^2*d-18*ln(1/2*(2*
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^4*b^3*c^2*d+108*a^(5/2)*(
(a*x+b)*x)^(1/2)*x*b^4*c^2*d+15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)
)/a^(1/2))*b^7*c^3+9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*
x*a^4*b^3*d^3+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*
b^6*c^3-9*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^4*b^3*d
^3-18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^6*c^2*d-18*
(a*x^2+b*x)^(1/2)*a^(9/2)*x*b^2*d^3-12*a^(7/2)*((a*x+b)*x)^(3/2)*b^2*c*d^2-
24*a^(5/2)*((a*x+b)*x)^(3/2)*b^3*c^2*d-18*a^(9/2)*((a*x+b)*x)^(1/2)*x*b^2*d
^3-90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^5*c^3+36*a^(3/2)*((a*x+b)*x)^(1/2)*b^5*
c^2*d-90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^4*c^3+24*a^(5/2)*((a*x+b)*x)^(3/2)
*x*b^3*c^3-18*a^(11/2)*((a*x+b)*x)^(1/2)*x^2*b*d^3-18*(a*x^2+b*x)^(1/2)*a^(
11/2)*x^2*b*d^3-30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^3+3*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^6*b*d^3+15*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^4*c^3-3*ln(1/2*(2*a*x+b+2*(a*x^
2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^6*b*d^3+9*ln(1/2*(2*a*x+b+2*((a*x+b)*x
)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^5*b^2*d^3+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^
(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b^5*c^3-9*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/
2)*a^(1/2))/a^(1/2))*x^2*a^5*b^2*d^3)/((a*x+b)*x)^(1/2)/b^3/(a*x+b)^3
```

maxima [A] time = 1.34, size = 228, normalized size = 1.59

$$\frac{1}{6}c^3\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-c^2d\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c^3*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - c^2*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
```

```
+ b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3*d
^3*(3/(sqrt(a + b/x)*b^2) - a/((a + b/x)^(3/2)*b^2)) + 2*c*d^2/((a + b/x)^(
3/2)*b)
```

mupad [B] time = 2.05, size = 194, normalized size = 1.36

$$\frac{\frac{2\left(a^3 d^3-3 a^2 b c d^2+3 a b^2 c^2 d-b^3 c^3\right)}{3 a}+\frac{\left(a+\frac{b}{x}\right)^2\left(2 a^3 d^3-6 a b^2 c^2 d+5 b^3 c^3\right)}{a^3}-\frac{2\left(a+\frac{b}{x}\right)\left(4 a^3 d^3-3 a^2 b c d^2-6 a b^2 c^2 d+5 b^3 c^3\right)}{3 a^2}}{b^2\left(a+\frac{b}{x}\right)^{5/2}-a b^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2 \operatorname{atanh}\left(\frac{\sqrt{a+\cdots}}{\sqrt{a}}\right.\cdots}{a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x)
^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3 +
 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/(b^2*(a + b/x)^(5/2)
- a*b^2*(a + b/x)^(3/2)) + (c^2*atanh((a + b/x)^(1/2)/a^(1/2))*(6*a*d - 5*b
*c))/a^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

3.260 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 1/3*(2*a^2*d^2+b*c*(-4*a*d+5*b*c))/a^2/b/(a+b/x)^(3/2)+c^2*x/a/(a+b/x)^(3/2
)-c*(-4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+c*(-4*a*d+5*b*c)/
a^3/(a+b/x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 78, 51, 63, 208}

$$\frac{\frac{c(5bc-4ad)}{a^2}+\frac{2d^2}{b}}{3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] ((2*d^2)/b + (c*(5*b*c - 4*a*d))/a^2)/(3*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a
*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)) - (c*(5*b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}c(5bc-4ad)+ad^2x}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}$$

$$= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.08, size = 97, normalized size = 0.80

$$\frac{ax\left(2a^2d^2+abc(3cx-4d)+5b^2c^2\right)+3bc(ax+b)(5bc-4ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)}{3a^3b\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (a*x*(5*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + 3*c*x)) + 3*b*c*(5*b*c - 4*a*d)
*(b + a*x)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(3*a^3*b*Sqrt[a +
b/x]*(b + a*x))
```

fricas [A] time = 1.01, size = 407, normalized size = 3.34

$$\left[-\frac{3\left(5\,b^4c^2-4\,ab^3cd+\left(5\,a^2b^2c^2-4\,a^3bcd\right)x^2+2\left(5\,ab^3c^2-4\,a^2b^2cd\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(3}{6\left(a^6bx^2+2\,a^5b^2x+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*(
5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4*d^
2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x^2 +
 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 -
 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*
c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/
(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

giac [A] time = 0.21, size = 163, normalized size = 1.34

$$-\frac{\frac{3\,b^2c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\left(5\,b^2c^2-4\,abcd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\frac{6\,(ax+b)b^2c^2}{x}-\frac{6\,(ax+b)abcd}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c^2 -
4*a*b*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^2*c^2
 - 2*a^2*b*c*d + a^3*d^2 + 6*(a*x + b)*b^2*c^2/x - 6*(a*x + b)*a*b*c*d/x)*x
/((a*x + b)*a^3*sqrt((a*x + b)/x)))/b
```

maple [B] time = 0.06, size = 588, normalized size = 4.82

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-12a^4bcd\,x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b^2c^2x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-36a^3b^2cd\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(24*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*c*d-30*a^(7/2)*(
(a*x+b)*x)^(1/2)*x^3*b*c^2-24*a^(7/2)*((a*x+b)*x)^(3/2)*x*c*d+72*((a*x+b)*x
)^(1/2)*a^(7/2)*b*c*d*x^2-4*((a*x+b)*x)^(3/2)*a^(7/2)*d^2+24*a^(5/2)*((a*x+
b)*x)^(3/2)*x*b*c^2-90*((a*x+b)*x)^(1/2)*a^(5/2)*b^2*c^2*x^2-16*((a*x+b)*x)
^(3/2)*a^(5/2)*b*c*d+72*((a*x+b)*x)^(1/2)*a^(5/2)*b^2*c*d*x-12*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^4*b*c*d+15*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^2*c^2+20*((a*x+b)*x)^(3/2)*
a^(3/2)*b^2*c^2-90*((a*x+b)*x)^(1/2)*a^(3/2)*b^3*c^2*x-36*a^3*b^2*c*d*x^2*l
n(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+45*a^2*b^3*c^2*x^2*ln(
1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+24*((a*x+b)*x)^(1/2)*a^(
3/2)*b^3*c*d-36*a^2*b^3*c*d*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/
a^(1/2))+45*a*b^4*c^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))-30*((a*x+b)*x)^(1/2)*a^(1/2)*b^4*c^2-12*a*b^4*c*d*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+15*b^5*c^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1
/2)*a^(1/2))/a^(1/2)))/a^(7/2)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

maxima [A] time = 1.31, size = 190, normalized size = 1.56

$$\frac{1}{6}\,c^{2}\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^{2}b-10\left(a+\frac{b}{x}\right)ab-2\,a^{2}b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{3}-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{4}}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{2}{3}\,cd\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c^2*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - 2/3*c*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3
*d^2/((a + b/x)^(3/2)*b)
```

mupad [B] time = 2.22, size = 144, normalized size = 1.18

$$\frac{\frac{2\left(a+\frac{b}{x}\right)\left(a^2\,d^2+4\,a\,b\,c\,d-5\,b^2\,c^2\right)}{3\,a^2}-\frac{2\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a}+\frac{b\left(a+\frac{b}{x}\right)^2\left(5\,b\,c^2-4\,a\,c\,d\right)}{a^3}}{b\left(a+\frac{b}{x}\right)^{5/2}-a\,b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\left(4\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a + b/x)*(a^2*d^2 - 5*b^2*c^2 + 4*a*b*c*d))/(3*a^2) - (2*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(3*a) + (b*(a + b/x)^2*(5*b*c^2 - 4*a*c*d))/a^3)/(b*(a
+ b/x)^(5/2) - a*b*(a + b/x)^(3/2)) + (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*
a*d - 5*b*c))/a^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

3.261 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 1/3*(-2*a*d+5*b*c)/a^2/(a+b/x)^(3/2)+c*x/a/(a+b/x)^(3/2)-(-2*a*d+5*b*c)*arc
tanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)+(-2*a*d+5*b*c)/a^3/(a+b/x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 51, 63, 208}

$$\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sqrt[a + b/x
]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(-\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.04, size = 60, normalized size = 0.58

$$\frac{x\left((5bc-2ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)+3acx\right)}{3a^2\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (x*(3*a*c*x + (5*b*c - 2*a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)]
))/(3*a^2*Sqrt[a + b/x]*(b + a*x))
```

fricas [A] time = 0.92, size = 331, normalized size = 3.21

$$\left[-\frac{3\left(5\,b^{3}c-2\,ab^{2}d+\left(5\,a^{2}bc-2\,a^{3}d\right)x^{2}+2\left(5\,ab^{2}c-2\,a^{2}bd\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,a^{3}cx^{3}+\cdots\right.}{6\left(a^{6}x^{2}+2\,a^{5}bx+a^{4}b^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3
*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqr
t((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*b^2*
d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^
2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x +
a^4*b^2)]
```

giac [A] time = 0.22, size = 145, normalized size = 1.41

$$-\frac{\frac{3\,b^{2}c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^{3}}-\frac{3\left(5\,b^{2}c-2\,abd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}}-\frac{2\left(ab^{2}c-a^{2}bd+\frac{6\,(ax+b)b^{2}c}{x}-\frac{3\,(ax+b)abd}{x}\right)x}{(ax+b)a^{3}\sqrt{\frac{ax+b}{x}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c - 2*a*
b*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^2*c - a^2*b
*d + 6*(a*x + b)*b^2*c/x - 3*(a*x + b)*a*b*d/x)*x/((a*x + b)*a^3*sqrt((a*x
+ b)/x)))/b
```

maple [B] time = 0.06, size = 541, normalized size = 5.25

$$\frac{\sqrt{\frac{ax+b}{x}}\;\left(6a^{4}bd\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\;\sqrt{a}}{2\sqrt{a}}\right)-15a^{3}b^{2}c\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\;\sqrt{a}}{2\sqrt{a}}\right)+18a^{3}b^{2}d\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\;\sqrt{a}}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*x^3*a^4*b*d-15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*x^3*a^3*b^2*c-12*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*d+30*a^(7/2)*((a*x+b
)*x)^(1/2)*x^3*b*c+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))
*x^2*a^3*b^2*d-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2
*a^2*b^3*c+12*a^(7/2)*((a*x+b)*x)^(3/2)*x*d-24*a^(5/2)*((a*x+b)*x)^(3/2)*x*
b*c-36*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*d+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b
^2*c+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^2*b^3*d-4
5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*b^4*c+8*a^(5/2)
*((a*x+b)*x)^(3/2)*b*d-20*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c-36*a^(5/2)*((a*x+
b)*x)^(1/2)*x*b^2*d+90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c+6*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^4*d-15*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*b^5*c-12*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*d+30*a^(
1/2)*((a*x+b)*x)^(1/2)*b^4*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

maxima [A] time = 1.25, size = 170, normalized size = 1.65

$$\frac{1}{6}\,c\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^{2}b-10\left(a+\frac{b}{x}\right)ab-2\,a^{2}b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{3}-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{4}}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{3}\,d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a
^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/a^(7/2)) - 1/3*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2))
```

mupad [B] time = 2.91, size = 87, normalized size = 0.84

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{2\,d\left(a+\frac{b}{x}\right)}{a^{2}}}{\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2\,c\,x\left(\frac{a\,x}{b}+1\right)^{5/2}{}_{2}\mathrm{F}_{1}\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(5/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*d)/(3*a) + (2*d*(a + b/x
))/a^2)/(a + b/x)^(3/2) + (2*c*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2],
9/2, -(a*x)/b))/(7*(a + b/x)^(5/2))
```

sympy [B] time = 155.82, size = 1479, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x)
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*
x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1
 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)
*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*
a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*
x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*
a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*
a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*
x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*
x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) +
 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) +
1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17
/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*
x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/
2)*b**3) + 6*a**7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**
(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqr
t(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x
+ 3*a**(13/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(
17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(
sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)
*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*
x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5
*b**2*x*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b
**2*x + 3*a**(13/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a*
*(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3)
- 3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(1
```

```
5/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3
*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**
3))
```

3.262 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal. Leaf size=79

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] 5/3*b/a^2/(a+b/x)^(3/2)+x/a/(a+b/x)^(3/2)-5*b*arctanh((a+b/x)^(1/2)/a^(1/2)
)/a^(7/2)+5*b/a^3/(a+b/x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-5/2),x]
```

```
[Out] (-2*x)/(3*a*(a + b/x)^(3/2)) - (10*x)/(3*a^2*Sqrt[a + b/x]) + (5*Sqrt[a + b
/x]*x)/a^3 - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.48

$$\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{a+\frac{b}{x}}{a}\right)}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-5/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, 2, -1/2, (a + b/x)/a])/(3*a^2*(a + b/x)^(3/2))
```

fricas [A] time = 0.59, size = 225, normalized size = 2.85

$$\left[\frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^3x^3+20\,a^2bx^2+15\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)},\ \frac{15\left(a^2bx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x +
b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x + b^
3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*b*x^
2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

giac [A] time = 0.24, size = 98, normalized size = 1.24

$$\frac{1}{3}\,b\left(\frac{2\left(a+\frac{6\,(ax+b)}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}+\frac{15\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{3\,\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(2*(a + 6*(a*x + b)/x)*x/((a*x + b)*a^3*sqrt((a*x + b)/x)) + 15*arcta
n(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 3*sqrt((a*x + b)/x)/((a - (a
*x + b)/x)*a^3))
```

maple [B] time = 0.07, size = 271, normalized size = 3.43

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-15a^3b\,x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-45a^2b^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+30\sqrt{(ax+b)\,x}\;a^{\frac{7}{2}}x^3-45a\,b^3x\ln\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x*(30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3-24*a^(5/2)*((a*x+
b)*x)^(3/2)*x+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b-15*ln(1/2*(2*a*x+b+2*((a*x
+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b-20*b*a^(3/2)*((a*x+b)*x)^(3/2)+90*
a^(3/2)*((a*x+b)*x)^(1/2)*x*b^2-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*x^2*a^2*b^2-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*x*a*b^3+30*a^(1/2)*((a*x+b)*x)^(1/2)*b^3-15*ln(1/2*(2*a*x+b+2*((a*x
+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^4)/a^(7/2)/((a*x+b)*x)^(1/2)/(a*x+b)^3
```

maxima [A] time = 1.29, size = 101, normalized size = 1.28

$$\frac{15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4\right)}+\frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
(a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
 sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.72, size = 34, normalized size = 0.43

$$\frac{2\,x\left(\frac{a\,x}{b}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(5/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x)
^(5/2))
```

sympy [B] time = 7.92, size = 774, normalized size = 9.80

$$\frac{6a^{17}x^4\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}+\frac{46a^{16}bx^3\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}+\frac{15a^{16}bx^3\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18
*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*
b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**
2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 +
b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
 + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**
15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*x))
 + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**
(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**
(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15
*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/
(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3)
```

3.263 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{\quad}{ac}$$

```
[Out] 1/3*b*(-3*a*d+5*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^(3/2)+x/a/c/(a+b/x)^(3/2)-2*d
^(7/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(5/2)-
(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^2+b*(a^2*d^2-8*a*b*c
*d+5*b^2*c^2)/a^3/c/(-a*d+b*c)^2/(a+b/x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 152, 156, 63, 208, 205}

$$\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{\quad}{ac\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5*b^2*c^2 -
 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/x]) + x/(a*c*(a + b/
x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^
2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^
(7/2)*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+2ad)+\frac{5bdx}{2}}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{3}{4}(bc-ad)(5bc+2ad)+\frac{3}{4}bd(5bc-3ad)x}{x(a+bx)^{3/2}(c+dx)}\,dx \cdots\right.}{3a^2c(bc-ad)}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{4\,\text{Subst}\left(\int \frac{\frac{3}{8}\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{d^4\,\text{Subst}\left(\int \cdots\right.}{c^2(\cdots}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(2d^4\right)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2}\tan^{-1}\left(\cdots\right.}{c^2(bc-a\cdots}$$

Mathematica [C] time = 0.08, size = 118, normalized size = 0.59

$$\frac{x\left((ad-bc)\left((2ad+5bc)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)+3acx\right)-2a^2d^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)\right)}{3a^2c^2\sqrt{a+\frac{b}{x}}\,(ax+b)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (x*(-2*a^2*d^2*Hypergeometric2F1[-3/2, 1, -1/2, (d*(a + b/x))/(-(b*c) + a*d
)] + (-(b*c) + a*d)*(3*a*c*x + (5*b*c + 2*a*d)*Hypergeometric2F1[-3/2, 1, -
1/2, 1 + b/(a*x)])))/(3*a^2*c^2*(-(b*c) + a*d)*Sqrt[a + b/x]*(b + a*x))
```

fricas [B] time = 5.60, size = 1990, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2
*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3
- 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*
d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((
a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 - 2*a^4
*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*
c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x
 + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 -
 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*
b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^
2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2
*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4
*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(a^3*b^2*c^3 - 2*
a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4
*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((
a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^
4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a
^7*b*c^2*d^2)*x), -1/6*(12*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt
(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)
/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*
d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(
5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c
^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^
2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b
)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a
^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^
2*d^2)*x), -1/3*(6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c
- a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x
+ b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5
*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*
```

```
c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x
^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4
*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4
- 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^
2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x)]
```

giac [A] time = 0.22, size = 247, normalized size = 1.23

$$-\frac{1}{3}\left(\frac{6\,d^4\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2\right)\sqrt{bcd-ad^2}}-\frac{2\left(abc-a^2d+\frac{6\,(ax+b)bc}{x}-\frac{9\,(ax+b)ad}{x}\right)x}{\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3bc}-\cdots 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] -1/3*(6*d^4*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^4*c^4 - 2*a
*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(b*c*d - a*d^2)) - 2*(a*b*c - a^2*d + 6*(
a*x + b)*b*c/x - 9*(a*x + b)*a*d/x)*x/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2
)*(a*x + b)*sqrt((a*x + b)/x)) + 3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3
*b*c) - 3*(5*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*
b^2*c^2))*b^2
```

maple [B] time = 0.07, size = 1767, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] -1/6*(15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c
^2*d)^(1/2)*b^6*c^4+6*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(
1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*d^4+6*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d
+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*d^4+3*ln(1/2*(
2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^2*b
^4*c^2*d^2+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b
*c)/c^2*d)^(1/2)*x^2*a^2*b^4*c^4+6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^6*c*d^3+15*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^3*b^3*c^4-3
0*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4+24*a^(5/2)*
((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4-90*a^(5/2)*((a*x+b)*x)^
(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^4-32*a^(5/2)*((a*x+b)*x)^(3/2)*((a*
```

```
d-b*c)/c^2*d)^(1/2)*b^2*c^3*d-90*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d
)^(1/2)*x*b^4*c^4-6*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c
^2*d^2+48*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d-6*a^(
11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^2*d^2+45*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a*b^5*c
^4+6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d
)^(1/2)*a^3*b^3*c*d^3-24*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
2))*((a*d-b*c)/c^2*d)^(1/2)*a*b^5*c^3*d+144*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d
-b*c)/c^2*d)^(1/2)*x^2*b^2*c^3*d-18*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*x*b^2*c^2*d^2+144*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1
/2)*x*b^3*c^3*d-18*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*
c^2*d^2+48*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^3*d-36
*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d+3*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^5*b
*c^2*d^2-24*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c
)/c^2*d)^(1/2)*x^3*a^4*b^2*c^3*d+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^5*b*c*d^3+9*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^4*b^2*c^2
*d^2-72*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^
2*d)^(1/2)*x^2*a^3*b^3*c^3*d+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)
)/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^4*b^2*c*d^3+9*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^3*b^3*c^2*d^2-7
2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*x*a^2*b^4*c^3*d+18*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)
^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*d^4-30*a^(1/2)*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4+20*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^
2*d)^(1/2)*b^3*c^4+18*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1
/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*d^4)*x*((a*x+b)/x)^(1/2)/a^(7/2)/(a
*x+b)^3/((a*d-b*c)/c^2*d)^(1/2)/c^3/(a*d-b*c)^2/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

mupad [B] time = 4.62, size = 5387, normalized size = 26.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)),x)
```

```
[Out] - ((2*b^2)/(3*(a^2*d - a*b*c)) + (2*b^2*(a + b/x)*(8*a*d - 5*b*c))/(3*(a^2*
d - a*b*c)^2) + (b*(a + b/x)^2*(a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(a^2*c*(a
^2*d - a*b*c)*(a*d - b*c)))/(a*(a + b/x)^(3/2) - (a + b/x)^(5/2)) - (atan((
(((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^1
1*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*
a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^1
7*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^
3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^
2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*
d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11
*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d
^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1/2)*(2*a*d +
 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*
d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13
*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10
*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^1
3))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c)*1i)/(2*c^2*(
a^7)^(1/2)) + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*
d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^
11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*
d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^1
3 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d + 5*b*c)*(20*a^1
2*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a
^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768
*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*
a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((a + b/x)^
(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a
^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376
*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280
*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^
26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c
)*1i)/(2*c^2*(a^7)^(1/2)))/(100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^10*d^7
+ 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*d^10 -
 1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^14 + 8*
a^18*b^3*c^2*d^15 + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13
*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b
^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^
7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c
^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(
20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 -
2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7
+ 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10
- 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a +
```

b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c))/(2*c^2*(a^7)^(1/2)) - (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((a + b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c))/(2*c^2*(a^7)^(1/2))))*(2*a*d + 5*b*c)*1i)/(c^2*(a^7)^(1/2)) - (atan((((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5))*1i)/(c^2*(a*d - b*c)^5) + ((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c

```
^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*
c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*
c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c
^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5))
*1i)/(c^2*(a*d - b*c)^5))/(((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*
a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280
*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 21
08*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a
^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b
^2*c^3*d^15) - ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13
*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a
^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*
a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22
*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^
(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^
4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d
^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d
^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13)
)/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5) - ((d^7*(
a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*
c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^
10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7
*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^
5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((d^7*(a*d - b*c)^5
)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^
15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*
c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*
c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^1
3 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a
^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600
*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320
*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*
a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d
- b*c)^5)))/(c^2*(a*d - b*c)^5) + 100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^1
0*d^7 + 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*
d^10 - 1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^1
4 + 8*a^18*b^3*c^2*d^15))*(d^7*(a*d - b*c)^5)^(1/2)*2i)/(c^2*(a*d - b*c)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x),x)

[Out] Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

3.264 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=287

$$-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}+\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^3}-\frac{d^{7/2}(9bc-4ad)\,\mathrm{t}}{c^3(bc-}$$

```
[Out] 1/3*b*(6*a^2*d^2-6*a*b*c*d+5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(3/2)+d*
(-2*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+
d/x)-d^(7/2)*(-4*a*d+9*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/
c^3/(-a*d+b*c)^(7/2)-(4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c
^3+b*(-2*a*d+b*c)*(a^2*d^2-a*b*c*d+5*b^2*c^2)/a^3/c^2/(-a*d+b*c)^3/(a+b/x)^
(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}-\frac{d^{7/2}(9bc-4ad)\,\mathrm{ta}}{c^3(bc-a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^
(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c -
a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2
)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) -
 ((5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+4ad)+\frac{7bdx}{2}}{x(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(5bc+4ad)-\frac{5}{2}\cdots}{x(a+bx)^{5/2}(c+\cdots}\right.}{ac^2(bc-a\cdots}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} \cdots$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\frac{b}{x}\cdots\right.}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\frac{b}{x}\cdots\right.}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\frac{b}{x}\cdots\right.}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\frac{b}{x}\cdots\right.}$$

Mathematica [C] time = 0.19, size = 178, normalized size = 0.62

$$\frac{x\left((ad-bc)\left(3acx(ad(cx+2d)-bc(cx+d))-(cx+d)\left(-4a^2d^2-abcd+5b^2c^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)\right)+a^2d^2(c\cdots\right.}{3a^2c^3\sqrt{a+\frac{b}{x}}\,(ax+b)(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (x*(a^2*d^2*(9*b*c - 4*a*d)*(d + c*x)*Hypergeometric2F1[-3/2, 1, -1/2, (d*(
a + b/x))/(-(b*c) + a*d)] + (-(b*c) + a*d)*(3*a*c*x*(-(b*c*(d + c*x)) + a*d
*(2*d + c*x)) - (5*b^2*c^2 - a*b*c*d - 4*a^2*d^2)*(d + c*x)*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + b/(a*x)])))/(3*a^2*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(b
 + a*x)*(d + c*x))
```

fricas [B] time = 5.60, size = 3887, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d
^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2
+ 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5
*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b
^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2
*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)
*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*
d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c
- a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c
*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*
d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b
*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3
*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11
*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x
))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4
 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2
*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^
3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c
^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9
*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a
^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(
d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/
(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*
a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b
^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^
4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)
*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 +
```

10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), 1/6*(6*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/3*(3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^

```
4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2
- a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 +
 a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^
3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x)]
```

giac [B] time = 0.33, size = 576, normalized size = 2.01

$$-\frac{1}{3}\,b^{3}\left(\frac{3\left(9\,bcd^{4}-4\,ad^{5}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{6}c^{6}-3\,ab^{5}c^{5}d+3\,a^{2}b^{4}c^{4}d^{2}-a^{3}b^{3}c^{3}d^{3}\right)\sqrt{bcd-ad^{2}}}-\frac{2\left(abc-a^{2}d+\frac{6\,(ax+b)bc}{x}-\frac{12\,(ax+b)ad}{x}\right)x}{\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}\right)(ax+b)\sqrt{\ }}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^3*(3*(9*b*c*d^4 - 4*a*d^5)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a
*d^2))/((b^6*c^6 - 3*a*b^5*c^5*d + 3*a^2*b^4*c^4*d^2 - a^3*b^3*c^3*d^3)*sqr
t(b*c*d - a*d^2)) - 2*(a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 12*(a*x + b)*a*d
/x)*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(a*x + b)*
sqrt((a*x + b)/x)) + 3*(b^4*c^4*sqrt((a*x + b)/x) - 4*a*b^3*c^3*d*sqrt((a*x
 + b)/x) + 6*a^2*b^2*c^2*d^2*sqrt((a*x + b)/x) - 4*a^3*b*c*d^3*sqrt((a*x +
b)/x) + 2*a^4*d^4*sqrt((a*x + b)/x) + (a*x + b)*b^3*c^3*d*sqrt((a*x + b)/x)
/x - 3*(a*x + b)*a*b^2*c^2*d^2*sqrt((a*x + b)/x)/x + 3*(a*x + b)*a^2*b*c*d^
3*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a^3*d^4*sqrt((a*x + b)/x)/x)/((a^3*b^5*
c^5 - 3*a^4*b^4*c^4*d + 3*a^5*b^3*c^3*d^2 - a^6*b^2*c^2*d^3)*(a*b*c - a^2*d
 - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2*d/x^2)) - 3*(5*b*c + 4
*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b^3*c^3))
```

maple [B] time = 0.08, size = 4644, normalized size = 16.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(12*a^(19/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c
^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*d^7+12*a^(13/2)*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*d^7+3
0*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d^4-84*a^(7/2)*
((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4*d^3+96*a^(5/2)*((a*x+b)*x
)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^6*c^5*d^2+12*a^6*ln(1/2*(2*a*x+b+2*((a*x+
b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c*d^6-33*a^5*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b
```

^4*c^2*d^5+12*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a
*d-b*c)/c^2*d)^(1/2)*b^5*c^3*d^4-6*a^(17/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*x^4*c^3*d^4-30*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)
*x^4*b^4*c^7-39*a^(17/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))*x^4*b*c^2*d^5+27*a^(15/2)*ln((-2*a*d*x+b*c*x-b
d+2((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^4*b^2*c^3*d^4+
12*a^9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2
*d)^(1/2)*x^4*c^2*d^5+15*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^5*c^7+24*a^(7/2)*((a*x+b)*x)^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^7-12*a^(17/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/
c^2*d)^(1/2)*x^3*c^2*d^5-90*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/
2)*x^3*b^5*c^7-3*a^(17/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*
((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*b*c*d^6-90*a^(15/2)*ln((-2*a*d*x+b*c*x-b*
d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*b^2*c^2*d^5+8
1*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/
2)*c)/(c*x+d))*x^3*b^3*c^3*d^4+12*a^9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a
^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*c*d^6+45*a^3*ln(1/2*(2*a*x+b+2
*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^6*c^7+20
a^(5/2)((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^7-90*a^(5/2)*((a
*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^6*c^7-81*a^(15/2)*ln((-2*a*d*x
+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^2*
c*d^6-36*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)
*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^2*d^5+81*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*
((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^4*c^3*d^4+45*a^
2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*x^2*b^7*c^7+38*a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c
^4*d^3-64*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^5*d^2+20*
a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^6*d-105*a^(13/2)*ln
((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d)
)*x*b^3*c*d^6+42*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*
((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^4*c^2*d^5+27*a^(9/2)*ln((-2*a*d*x+b*c*x-b
d+2((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^5*c^3*d^4-12
a^(11/2)((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^2*d^5+15*a*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b
^8*c^7-30*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^7*c^6*d+15*a*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/
2)*b^8*c^6*d+42*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*(
(a*d-b*c)/c^2*d)^(1/2)*b^6*c^4*d^3-48*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^7*c^5*d^2+6*a^(17/2)*((a*x+b
)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^5*c^4*d^3-6*a^(15/2)*((a*x+b)*x)^(3/2)
*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^4*d^3-30*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c
)/c^2*d)^(1/2)*x*b^7*c^7-102*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^6*c^5*d^2-3*a^2*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^7*c^6*d+36*a
^8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^

(1/2)*x^2*b*c*d^6-63*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^2*d^5+12*a^(15/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^3*d^4+24*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4*d^3-33*a^8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b*c^3*d^4+12*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^2*c^4*d^3+42*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^3*c^5*d^2-48*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^4*c^6*d+48*a^(15/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*b*c^4*d^3-84*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^2*c^5*d^2+96*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^3*c^6*d-36*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^2*d^5-63*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^3*d^4+162*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^4*d^3-18*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^5*c^5*d^2-99*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^6*c^6*d+198*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^5*c^6*d-28*a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^5*d^2-40*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^6*d-36*a^(15/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^2*d^5+72*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^3*d^4-156*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^4*d^3+36*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^5*d^2+78*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^4*c^5*d^2-129*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^5*c^6*d+30*a^(11/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4*d^3+3*a^8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^2*d^5-87*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^3*d^4+78*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^4*d^3-156*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^5*d^2+258*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^4*c^6*d-18*a^(13/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*d^3+48*a^(11/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^5*d^2-72*a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^6*d+84*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^3*d^4-222*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^4*d^3+204*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^5*d^2+6*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^6*c^6*d+36*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c*d^6-87*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^2*d^5+3*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^3*d^4+138*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*

```
b^5*c^4*d^3+12*a^(19/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))*x^4*c*d^6+36*a^(17/2)*ln((-2*a*d*x+b*c*x-b*d+2*
((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*d^7+36*a^(15/2)
*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x
+d))*x*b^2*d^7-39*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)
*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^4*c*d^6+27*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+
2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^5*c^2*d^5)/a^(9/2
)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)^4/(c*x+d)/((a*d-b*c)/c^2*d)^(1/2)/(a*x+b)
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)
```

mupad [B] time = 8.73, size = 5789, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(3*(a^2*d - a*b*c)) + (10*b^3*(a + b/x)*(2*a*d - b*c))/(3*(a^2*d -
 a*b*c)^2) - (b*(a + b/x)^2*(6*a^4*d^4 + 15*b^4*c^4 + 64*a^2*b^2*c^2*d^2 -
58*a*b^3*c^3*d - 12*a^3*b*c*d^3))/(3*c^2*(a^2*d - a*b*c)^3) + (b*(a + b/x)^
3*(2*a*d - b*c)*(a^2*d^3 + 5*b^2*c^2*d - a*b*c*d^2))/(c^2*(a^2*d - a*b*c)^3
))/(d*(a + b/x)^(7/2) + (a + b/x)^(3/2)*(a^2*d - a*b*c) - (a + b/x)^(5/2)*(
2*a*d - b*c)) + (atan((a^15*b^19*c^19*(a + b/x)^(1/2)*125i + a^17*b^17*c^17
*d^2*(a + b/x)^(1/2)*10440i - a^18*b^16*c^16*d^3*(a + b/x)^(1/2)*37776i + a
^19*b^15*c^15*d^4*(a + b/x)^(1/2)*87276i - a^20*b^14*c^14*d^5*(a + b/x)^(1/
2)*126720i + a^21*b^13*c^13*d^6*(a + b/x)^(1/2)*91560i + a^22*b^12*c^12*d^7
*(a + b/x)^(1/2)*40965i - a^23*b^11*c^11*d^8*(a + b/x)^(1/2)*184563i + a^24
*b^10*c^10*d^9*(a + b/x)^(1/2)*212608i - a^25*b^9*c^9*d^10*(a + b/x)^(1/2)*
107740i - a^26*b^8*c^8*d^11*(a + b/x)^(1/2)*19530i + a^27*b^7*c^7*d^12*(a +
 b/x)^(1/2)*71070i - a^28*b^6*c^6*d^13*(a + b/x)^(1/2)*52836i + a^29*b^5*c^
5*d^14*(a + b/x)^(1/2)*20916i - a^30*b^4*c^4*d^15*(a + b/x)^(1/2)*4515i + a
^31*b^3*c^3*d^16*(a + b/x)^(1/2)*420i - a^16*b^18*c^18*d*(a + b/x)^(1/2)*17
00i)/(a^7*(a^7)^(1/2)*(a^7*(a^7*(212608*b^10*c^10*d^9 - 107740*a*b^9*c^9*d^
```

10 - 19530*a^2*b^8*c^8*d^11 + 71070*a^3*b^7*c^7*d^12 - 52836*a^4*b^6*c^6*d^
13 + 20916*a^5*b^5*c^5*d^14 - 4515*a^6*b^4*c^4*d^15 + 420*a^7*b^3*c^3*d^16)
 + 10440*b^17*c^17*d^2 - 37776*a*b^16*c^16*d^3 + 87276*a^2*b^15*c^15*d^4 -
126720*a^3*b^14*c^14*d^5 + 91560*a^4*b^13*c^13*d^6 + 40965*a^5*b^12*c^12*d^
7 - 184563*a^6*b^11*c^11*d^8) + 125*a^5*b^19*c^19 - 1700*a^6*b^18*c^18*d)))
*(4*a*d + 5*b*c)*1i)/(c^3*(a^7)^(1/2)) - (atan((((d^7*(a*d - b*c)^7)^(1/2)*
((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11
*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 5551
0*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10
 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c
^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22
*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^2
5*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d
- 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*b^17*c
^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*a^17*b
^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 - 95700
*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^16*d^12
 + 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*c^13*d
^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^10*d^1
8 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18
*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^
15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a
^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10
- 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^
17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^
4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10
 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3
+ 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10
- a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 +
 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)
1i)/(2(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35
*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
) + ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 5
0*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 -
35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*
d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^
10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*
a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4
820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d
^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^13*b
^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476*a^
16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 - 10
9648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^17*
d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^7*c
^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b^4*

c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/
2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^
17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 458
64*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^
9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^
8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^
28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^
31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^
5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 -
7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5
*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7
*a*b^6*c^9*d)))*(4*a*d - 9*b*c)*1i)/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^
4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a
^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/(4880*a^10*b^16*c^17*d^7 - 450*a^9*b^17*c
^18*d^6 - 23428*a^11*b^15*c^16*d^8 + 65234*a^12*b^14*c^15*d^9 - 115136*a^13
*b^13*c^14*d^10 + 129800*a^14*b^12*c^13*d^11 - 83040*a^15*b^11*c^12*d^12 +
5916*a^16*b^10*c^11*d^13 + 45702*a^17*b^9*c^10*d^14 - 51528*a^18*b^8*c^9*d^
15 + 32500*a^19*b^7*c^8*d^16 - 13790*a^20*b^6*c^7*d^17 + 4012*a^21*b^5*c^6*
d^18 - 736*a^22*b^4*c^5*d^19 + 64*a^23*b^3*c^4*d^20 + ((d^7*(a*d - b*c)^7)^
(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 408
2*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7
+ 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^1
6*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19
*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 3449
0*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 8
32*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(
4*a*d - 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*
b^17*c^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*
a^17*b^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 -
95700*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^1
6*d^12 + 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*
c^13*d^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^
10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^1
5*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a
^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 8
8088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20
*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*
b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a
^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^
7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^
7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7
*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7
*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d -
9*b*c))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 -
35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9

```
*d)) - ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4
- 50*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6
 - 35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^
17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18
*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 454
30*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17
+ 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) -
((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^1
3*b^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476
*a^16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 -
 109648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^
17*d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^
7*c^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b
^4*c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^
(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080
*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 -
45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21
*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25
*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120
*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16
*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2
*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5
 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*
b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5
- 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^
4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a
^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))*(d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c
)*1i)/(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a
^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
[Out] Timed out
```

3.265 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3} dx$

Optimal. Leaf size=409

$$-\frac{(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}-\frac{d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

```
[Out] 1/12*b*(-36*a^3*d^3+87*a^2*b*c*d^2-36*a*b^2*c^2*d+20*b^3*c^3)/a^2/c^3/(-a*d
+b*c)^3/(a+b/x)^(3/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(
c+d/x)^2+1/4*d*(12*a^2*d^2-23*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b/x)
^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+d/x)^2-1/4*d^(7/2)*(24*a^2*d^2-88*a*b
*c*d+99*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b
*c)^(9/2)-(6*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^4+1/4*b*(1
2*a^4*d^4-35*a^3*b*c*d^3+24*a^2*b^2*c^2*d^2-56*a*b^3*c^3*d+20*b^4*c^4)/a^3/
c^3/(-a*d+b*c)^4/(a+b/x)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b\left(24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4-56ab^3c^3d+20b^4c^4\right)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4}+\frac{b\left(87a^2bcd^2-36a\right.}{12a^2c^3\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3
*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*Sqrt[a
 + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/
x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a
 + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(
99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b
*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^{3}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+6ad)+\frac{9bdx}{2}}{x(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{2}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{2}} - \frac{\text{Subst}\left(\int \frac{-(bc-ad)(5bc+6a}{x(a+bx)^{5}}\right.}{2ac^2(b}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{2}} + \frac{d\left(4b^2c^2-23abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^{2}} + $$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a}}$$

Mathematica [C] time = 0.39, size = 239, normalized size = 0.58

$$\frac{(cx+d)\left(2(cx+d)\left(\frac{1}{4}a^2d^2\left(24a^2d^2-88abcd+99b^2c^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)+(6ad+5bc)(bc-ad)^3{}_2F_1\left(-\frac{3}{2},1;\right.\right.\right.}{6a^2c^4\left(a+\frac{b}{x}\right)^{3/2}(cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (-3*a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2 + 6*a*c^3*(b*c - a*d)^3*x^3
+ (d + c*x)*((-3*a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2)
*x)/2 + 2*(d + c*x)*((a^2*d^2*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*Hyperg
eometric2F1[-3/2, 1, -1/2, (d*(a + b/x))/(-(b*c) + a*d)])/4 + (b*c - a*d)^3
*(5*b*c + 6*a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)])))/(6*a^2*c^
4*(b*c - a*d)^3*(a + b/x)^(3/2)*(d + c*x)^2)
```

fricas [B] time = 14.87, size = 6171, normalized size = 15.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^
4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*
c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c
^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4
*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5
*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4
*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b
^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4
*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 +
24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x
^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8
*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d
^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c
^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4
*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 4
8*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5
)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3
```

*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x), 1/24*(24*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c* d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 6*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c

```
^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^4*b^3*c^5*d^2 + 48
*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7
- 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2
*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2
*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^
5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3
*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*sqrt((a*x + b)/x))
/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d
^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2
- 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d +
 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^
3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*
d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*
c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x)]
```

giac [A] time = 0.29, size = 523, normalized size = 1.28

$$-\frac{1}{12}\,b^4\left(\frac{3\left(99\,b^2c^2d^4-88\,abcd^5+24\,a^2d^6\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^8c^8-4\,ab^7c^7d+6\,a^2b^6c^6d^2-4\,a^3b^5c^5d^3+a^4b^4c^4d^4\right)\sqrt{bcd-ad^2}}-\frac{8\left(abc-a^2d+\frac{6\,(ax\cdots}{x}\right.}{\left(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/12*b^4*(3*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*arctan(d*sqrt((a*
x + b)/x)/sqrt(b*c*d - a*d^2))/((b^8*c^8 - 4*a*b^7*c^7*d + 6*a^2*b^6*c^6*d^
2 - 4*a^3*b^5*c^5*d^3 + a^4*b^4*c^4*d^4)*sqrt(b*c*d - a*d^2)) - 8*(a*b*c -
a^2*d + 6*(a*x + b)*b*c/x - 15*(a*x + b)*a*d/x)*x/((a^3*b^4*c^4 - 4*a^4*b^3
*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*(a*x + b)*sqrt((a*x +
 b)/x)) + 3*(21*b^2*c^2*d^4*sqrt((a*x + b)/x) - 29*a*b*c*d^5*sqrt((a*x + b)
/x) + 8*a^2*d^6*sqrt((a*x + b)/x) + 19*(a*x + b)*b*c*d^5*sqrt((a*x + b)/x)/
x - 8*(a*x + b)*a*d^6*sqrt((a*x + b)/x)/x)/((b^7*c^7 - 4*a*b^6*c^6*d + 6*a^
2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4)*(b*c - a*d + (a*x + b)
*d/x)^2) + 12*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3*b^3*c^3) - 12*(5*b*c
 + 6*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b^4*c^4))
```

maple [B] time = 0.08, size = 7300, normalized size = 17.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

mupad [B] time = 8.23, size = 4284, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^3),x)
```

```
[Out] ((2*b^4)/(3*(a^2*d - a*b*c)) + (2*b^4*(a + b/x)*(12*a*d - 5*b*c))/(3*(a^2*d
 - a*b*c)^2) + (b*(a + b/x)^2*(36*a^5*d^5 - 60*b^5*c^5 - 456*a^2*b^3*c^3*d^
2 + 120*a^3*b^2*c^2*d^3 + 308*a*b^4*c^4*d - 123*a^4*b*c*d^4))/(12*a^2*c^3*(
a^2*d - a*b*c)*(a*d - b*c)^2) + (b*(a + b/x)^4*(12*a^4*d^6 + 20*b^4*c^4*d^2
 - 56*a*b^3*c^3*d^3 + 24*a^2*b^2*c^2*d^4 - 35*a^3*b*c*d^5))/(4*a^2*c^3*(a^2
*d - a*b*c)*(a*d - b*c)^3) - (b*(a + b/x)^3*(72*a^5*d^6 - 120*b^5*c^5*d + 4
96*a*b^4*c^4*d^2 - 592*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 - 264*a^4*b*c*
d^5))/(12*a^2*c^3*(a^2*d - a*b*c)*(a*d - b*c)^3))/((a + b/x)^(5/2)*(3*a^2*d
^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(7/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
b/x)^(9/2) - (a + b/x)^(3/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan((
a^15*b^24*c^24*(a + b/x)^(1/2)*2000i + a^17*b^22*c^22*d^2*(a + b/x)^(1/2)*2
77440i - a^18*b^21*c^21*d^3*(a + b/x)^(1/2)*1325984i + a^19*b^20*c^20*d^4*(
a + b/x)^(1/2)*4135824i - a^20*b^19*c^19*d^5*(a + b/x)^(1/2)*8371440i + a^2
1*b^18*c^18*d^6*(a + b/x)^(1/2)*9129120i + a^22*b^17*c^17*d^7*(a + b/x)^(1/
2)*3058605i - a^23*b^16*c^16*d^8*(a + b/x)^(1/2)*32337558i + a^24*b^15*c^15
*d^9*(a + b/x)^(1/2)*63677218i - a^25*b^14*c^14*d^10*(a + b/x)^(1/2)*666652
80i + a^26*b^13*c^13*d^11*(a + b/x)^(1/2)*24871035i + a^27*b^12*c^12*d^12*(
a + b/x)^(1/2)*40203170i - a^28*b^11*c^11*d^13*(a + b/x)^(1/2)*85652532i +
a^29*b^10*c^10*d^14*(a + b/x)^(1/2)*88170192i - a^30*b^9*c^9*d^15*(a + b/x)
^(1/2)*60362445i + a^31*b^8*c^8*d^16*(a + b/x)^(1/2)*29178270i - a^32*b^7*c
^7*d^17*(a + b/x)^(1/2)*9940590i + a^33*b^6*c^6*d^18*(a + b/x)^(1/2)*228782
4i - a^34*b^5*c^5*d^19*(a + b/x)^(1/2)*320859i + a^35*b^4*c^4*d^20*(a + b/x
)^(1/2)*20790i - a^16*b^23*c^23*d*(a + b/x)^(1/2)*34800i)/(a^7*(a^7)^(1/2)*
(a^7*(a^7*(a^7*(29178270*b^8*c^8*d^16 - 9940590*a*b^7*c^7*d^17 + 2287824*a^
```

2*b^6*c^6*d^18 - 320859*a^3*b^5*c^5*d^19 + 20790*a^4*b^4*c^4*d^20) + 636772
18*b^15*c^15*d^9 - 66665280*a*b^14*c^14*d^10 + 24871035*a^2*b^13*c^13*d^11
+ 40203170*a^3*b^12*c^12*d^12 - 85652532*a^4*b^11*c^11*d^13 + 88170192*a^5*
b^10*c^10*d^14 - 60362445*a^6*b^9*c^9*d^15) + 277440*b^22*c^22*d^2 - 132598
4*a*b^21*c^21*d^3 + 4135824*a^2*b^20*c^20*d^4 - 8371440*a^3*b^19*c^19*d^5 +
9129120*a^4*b^18*c^18*d^6 + 3058605*a^5*b^17*c^17*d^7 - 32337558*a^6*b^16*
c^16*d^8) + 2000*a^5*b^24*c^24 - 34800*a^6*b^23*c^23*d)))*(6*a*d + 5*b*c)*1
i)/(c^4*(a^7)^(1/2)) + (log(400*b^25*c^25*d^4 - 8240*a*b^24*c^24*d^5 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 79696*a^2*b^23*c^23*d^6 - 478768*a^3*
b^22*c^22*d^7 + 1987568*a^4*b^21*c^21*d^8 - 5978896*a^5*b^20*c^20*d^9 + 131
76240*a^6*b^19*c^19*d^10 - 20525703*a^7*b^18*c^18*d^11 + 18765714*a^8*b^17*
c^17*d^12 + 3763331*a^9*b^16*c^16*d^13 - 49787452*a^10*b^15*c^15*d^14 + 104
120705*a^11*b^14*c^14*d^15 - 140185682*a^12*b^13*c^13*d^16 + 139985251*a^13
*b^12*c^12*d^17 - 108046616*a^14*b^11*c^11*d^18 + 65184867*a^15*b^10*c^10*d
^19 - 30607170*a^16*b^9*c^9*d^20 + 10996689*a^17*b^8*c^8*d^21 - 2926572*a^1
8*b^7*c^7*d^22 + 544467*a^19*b^6*c^6*d^23 - 63294*a^20*b^5*c^5*d^24 + 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
(d^7(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
c)^9)^(1/2)(3*a^2*d^2 + (99*b^2*c^2)/8 - 11*a*b*c*d))/(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d) - (log(8240*a*b^24*c^24*d^5 - 400*b^25*c^25*d^4 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(

```
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 79696*a^2*b^23*c^23*d^6 + 478768*a^3*
b^22*c^22*d^7 - 1987568*a^4*b^21*c^21*d^8 + 5978896*a^5*b^20*c^20*d^9 - 131
76240*a^6*b^19*c^19*d^10 + 20525703*a^7*b^18*c^18*d^11 - 18765714*a^8*b^17*
c^17*d^12 - 3763331*a^9*b^16*c^16*d^13 + 49787452*a^10*b^15*c^15*d^14 - 104
120705*a^11*b^14*c^14*d^15 + 140185682*a^12*b^13*c^13*d^16 - 139985251*a^13
*b^12*c^12*d^17 + 108046616*a^14*b^11*c^11*d^18 - 65184867*a^15*b^10*c^10*d
^19 + 30607170*a^16*b^9*c^9*d^20 - 10996689*a^17*b^8*c^8*d^21 + 2926572*a^1
8*b^7*c^7*d^22 - 544467*a^19*b^6*c^6*d^23 + 63294*a^20*b^5*c^5*d^24 - 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
 b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
*c)^9)^(1/2)*(24*a^2*d^2 + 99*b^2*c^2 - 88*a*b*c*d))/(8*(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.266 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=123

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

```
[Out] (a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/a^(1/2)/c^(1
/2)-2*arctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*b^(1/2)*d^(1/2)+
x*(a+b/x)^(1/2)*(c+d/x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {375, 97, 157, 63, 217, 206, 93, 208}

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x]
)/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - \text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - (bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right) - \frac{1}{2}(bc+ad)\,\text{Subst}\Big(\int$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - (2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right) - (bc+ad)\,\text{Subst}$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - (2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+}}{\sqrt{c+}}\right.$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Mathematica [A] time = 1.40, size = 167, normalized size = 1.36

$$\frac{\sqrt{a+\frac{b}{x}}\,(cx+d) - 2\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{bcx+bd}{bcx-adx}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{\sqrt{c+\frac{d}{x}}\,(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}}{\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*(d + c*x) - 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*d + b*c*x)/(b*
c*x - a*d*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]] + ((b*c + a*
d)*Sqrt[c + d/x]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/
(Sqrt[a]*Sqrt[c]))/Sqrt[c + d/x]
```

fricas [A] time = 2.16, size = 890, normalized size = 7.24

$$\left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+2\sqrt{bd}\,ac\log\left(-\frac{8\,b^2d^2+\left(b^2c^2+6\,abcd+a^2d^2\right)x^2-4\left(2\,bdx+(bc+ad)x^2\right)\sqrt{bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\left(b^2cd+abd^2\right)x}{x^2}\right)+\sqrt{ac}\cdots}{4\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(8
*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x
^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x
)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a
^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*
x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x + b)/x)
*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2
)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (
b^2*c*d + a*b*d^2)*x)) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2
 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x
+ b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/2*(2*a*c*x*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt(b*d)*a*c*log(-(8*b^2*d^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2) - sqrt(-a*
c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2
*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/
x) + 2*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (b^2*c*d + a*b*d^
2)*x)) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

maple [B] time = 0.10, size = 253, normalized size = 2.06

$$\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(\sqrt{bd}\,ad\ln\left(\frac{2acx+ad+bc+2\sqrt{ac\,x^2+adx+bcx+bd}\,\sqrt{ac}}{2\sqrt{ac}}\right)+\sqrt{bd}\,bc\ln\left(\frac{2acx+ad+bc+2\sqrt{ac\,x^2+adx+bcx+bd}\,\sqrt{ac}}{2\sqrt{ac}}\right)-2\right.}{2\sqrt{ac\,x^2+adx+bcx+bd}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(a
*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+(b*d)^(
1/2)*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)
/(a*c)^(1/2))*b*c-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(1/2)*(a*c*x^2+a*d*x+b*c*x+
b*d)^(1/2)+2*b*d)/x)*(a*c)^(1/2)+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1
/2)*(b*d)^(1/2))/(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+\frac{b}{x}}\,\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

mupad [B] time = 22.22, size = 4674, normalized size = 38.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
```

```
[Out] atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/
2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3
 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2)
)*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*
c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^
6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5
))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) -
 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b
^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)
```

*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i - (b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i)/((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d +

16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (7*a^(7/2)*b^12*c^(21/2)*d - 7*a^(9/2)*b^11*c^(19/2)*d^2 - 21*a^(11/2)*b^10*c^(17/2)*d^3 + 21*a^(13/2)*b^9*c^(15/2)*d^4 + 21*a^(15/2)*b^8*c^

```
(13/2)*d^5 - 21*a^(17/2)*b^7*c^(11/2)*d^6 - 7*a^(19/2)*b^6*c^(9/2)*d^7 + 7*
a^(21/2)*b^5*c^(7/2)*d^8)/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(112
*a^5*b^10*c^9*d^3 - 56*a^4*b^11*c^10*d^2 + 56*a^6*b^9*c^8*d^4 - 224*a^7*b^8
*c^7*d^5 + 56*a^8*b^7*c^6*d^6 + 112*a^9*b^6*c^5*d^7 - 56*a^10*b^5*c^4*d^8))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*(b*d)^(1/2)*4i - ((((a + b/x
)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(
1/2) - c^(1/2))) - b^2/(4*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d^2)/4
+ (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c*d*((c + d/x)^(1/2) - c^(1/2))^2))/(((a
 + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a + b/x)^(1
/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2) - a^(1/
2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2)) + (d*
((a + b/x)^(1/2) - a^(1/2)))/(4*((c + d/x)^(1/2) - c^(1/2))) + (log(((a + b
/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a*d + b*c))/(2*a^(1/2)*c
^(1/2)) - (log(((c^(1/2)*(a + b/x)^(1/2) - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1
/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1/2)))/((c + d/x)^(1/2) - c^(1/2))))
/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(2*a
*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x}}\,\sqrt{c + \frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

3.267 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$

Optimal. Leaf size=81

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

```
[Out] (-a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(3/2)/a^(
1/2)+x*(a+b/x)^(1/2)*(c+d/x)^(1/2)/c
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {375, 94, 93, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.00

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

fricas [A] time = 0.93, size = 247, normalized size = 3.05

$$\left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\sqrt{ac}\,(bc-ad)\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\left(2\,acx^2+(bc+ad)x\right)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\right.}{4\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*l
og(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a
*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d
)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(-a*c
)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*
a*c*x + b*c + a*d)))/(a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

maple [B] time = 0.10, size = 155, normalized size = 1.91

$$\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(-ad\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+bc\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+2\sqrt{(ax+b)\,(cx+d)}\right.}{2\sqrt{(ax+b)\,(cx+d)}\,\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x
+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+ln(1/2*(2*a*c*x+2*((a*x+b)
*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c+2*((a*x+b)*(c*x+d))^(
1/2)*(a*c)^(1/2))/((a*x+b)*(c*x+d))^(1/2)/c/(a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

mupad [B] time = 6.58, size = 478, normalized size = 5.90

$$\frac{d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{4\,c\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{4}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{b^2}{4\,c\,d}+\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{4}-\frac{3\,a\,b\,c\,d}{4}+\frac{b^2\,c^2}{4}\right)}{a\,c^2\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^3}{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\right.}{2\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(1/2),x)
```

```
[Out] (d*((a + b/x)^(1/2) - a^(1/2)))/(4*c*((c + d/x)^(1/2) - c^(1/2))) - ((((a +
 b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(3/2)*d*((c + d/
x)^(1/2) - c^(1/2))) - b^2/(4*c*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d
^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c^2*d*((c + d/x)^(1/2) - c^(1/2))^
2))/(((a + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a +
 b/x)^(1/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2)
 - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2
)) + (log(((a + b/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)
*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2) - (log(((c^(1/2)*(a + b/x)^(1/2)
 - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1
/2)))/((c + d/x)^(1/2) - c^(1/2))))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b
*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

3.268 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

```
[Out] (-3*a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/c^(5/2)/a
^(1/2)+(a+b/x)^(3/2)*x/a/c/(c+d/x)^(1/2)-(-3*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(
c+d/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {375, 96, 94, 93, 208}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/x)^(3/2)*x
)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqr
t[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
```

```
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{\left(-\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}$$

Mathematica [A] time = 0.11, size = 87, normalized size = 0.71

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}} + \frac{\sqrt{a+\frac{b}{x}}\,(cx+3d)}{c^2\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(3*d + c*x))/(c^2*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

fricas [A] time = 1.30, size = 319, normalized size = 2.61

$$\left[-\frac{\left(bcd-3\,ad^2+\left(bc^2-3\,acd\right)x\right)\sqrt{ac}\,\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\left(2\,acx^2+(bc+ad)x\right)\sqrt{ac}\,\sqrt{\frac{ax+b}{x}}\right.}{4\left(ac^4x+ac^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^2
 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*
sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^2*x
^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d), -
1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(-a*c)
*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*c^2*x^
2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Unable to divide, perhap
s due to rounding error%%%{%%%{1,[1]%%%},[2,1,2]%%%}+%%%{%%{[-2,0]:[1,0,%%%
{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{1,[0,1,4]%%%} / %%%{%%%{1,[2]%%%},[2,0,0]%%
%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,
[1]%%%},[0,0,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.08, size = 280, normalized size = 2.30

$$\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(-3acdx\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+b\,c^{2}x\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right)-3a\,d^{2}\ln\Bigl(2ac\right.}{2\sqrt{ac}\,\;(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+a*d+b*c+2*((a
*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*x*a*c*d+ln(1/2*(2*a*c*x+a*d+
b*c+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*x*b*c^2+2*x*c*((a*x
+b)*(c*x+d))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*(c*x+d)
)^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*
(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*b*c*d+6*d*((a*x+b)*(c*x+d))^(1/2)*
(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((a*x+b)*(c*x+d))^(1/2)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(3/2),x)
[Out] int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
[Out] Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

3.269 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \,dx$

Optimal. Leaf size=96

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

```
[Out] -b*(a+b/x)^(1+p)*(c+d/x)^q*AppellF1(1+p,2,-q,2+p,(a+b/x)/a,-d*(a+b/x)/(-a*d
+b*c))/a^2/(1+p)/((b*(c+d/x)/(-a*d+b*c))^q)
```

Rubi [A] time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 137, 136}

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d))^
q))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
```

```
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx &= -\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= -\left(\left(\left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x}\right)\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}\end{aligned}$$

Mathematica [B] time = 0.37, size = 206, normalized size = 2.15

$$\frac{bdx(p+q-2)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q F_1\left(-p-q+1;-p,-q;-p-q\right.}{(p+q-1)\left(x\left(adpF_1\left(-p-q+2;1-p,-q;-p-q+3;-\frac{ax}{b},-\frac{cx}{d}\right)+bcqF_1\left(-p-q+2;-p,1-q;-p-q+3;-\frac{ax}{b},\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] (b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -p, -q, 2 -
 p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(-2 + p + q)*AppellF1
[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)]) + x*(a*d*p*AppellF1
[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*AppellF1[
2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)])))
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax+b}{x}\right)^p\left(\frac{cx+d}{x}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="giac")
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^p*(c+d/x)^q,x)
[Out] int((a+b/x)^p*(c+d/x)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^p*(c + d/x)^q,x)
[Out] int((a + b/x)^p*(c + d/x)^q, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**p*(c+d/x)**q,x)
```

```
[Out] Integral((a + b/x)**p*(c + d/x)**q, x)
```

3.270 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

```
[Out] a*x/c+(-a*d+b*c)*arctan(x*c^(1/2)/d^(1/2))/c^(3/2)/d^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \int \frac{b+ax^2}{d+cx^2}\,dx$$
$$= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^2}\,dx}{c}$$
$$= \frac{ax}{c} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.03

$$\frac{ax}{c} - \frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

fricas [A] time = 0.88, size = 98, normalized size = 2.51

$$\left[\frac{2\,acdx + (bc-ad)\sqrt{-cd}\,\log\left(\frac{cx^2+2\,\sqrt{-cd}\,x-d}{cx^2+d}\right)}{2\,c^2d}, \frac{acdx + (bc-ad)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{d}\right)}{c^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/(
c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/d
))/(c^2*d)]
```

giac [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

maple [A] time = 0.05, size = 45, normalized size = 1.15

$$-\frac{ad\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c}+\frac{b\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{ax}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2),x)
[Out] a*x/c-1/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))*a*d+1/(c*d)^(1/2)*arctan(c*x/
(c*d)^(1/2))*b
```

maxima [A] time = 1.30, size = 33, normalized size = 0.85

$$\frac{ax}{c}+\frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

mupad [B] time = 0.07, size = 32, normalized size = 0.82

$$\frac{a\,x}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\,\left(a\,d-b\,c\right)}{c^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2),x)
[Out] (a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))
```

sympy [B] time = 0.33, size = 82, normalized size = 2.10

$$\frac{ax}{c}+\frac{\sqrt{-\frac{1}{c^3d}}\,(ad-bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{c^3d}}\,(ad-bc)\log\left(cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - s
qrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

3.271 $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=233

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\right.\right.}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] -2*d*(a+b/x^2)^(1/2)/x/(c+d/x^2)^(1/2)-(a*d+b*c)*(x^2*c/d/(1+x^2*c/d))^(1/2
)/x*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b
/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1
+x^2*c/d))^(1/2)/x*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c
/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2
)+x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {375, 473, 531, 418, 492, 411}

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\right.\right.}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt[c + d/x^2
]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[
d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^
2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqr
t[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x - 2\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x - (2bd)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right) - (bc+ad)\operatorname{Sub}$$
$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x - \frac{\sqrt{c}\,(bc+ad)\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$
$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \cdots$$

Mathematica [C] time = 0.38, size = 205, normalized size = 0.88

$$-\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\left(\sqrt{\frac{a}{b}}\left(ax^2+b\right)\left(cx^2+d\right)+ix\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}\,(bc-ad)F\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{bc}{ad}\right)+2iadx\sqrt{\cdots}\right.}{\sqrt{\frac{a}{b}}\left(ax^2+b\right)\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] -((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) + (
2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt
[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^
2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b + a*x^
2)*(d + c*x^2)))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^2}}\,\sqrt{c + \frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

maple [A] time = 0.14, size = 277, normalized size = 1.19

$$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(-\sqrt{-\frac{c}{d}}\,ac\,x^4 - \sqrt{-\frac{c}{d}}\,ad\,x^2 + \sqrt{\frac{c\,x^2+d}{d}}\,\sqrt{\frac{a\,x^2+b}{b}}\,adx\,\mathrm{EllipticF}\left(\sqrt{-\frac{c}{d}}\,x, \sqrt{\frac{ad}{bc}}\right) - \sqrt{-\frac{c}{d}}\,bc\,x^2 + 2\sqrt{}\right.}{\left(ac\,x^4 + ad\,x^2 + bc\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)*x*((c*x^2+d)/x^2)^(1/2)*(-(-c/d)^(1/2)*x^4*a*c+((c*x^
2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a/b/c*d)^(1/2))
*x*a*d-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticF(x*(-c/d)^(1/
2),(a/b/c*d)^(1/2))+2*c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*Ellipti
cE(x*(-c/d)^(1/2),(a/b/c*d)^(1/2))-(-c/d)^(1/2)*x^2*a*d-(-c/d)^(1/2)*x^2*b*
c-(-c/d)^(1/2)*b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^2}}\,\sqrt{c + \frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{b}{x^2}}\,\sqrt{c + \frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2),x)
```

```
[Out] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

3.272 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}} dx$

Optimal. Leaf size=232

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] -d*(a+b/x^2)^(1/2)/c/x/(c+d/x^2)^(1/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x*(1+x
^2*c/d)^(1/2)*EllipticF(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1
/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+(x^2*c/d/(1+x^2*c/d))
^(1/2)/x/c*d*(1+x^2*c/d)^(1/2)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1
/2))*(a+b/x^2)^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+x*(a+b
/x^2)^(1/2)*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.20, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {375, 475, 21, 422, 418, 492, 411}

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*Sqrt[c + d/
x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c
+ d/x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) - \frac{(bd)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,d\right.}{c}$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + d\,\text{Subst}\left(\int \frac{\sqrt{a+}}{(c+d}\right.$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} + \frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,F\left(\text{co}\right.}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x}\right.}{a\left(c+\frac{c}{x}\right.}}}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.37

$$\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{cx^2+d}{d}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{c}{d}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{\frac{ax^2+b}{b}}\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*d
)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] integral(x^2*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

maple [A] time = 0.07, size = 94, normalized size = 0.41

$$\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,\sqrt{\frac{a\,x^2+b}{b}}\,\sqrt{\frac{c\,x^2+d}{d}}\,b\,\mathrm{EllipticE}\left(\sqrt{-\frac{c}{d}}\,x, \sqrt{\frac{ad}{bc}}\right)}{\left(a\,x^2+b\right)\sqrt{-\frac{c}{d}}\,\sqrt{\frac{c\,x^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x)
[Out] ((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE((-c/d)^(1/2)*x,(a/b/c*d)^(1/2))*(
(a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/((c*x^2+d)/x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2),x)
```

```
[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

3.273 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] -2*d*(a+b/x^2)^(1/2)/c^2/x/(c+d/x^2)^(1/2)-x*(a+b/x^2)^(1/2)/c/(c+d/x^2)^(1
/2)-b*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c*(1+x^2*c/d)^(1/2)*EllipticF(1/(1+x^2*
c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/a/(c*(a+b/x^2)/a/(c+d/x^2))^(
1/2)/(c+d/x^2)^(1/2)+2*(x^2*c/d/(1+x^2*c/d))^(1/2)/x/c^2*d*(1+x^2*c/d)^(1/2
)*EllipticE(1/(1+x^2*c/d)^(1/2),(1-b*c/a/d)^(1/2))*(a+b/x^2)^(1/2)/(c*(a+b/
x^2)/a/(c+d/x^2))^(1/2)/(c+d/x^2)^(1/2)+2*x*(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)
/c^2
```

Rubi [A] time = 0.28, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {375, 469, 583, 531, 418, 492, 411}

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]*x)/(c*Sqr
t[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c^2 + (2*Sqrt[d]*Sqrt
[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(c^(3/
2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x
^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{\operatorname{Subst}\left(\int \frac{-2a-bx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} - \frac{\operatorname{Subst}\left(\int \frac{abc+2abdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(2bd)\operatorname{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x} - \frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} - \frac{b\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + \frac{(2}{}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x} - \frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} + \frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} -$$

Mathematica [C] time = 0.33, size = 191, normalized size = 0.73

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(i\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}\,(bc-2ad)F\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{bc}{ad}\right)+2iad\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}\,x\right)|\frac{b}{a}\right.\right.}{c^2\sqrt{\frac{a}{b}}\left(ax^2+b\right)\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] -((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1 + (a*x^2)/
b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I*(
b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh[Sq
rt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^4\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}}}{c^2x^4+2\,cdx^2+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2)/(c^2*x^4 + 2*c*d*x
^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

maple [A] time = 0.17, size = 185, normalized size = 0.71

$$-\frac{\sqrt{\frac{a x^2+b}{x^2}}\left(\sqrt{-\frac{c}{d}}\, a x^3+\sqrt{-\frac{c}{d}}\, b x-2\sqrt{\frac{c x^2+d}{d}}\sqrt{\frac{a x^2+b}{b}}\, b\,\mathrm{EllipticE}\left(\sqrt{-\frac{c}{d}}\, x, \sqrt{\frac{ad}{bc}}\right)+\sqrt{\frac{c x^2+d}{d}}\sqrt{\frac{a x^2+b}{b}}\, b\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.\right.}{\left(a x^2+b\right)\sqrt{-\frac{c}{d}}\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x)
```

```
[Out] -((a*x^2+b)/x^2)^(1/2)/x^2/(a*x^2+b)*(x^3*a*(-c/d)^(1/2)+EllipticF((-c/d)^(
1/2)*x,(a/b/c*d)^(1/2))*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)-2*Ellipti
```

```
cE((-c/d)^(1/2)*x,(a/b/c*d)^(1/2))*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2
)+x*b*(-c/d)^(1/2))*(c*x^2+d)/(-c/d)^(1/2)/c/((c*x^2+d)/x^2)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2),x)
```

```
[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)
```

3.274 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/
a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 511, 510}

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx &= -\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q}{x^2} dx, x, \frac{1}{x}\right)\\
&= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q}{x^2} dx, x, \frac{1}{x}\right)\right)\\
&= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^2}\right.\right.\\
&= \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 104, normalized size = 1.32

$$
-\frac{x\left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{1}{2};-p,-q;-p-q+\frac{3}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2p+2q-1}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$
\operatorname{integral}\left(\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] Timed out
```

3.275 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}} \, dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}\,x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}}+\frac{ax}{c}$$

```
[Out] a*x/c+1/3*(-a*d+b*c)*ln(d^(1/3)+c^(1/3)*x)/c^(4/3)/d^(2/3)-1/6*(-a*d+b*c)*l
n(d^(2/3)-c^(1/3)*d^(1/3)*x+c^(2/3)*x^2)/c^(4/3)/d^(2/3)-1/3*(-a*d+b*c)*arc
tan(1/3*(d^(1/3)-2*c^(1/3)*x)/d^(1/3)*3^(1/2))/c^(4/3)/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {374, 388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}\,x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}\,x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(
Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c^(1/3)*x])/(3*c^(4/3
)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \int \frac{b+ax^3}{d+cx^3}\,dx$$

$$= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^3}\,dx}{c}$$

$$= \frac{ax}{c} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{c}\,x}\,dx}{3cd^{2/3}} + \frac{(bc-ad)\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}\,x}{d^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+c^{2/3}x^2}\,dx}{3cd^{2/3}}$$

$$= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2c^{2/3}x}{d^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+c^{2/3}x^2}\,dx}{6c^{4/3}d^{2/3}} + \frac{(bc-ad)\int \frac{1}{d^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+c^{2/3}x^2}}{2c\sqrt[3]{d}}$$

$$= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int -\right.}{c^{4/3}}$$

$$= \frac{ax}{c} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}\,x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+c\right.}{6c^{4/3}d^{2/3}}$$

Mathematica [A] time = 0.10, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}\right)+2(bc-ad)\log\left(\sqrt[3]{c}\,x+\sqrt[3]{d}\right)-2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+6a\sqrt[3]{c}\,d^{2}}{6c^{4/3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Log[
d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

fricas [A] time = 0.66, size = 390, normalized size = 2.69

$$\left[\frac{6\,acd^2x-3\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{\frac{\left(-cd^2\right)^{\frac{1}{3}}}{c}}\log\left(\frac{2\,cdx^3+3\left(-cd^2\right)^{\frac{1}{3}}dx-d^2-3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(-cd^2\right)^{\frac{2}{3}}x+\left(-cd^2\right)^{\frac{1}{3}}d\right)\sqrt{\frac{\left(-cd^2\right)^{\frac{1}{3}}}{c}}}{cx^3+d}\right)-\left(-cd^2\right)}{6\,c^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)*
log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-c*
d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) - (-c
*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)*d)
+ 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2), 1/6*
(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c)*arct
an(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^(1/3)/c
)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^
2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^
2*d^2)]
```

giac [A] time = 0.22, size = 133, normalized size = 0.92

$$-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\left(-c^2d\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(-c^2d\right)^{\frac{2}{3}}}+\frac{ax}{c}-\frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\right.\right.\right.}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1/
3))/(-c^2*d)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2/3
))/(-c^2*d)^(2/3) + a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d/c)
^(1/3)))/(c*d)
```

maple [A] time = 0.05, size = 195, normalized size = 1.34

$$\frac{ax}{c}-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c^{2}}-\frac{ad\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c^{2}}+\frac{ad\ln\left(x^{2}-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{c}\right)^{\frac{2}{3}}c^{2}}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c}+\frac{b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)/(c+d/x^3),x)
```

```
[Out] a/c*x-1/3/c^2/(1/c*d)^(2/3)*ln(x+(1/c*d)^(1/3))*a*d+1/3/c/(1/c*d)^(2/3)*ln(
x+(1/c*d)^(1/3))*b+1/6/c^2/(1/c*d)^(2/3)*ln(x^2-(1/c*d)^(1/3)*x+(1/c*d)^(2/
3))*a*d-1/6/c/(1/c*d)^(2/3)*ln(x^2-(1/c*d)^(1/3)*x+(1/c*d)^(2/3))*b-1/3/c^2
/(1/c*d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c*d)^(1/3)*x-1))*a*d+1/3/c/
(1/c*d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c*d)^(1/3)*x-1))*b
```

maxima [A] time = 1.29, size = 128, normalized size = 0.88

$$\frac{ax}{c}+\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3c^{2}\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^{2}-x\left(\frac{d}{c}\right)^{\frac{1}{3}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c^{2}\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c^{2}\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] a*x/c + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (d/c)^(1/3))/(d/c
)^(1/3))/(c^2*(d/c)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(d/c)^(1/3) + (d/c
)^(2/3))/(c^2*(d/c)^(2/3)) + 1/3*(b*c - a*d)*log(x + (d/c)^(1/3))/(c^2*(d/c
)^(2/3))
```

mupad [B] time = 0.27, size = 123, normalized size = 0.85

$$\frac{a\,x}{c}-\frac{\ln\left(c^{1/3}\,x+d^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}+\frac{\ln\left(d^{1/3}-2\,c^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}-\frac{\ln\left(2\,c^{1/3}\,x-d^{1/3}+\sqrt{3}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)/(c + d/x^3),x)
```

```
[Out] (a*x)/c - (log(c^(1/3)*x + d^(1/3))*(a*d - b*c))/(3*c^(4/3)*d^(2/3)) + (log
(3^(1/2)*d^(1/3)*1i - 2*c^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(4/3)*d^(2/3)) - (log(3^(1/2)*d^(1/3)*1i + 2*c^(1/3)*x - d^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(4/3)*d^(2/3))
```

sympy [A] time = 0.45, size = 71, normalized size = 0.49

$$\frac{ax}{c} + \operatorname{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2
*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

3.276 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{2\sqrt{x}\,(bc-ad)}{d^2}+\frac{bx}{d}$$

```
[Out] b*x/d+2*c*(-a*d+b*c)*ln(c+d*x^(1/2))/d^3-2*(-a*d+b*c)*x^(1/2)/d^2
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {376, 77}

$$-\frac{2\sqrt{x}\,(bc-ad)}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c + d*Sqrt[x]
])/d^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.84

$$\frac{2(ad-bc)\left(d\sqrt{x}-c\log\left(c+d\sqrt{x}\right)\right)}{d^3}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (b*x)/d + (2*(-(b*c) + a*d)*(d*Sqrt[x] - c*Log[c + d*Sqrt[x]]))/d^3
```

fricas [A] time = 0.97, size = 48, normalized size = 0.98

$$\frac{bd^2x+2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)-2\left(bcd-ad^2\right)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x)
)/d^3
```

giac [A] time = 0.19, size = 49, normalized size = 1.00

$$\frac{bdx-2\,bc\sqrt{x}+2\,ad\sqrt{x}}{d^2}+\frac{2\left(bc^2-acd\right)\log\left(\left|d\sqrt{x}+c\right|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] (b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*s
qrt(x) + c))/d^3
```

maple [A] time = 0.05, size = 59, normalized size = 1.20

$$-\frac{2ac\ln\left(d\sqrt{x}+c\right)}{d^2}+\frac{2b\,c^2\ln\left(d\sqrt{x}+c\right)}{d^3}+\frac{bx}{d}+\frac{2a\sqrt{x}}{d}-\frac{2bc\sqrt{x}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)/(c+d*x^(1/2)),x)
```

```
[Out] b/d*x+2/d*a*x^(1/2)-2/d^2*b*c*x^(1/2)-2*c/d^2*ln(c+d*x^(1/2))*a+2*c^2/d^3*l
n(c+d*x^(1/2))*b
```

maxima [A] time = 0.64, size = 47, normalized size = 0.96

$$\frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2}+\frac{2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)/
d^3
```

mupad [B] time = 0.07, size = 49, normalized size = 1.00

$$\sqrt{x}\,\left(\frac{2\,a}{d}-\frac{2\,b\,c}{d^2}\right)+\frac{\ln\left(c+d\,\sqrt{x}\right)\,\left(2\,b\,c^2-2\,a\,c\,d\right)}{d^3}+\frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/(c + d*x^(1/2)),x)
```

```
[Out] x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))/
d^3 + (b*x)/d
```

sympy [A] time = 0.30, size = 82, normalized size = 1.67

$$\begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2}+\frac{2a\sqrt{x}}{d}+\frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3}-\frac{2bc\sqrt{x}}{d^2}+\frac{bx}{d} & \text{for } d\neq 0 \\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c/
d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x**(
3/2)/3)/c, True))
```

3.277 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} dx$

Optimal. Leaf size=26

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(x^(1/3)+1)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{(-1+x)x^2}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(2-2x+x^2-\frac{2}{1+x}\right)dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

fricas [A] time = 0.91, size = 20, normalized size = 0.77

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

giac [A] time = 0.20, size = 20, normalized size = 0.77

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

maple [A] time = 0.04, size = 21, normalized size = 0.81

$$x-6\ln\left(x^{\frac{1}{3}}+1\right)-3x^{\frac{2}{3}}+6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)-1)/(x^(1/3)+1),x)
```

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(x^(1/3)+1)
```

maxima [A] time = 0.55, size = 20, normalized size = 0.77

$$x-3\,x^{\frac{2}{3}}+6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$x-6\,\ln\left(x^{1/3}+1\right)+6\,x^{1/3}-3\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) - 1)/(x^(1/3) + 1),x)
```

```
[Out] x - 6*log(x^(1/3) + 1) + 6*x^(1/3) - 3*x^(2/3)
```

sympy [A] time = 0.20, size = 24, normalized size = 0.92

$$-3x^{\frac{2}{3}}+6\sqrt[3]{x}+x-6\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

```
[Out] -3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

3.278 $\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx$

Optimal. Leaf size=17

$$x + 6\sqrt[3]{x} - 6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

[Out] 6*x^(1/3)+x-6*arctanh(x^(1/3))

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {376, 459, 321, 207}

$$x + 6\sqrt[3]{x} - 6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]

[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= x+6\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}+x-6\tanh^{-1}\left(\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

fricas [A] time = 0.69, size = 23, normalized size = 1.35

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

giac [A] time = 0.18, size = 24, normalized size = 1.41

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
```

```
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

maple [A] time = 0.05, size = 24, normalized size = 1.41

$$x-3\ln\left(x^{\frac{1}{3}}+1\right)+3\ln\left(x^{\frac{1}{3}}-1\right)+6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3)+1)/(x^(2/3)-1),x)
[Out] x+6*x^(1/3)+3*ln(x^(1/3)-1)-3*ln(x^(1/3)+1)
```

maxima [A] time = 0.56, size = 23, normalized size = 1.35

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

mupad [B] time = 1.46, size = 13, normalized size = 0.76

$$x-6\,\mathrm{atanh}\left(x^{1/3}\right)+6\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3) + 1)/(x^(2/3) - 1),x)
[Out] x - 6*atanh(x^(1/3)) + 6*x^(1/3)
```

sympy [A] time = 0.27, size = 27, normalized size = 1.59

$$6\sqrt[3]{x}+x+3\log\left(\sqrt[3]{x}-1\right)-3\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(2/3))/(-1+x**(2/3)),x)
[Out] 6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

3.279 $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

Optimal. Leaf size=104

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\,2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -128*x^(1/4)+x+256/3*2^(1/3)*ln(2*2^(1/3)+x^(1/4))-128/3*2^(1/3)*ln(4*2^(2/
3)-2*2^(1/3)*x^(1/4)+x^(1/2))-256/3*2^(1/3)*arctan(1/6*(2^(1/3)-x^(1/4))*2^
(2/3)*3^(1/2))*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {376, 459, 321, 200, 31, 634, 617, 204, 628}

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\,2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3 - (128*2^(1/3)*Log[4
*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = 4\,\text{Subst}\left(\int \frac{x^3\left(-16+x^3\right)}{16+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= x-128\,\text{Subst}\left(\int \frac{x^3}{16+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= -128\sqrt[4]{x}+x+2048\,\text{Subst}\left(\int \frac{1}{16+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= -128\sqrt[4]{x}+x+\frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+x}\,dx, x, \sqrt[4]{x}\right)+\frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{4\sqrt[3]{2}}{4\,2^{2/3}-2}\right.$$
$$= -128\sqrt[4]{x}+x+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{1}{3}\left(128\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{-2\sqrt[3]{2}+2x}{4\,2^{2/3}-2\sqrt[3]{2}\,x+x^2}\,dx, x,\right.$$
$$= -128\sqrt[4]{x}+x+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\,2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)+\left(256\sqrt[3]{2}\right)$$
$$= -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{2-2^{2/3}\sqrt[4]{x}}{2\sqrt{3}}\right)}{\sqrt{3}}+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\,2^{2/3}-2\right.$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.21

$$x-2x\,{}_2F_1\left(1,\frac{4}{3};\frac{7}{3};-\frac{x^{3/4}}{16}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] x - 2*x*Hypergeometric2F1[1, 4/3, 7/3, -1/16*x^(3/4)]
```

fricas [A] time = 0.82, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*l
og(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

giac [A] time = 0.17, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

maple [A] time = 0.04, size = 66, normalized size = 0.63

$$x+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1\right)}{3}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}-128x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16+x^(3/4))/(16+x^(3/4)),x)
```

```
[Out] x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)-
16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8*1
6^(2/3)*x^(1/4)-1))
```

maxima [A] time = 1.13, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

mupad [B] time = 1.50, size = 90, normalized size = 0.87

$$x+\frac{256\,2^{1/3}\,\ln\left(12288\,2^{1/3}+6144\,x^{1/4}\right)}{3}-128\,x^{1/4}+\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}+6144\,2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/4) - 16)/(x^(3/4) + 16),x)
```

```
[Out] x + (256*2^(1/3)*log(12288*2^(1/3) + 6144*x^(1/4)))/3 - 128*x^(1/4) + (128*
2^(1/3)*log(6144*x^(1/4) + 6144*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))
/3 - (128*2^(1/3)*log(6144*x^(1/4) - 6144*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2
)*1i + 1))/3
```

sympy [A] time = 5.72, size = 102, normalized size = 0.98

$$-128\sqrt[4]{x}+x+\frac{256\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{3}-\frac{128\sqrt[3]{2}\log\left(-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}+\frac{256\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

```
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/3
)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(3)
*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3
```

3.280 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

Optimal. Leaf size=30

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 376, 77}

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}\,dx$$
$$= 3\,\text{Subst}\left(\int \frac{x^2(1+x)}{1-x}\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right)dx, x, \sqrt[3]{x}\right)$$
$$= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.00

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

fricas [A] time = 0.59, size = 22, normalized size = 0.73

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

giac [A] time = 0.15, size = 23, normalized size = 0.77

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

maple [A] time = 0.04, size = 23, normalized size = 0.77

$$-x-6\ln\left(x^{\frac{1}{3}}-1\right)-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
```

```
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

maxima [A] time = 0.56, size = 22, normalized size = 0.73

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.73

$$-x - 6\,\ln\left(x^{1/3} - 1\right) - 6\,x^{1/3} - 3\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
```

```
[Out] - x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

sympy [A] time = 0.18, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

3.281 $\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$

Optimal. Leaf size=79

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] a^2*x*hypergeom([-3/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a
+b*x^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {253, 246, 245}

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
```

```
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx &= \frac{\left(\sqrt{a-bx^n}\,\sqrt{a+bx^n}\right)\int \left(a^2-b^2x^{2n}\right)^{3/2}\,dx}{\sqrt{a^2-b^2x^{2n}}}\\ &= \frac{\left(a^2\sqrt{a-bx^n}\,\sqrt{a+bx^n}\right)\int \left(1-\frac{b^2x^{2n}}{a^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\\ &= \frac{a^2x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 1.00

$$\frac{a^2x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), 1 +
 1/(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{n}+a\right)^{\frac{3}{2}} \left(b\,x^{n}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^(3/2)*(b*x^n+a)^(3/2),x)
[Out] int((a-b*x^n)^(3/2)*(b*x^n+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)^{\frac{3}{2}}(-bx^{n}+a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,x^n\right)}^{3/2}\,{\left(a-b\,x^n\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2),x)
[Out] int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a - bx^{n}\right)^{\frac{3}{2}} \left(a + bx^{n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
[Out] Integral((a - b*x**n)**(3/2)*(a + b*x**n)**(3/2), x)
```

3.282 $\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=76

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] x*hypergeom([-1/2, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)*(a-b*x^n)^(1/2)*(a+b*x
^n)^(1/2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {253, 246, 245}

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^
(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
```

```
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a-bx^n}\,\sqrt{a+bx^n}\,dx &= \frac{\left(\sqrt{a-bx^n}\,\sqrt{a+bx^n}\right)\int \sqrt{a^2-b^2x^{2n}}\,dx}{\sqrt{a^2-b^2x^{2n}}}\\
&= \frac{\left(\sqrt{a-bx^n}\,\sqrt{a+bx^n}\right)\int \sqrt{1-\frac{b^2x^{2n}}{a^2}}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}\\
&= \frac{x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 76, normalized size = 1.00

$$
\frac{x\sqrt{a-bx^n}\,\sqrt{a+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), 1 + 1/(
2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{bx^n+a}\,\sqrt{-bx^n+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \sqrt{-b\,x^{n}+a}\,\sqrt{b\,x^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^n+a)^(1/2)*(b*x^n+a)^(1/2),x)

[Out] int((-b*x^n+a)^(1/2)*(b*x^n+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^{n}+a}\,\sqrt{-bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x^{n}}\,\sqrt{a-b\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2),x)

[Out] int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a-bx^{n}}\,\sqrt{a+bx^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)

[Out] Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)

3.283 $\int (a-bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=72

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)/
((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {253, 246, 245}

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/
2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a-bx^n)^p (a+bx^n)^p \, dx &= \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \, dx \\ &= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \, dx \\ &= x (a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 1.00

$$x (a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n), (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n+a)^p(-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p(-bx^n+a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \left(-b\,x^n+a\right)^p \left(b\,x^n+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^n+a)^p*(b*x^n+a)^p,x)
```

```
[Out] int((-b*x^n+a)^p*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(-bx^n + a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x^n)^p\,(a - b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(a - b*x^n)^p,x)
```

```
[Out] int((a + b*x^n)^p*(a - b*x^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a - bx^n)^p\,(a + bx^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

3.284 $\int (a+bx^n)(c+dx^n)^4\,dx$

Optimal. Leaf size=132

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

```
[Out] a*c^4*x+c^3*(4*a*d+b*c)*x^(1+n)/(1+n)+2*c^2*d*(3*a*d+2*b*c)*x^(1+2*n)/(1+2*
n)+2*c*d^2*(2*a*d+3*b*c)*x^(1+3*n)/(1+3*n)+d^3*(a*d+4*b*c)*x^(1+4*n)/(1+4*n
)+b*d^4*x^(1+5*n)/(1+5*n)
```

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n) +
(d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^4\,dx &= \int \left(ac^4+c^3(bc+4ad)x^n+2c^2d(2bc+3ad)x^{2n}+2cd^2(3bc+2ad)x^{3n}+d^3(4bc+a\right.\\ &= ac^4x+\frac{c^3(bc+4ad)x^{1+n}}{1+n}+\frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n}+\frac{d^3(4b}{}\end{aligned}$$

Mathematica [A] time = 0.28, size = 110, normalized size = 0.83

$$\frac{bx(c+dx^n)^5-x\left(c^4+\frac{4c^3dx^n}{n+1}+\frac{6c^2d^2x^{2n}}{2n+1}+\frac{4cd^3x^{3n}}{3n+1}+\frac{d^4x^{4n}}{4n+1}\right)(bc-ad(5n+1))}{5dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n) +
 (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*n)
)/(1 + 4*n)))/(d + 5*d*n)
```

fricas [B] time = 0.68, size = 527, normalized size = 3.99

$$\frac{\left(24\,bd^4n^4 + 50\,bd^4n^3 + 35\,bd^4n^2 + 10\,bd^4n + bd^4\right)xx^{5\,n} + \left(4\,bcd^3 + ad^4 + 30\,\left(4\,bcd^3 + ad^4\right)n^4 + 61\,\left(4\,bcd^3 + ad^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] ((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*n
) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d^4
)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n) +
2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2
*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2
+ 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*
a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3*a*c^
2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4*a*c^3*d
 + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*(b*c^4 +
4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5 + 274*a*c
^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 2
74*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

giac [B] time = 0.22, size = 740, normalized size = 5.61

$$\frac{120\,ac^4n^5x + 24\,bd^4n^4xx^{5\,n} + 120\,bcd^3n^4xx^{4\,n} + 30\,ad^4n^4xx^{4\,n} + 240\,bc^2d^2n^4xx^{3\,n} + 160\,acd^3n^4xx^{3\,n} + 240\,bc^3dn}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] (120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30*
a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(3*
n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4*n^
4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5*n)
+ 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n^3*x*
x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642*a*c^2
*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 225*a*c^
4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a*d^4*n^2
```

```
*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*n) + 236*
b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^2*x*x^n +
284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) + 44*b*c*d^3*
n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) + 48*a*c*d^3*
n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n) + 14*b*c^4*
n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n) + 4*b*c*d^3*x
*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d^3*x*x^(3*n) +
4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n + 4*a*c^3*d*x*x^n
 + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

maple [A] time = 0.05, size = 138, normalized size = 1.05

$$\frac{b\,d^4 x\,\mathrm{e}^{5n\ln(x)}}{5n+1}+a\,c^4 x+\frac{(4ad+bc)\,c^3 x\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{2\,(3ad+2bc)\,c^2 dx\,\mathrm{e}^{2n\ln(x)}}{2n+1}+\frac{2\,(2ad+3bc)\,c\,d^2 x\,\mathrm{e}^{3n\ln(x)}}{3n+1}+\frac{(ad+4bc)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n)^4,x)
```

```
[Out] a*c^4*x+b*d^4/(1+5*n)*x*exp(n*ln(x))^5+c^3*(4*a*d+b*c)/(n+1)*x*exp(n*ln(x))
+d^3*(a*d+4*b*c)/(4*n+1)*x*exp(n*ln(x))^4+2*c*d^2*(2*a*d+3*b*c)/(3*n+1)*x*e
xp(n*ln(x))^3+2*c^2*d*(3*a*d+2*b*c)/(2*n+1)*x*exp(n*ln(x))^2
```

maxima [A] time = 0.61, size = 186, normalized size = 1.41

$$ac^4x+\frac{bd^4x^{5\,n+1}}{5\,n+1}+\frac{4\,bcd^3x^{4\,n+1}}{4\,n+1}+\frac{ad^4x^{4\,n+1}}{4\,n+1}+\frac{6\,bc^2d^2x^{3\,n+1}}{3\,n+1}+\frac{4\,acd^3x^{3\,n+1}}{3\,n+1}+\frac{4\,bc^3dx^{2\,n+1}}{2\,n+1}+\frac{6\,ac^2d^2x^{2\,n+1}}{2\,n+1}+\frac{bc^4x^{n+1}}{n+1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) + a
*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^3*
x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^(2*
n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.64, size = 131, normalized size = 0.99

$$a\,c^4\,x+\frac{x\,x^n\,\left(b\,c^4+4\,a\,d\,c^3\right)}{n+1}+\frac{x\,x^{4\,n}\,\left(a\,d^4+4\,b\,c\,d^3\right)}{4\,n+1}+\frac{b\,d^4\,x\,x^{5\,n}}{5\,n+1}+\frac{2\,c^2\,d\,x\,x^{2\,n}\,\left(3\,a\,d+2\,b\,c\right)}{2\,n+1}+\frac{2\,c\,d^2\,x\,x^{3\,n}\,\left(2\,a\,d+\right.}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^4,x)
```

```
[Out] a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*d
^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n)*(3*a*d +
2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n)*(2*a*d + 3*b*c))/(3*n + 1)
```

sympy [A] time = 3.67, size = 2744, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**2
 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 - 4*
b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*sqr
t(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt
(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/
(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**
2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)/2 +
12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3) - 3*b*d*
*4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3 + 12*a*c*
*2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4*x**(3/4)/
3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*
d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d*
*2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**3*x**(1/5)
 + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 284*a*c**3*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3*x*x**(2*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 354*a*c**2*d**2*n
**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a
*c**2*d**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 6*a*c**2*d**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 160*a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/(120*n**5 + 274
```

```
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d**3*n*x*x**(3*n)/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c*d**3*x*x**(3*n)/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 30*a*d**4*n**4*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*a*d**4*n**3*x*x
**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 41*a*d**4*n
**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 11*a
*d**4*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
a*d**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 1
20*b*c**4*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 154*b*c**4*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 71*b*c**4*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 14*b*c**4*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + b*c**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 240*b*c**3*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 428*b*c**3*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 236*b*c**3*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 52*b*c**3*d*n*x*x**(2*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c**3*d*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*b*c**2*d**2*n**4*x*x**(3*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*b*c**2*d**2*n*
*3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 294*b
*c**2*d**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 72*b*c**2*d**2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
*2 + 15*n + 1) + 6*b*c**2*d**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 120*b*c*d**3*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 244*b*c*d**3*n**3*x*x**(4*n)/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 164*b*c*d**3*n**2*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 44*b*c*d**3*n*x*x**(4*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c*d**3*x*x**
(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*d**4*n**
4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*b*d
**4*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 35*b*d**4*n**2*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 10*b*d**4*n*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + b*d**4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1), True))
```

3.285 $\int (a+bx^n)(c+dx^n)^3\,dx$

Optimal. Leaf size=99

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

```
[Out] a*c^3*x+c^2*(3*a*d+b*c)*x^(1+n)/(1+n)+3*c*d*(a*d+b*c)*x^(1+2*n)/(1+2*n)+d^2
*(a*d+3*b*c)*x^(1+3*n)/(1+3*n)+b*d^3*x^(1+4*n)/(1+4*n)
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1 +
 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(1
+ 4*n))/(1 + 4*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^3\,dx&=\int \left(ac^3+c^2(bc+3ad)x^n+3cd(bc+ad)x^{2n}+d^2(3bc+ad)x^{3n}+bd^3x^{4n}\right)dx\\&=ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}\end{aligned}$$

Mathematica [A] time = 0.13, size = 90, normalized size = 0.91

$$\frac{bx(c+dx^n)^4-x\left(c^3+\frac{3c^2dx^n}{n+1}+\frac{3cd^2x^{2n}}{2n+1}+\frac{d^3x^{3n}}{3n+1}\right)(bc-ad(4n+1))}{4dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n) +
 (3*c*d^2*x^(2*n))/(1 + 2*n) + (d^3*x^(3*n))/(1 + 3*n)))/(d + 4*d*n)
```

fricas [B] time = 1.01, size = 319, normalized size = 3.22

$$\left(6\,bd^3n^3 + 11\,bd^3n^2 + 6\,bd^3n + bd^3\right)xx^{4\,n} + \left(3\,bcd^2 + ad^3 + 8\left(3\,bcd^2 + ad^3\right)n^3 + 14\left(3\,bcd^2 + ad^3\right)n^2 + 7\left(3\,bcd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d
^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^
3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^
3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9
*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2
 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

giac [B] time = 0.19, size = 450, normalized size = 4.55

$$24\,ac^3n^4x + 6\,bd^3n^3xx^{4\,n} + 24\,bcd^2n^3xx^{3\,n} + 8\,ad^3n^3xx^{3\,n} + 36\,bc^2dn^3xx^{2\,n} + 36\,acd^2n^3xx^{2\,n} + 24\,bc^3n^3xx^{n} + 7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d^
3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 24*
b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x*x^
(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*n^2*
x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d*n^2*
x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n) + 7*a
*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) + 9*b*c^
3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3*b*c*d^2
*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x^(2*n) +
b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n +
 1)
```

maple [A] time = 0.06, size = 104, normalized size = 1.05

$$\frac{b\,d^3x\,\mathrm{e}^{4n\ln(x)}}{4n+1} + a\,c^3x + \frac{(3ad+bc)\,c^2x\,\mathrm{e}^{n\ln(x)}}{n+1} + \frac{3\,(ad+bc)\,cdx\,\mathrm{e}^{2n\ln(x)}}{2n+1} + \frac{(ad+3bc)\,d^2x\,\mathrm{e}^{3n\ln(x)}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n)^3,x)
```

```
[Out] a*c^3*x+b*d^3/(4*n+1)*x*exp(n*ln(x))^4+c^2*(3*a*d+b*c)/(n+1)*x*exp(n*ln(x))
+d^2*(a*d+3*b*c)/(3*n+1)*x*exp(n*ln(x))^3+3*c*d*(a*d+b*c)/(2*n+1)*x*exp(n*l
n(x))^2
```

maxima [A] time = 0.50, size = 140, normalized size = 1.41

$$ac^3x+\frac{bd^3x^{4\,n+1}}{4\,n+1}+\frac{3\,bcd^2x^{3\,n+1}}{3\,n+1}+\frac{ad^3x^{3\,n+1}}{3\,n+1}+\frac{3\,bc^2dx^{2\,n+1}}{2\,n+1}+\frac{3\,acd^2x^{2\,n+1}}{2\,n+1}+\frac{bc^3x^{n+1}}{n+1}+\frac{3\,ac^2dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) + a
*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*x^
(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.56, size = 99, normalized size = 1.00

$$a\,c^3\,x+\frac{x\,x^n\,\left(b\,c^3+3\,a\,d\,c^2\right)}{n+1}+\frac{x\,x^{3\,n}\,\left(a\,d^3+3\,b\,c\,d^2\right)}{3\,n+1}+\frac{b\,d^3\,x\,x^{4\,n}}{4\,n+1}+\frac{3\,c\,d\,x\,x^{2\,n}\,\left(a\,d+b\,c\right)}{2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^3,x)
```

```
[Out] a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*d
^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*c)
)/(2*n + 1)
```

sympy [A] time = 3.35, size = 1540, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(n,
 -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqrt(x
) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, E
q(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d*
*3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) -
```

```
 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*a*c*d
**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*sqrt(x)
+ 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n**4*x/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x
/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 5
7*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*
c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x
*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x**(3*
n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2*x*x**(3*n)/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n*x*x**(3*n)/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 24*b*c**3*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 26*b*c**3*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*
c**3*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x**(2*n)/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 3*b*c**2*d*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 24*b*c*d**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 42*b*c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3
*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*
*3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*d**3*n**2*x*x
**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*x*x**(4*n)/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1), True))
```

3.286 $\int (a+bx^n)(c+dx^n)^2\,dx$

Optimal. Leaf size=70

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

```
[Out] a*c^2*x+c*(2*a*d+b*c)*x^(1+n)/(1+n)+d*(a*d+2*b*c)*x^(1+2*n)/(1+2*n)+b*d^2*x
^(1+3*n)/(1+3*n)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^n+d(2bc+ad)x^{2n}+bd^2x^{3n}\right)dx\\ &= ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 1.00

$$\frac{bx(c+dx^n)^3-x\left(c^2+\frac{2cdx^n}{n+1}+\frac{d^2x^{2n}}{2n+1}\right)(bc-ad(3n+1))}{3dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] (b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) + (
d^2*x^(2*n))/(1 + 2*n)))/(d + 3*d*n)
```

fricas [B] time = 0.98, size = 175, normalized size = 2.50

$$\frac{\left(2\,bd^2n^2+3\,bd^2n+bd^2\right)xx^{3\,n}+\left(2\,bcd+ad^2+3\left(2\,bcd+ad^2\right)n^2+4\left(2\,bcd+ad^2\right)n\right)xx^{2\,n}+\left(bc^2+2\,acd+6\left(b\right.\right.}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c*
d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*(b
*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^
2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [B] time = 0.19, size = 232, normalized size = 3.31

$$\frac{6\,ac^2n^3x+2\,bd^2n^2xx^{3\,n}+6\,bcdn^2xx^{2\,n}+3\,ad^2n^2xx^{2\,n}+6\,bc^2n^2xx^n+12\,acdn^2xx^n+11\,ac^2n^2x+3\,bd^2nxx^{3\,n}+}{6\,n^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n^
2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3*b
*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n*x*
x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2*n)
+ a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

maple [A] time = 0.05, size = 74, normalized size = 1.06

$$\frac{b\,d^2x\,\mathrm{e}^{3n\ln(x)}}{3n+1}+a\,c^2x+\frac{(2ad+bc)\,cx\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{(ad+2bc)\,dx\,\mathrm{e}^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n)^2,x)
```

```
[Out] a*c^2*x+b*d^2/(3*n+1)*x*exp(n*ln(x))^3+c*(2*a*d+b*c)/(n+1)*x*exp(n*ln(x))+d
*(a*d+2*b*c)/(2*n+1)*x*exp(n*ln(x))^2
```

maxima [A] time = 0.44, size = 94, normalized size = 1.34

$$ac^2x + \frac{bd^2x^{3n+1}}{3n+1} + \frac{2\,bcdx^{2n+1}}{2n+1} + \frac{ad^2x^{2n+1}}{2n+1} + \frac{bc^2x^{n+1}}{n+1} + \frac{2\,acdx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*d
^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n +
 1)
```

mupad [B] time = 1.53, size = 71, normalized size = 1.01

$$a\,c^2\,x + \frac{x\,x^{2\,n}\,\left(a\,d^2 + 2\,b\,c\,d\right)}{2\,n+1} + \frac{x\,x^n\,\left(b\,c^2 + 2\,a\,d\,c\right)}{n+1} + \frac{b\,d^2\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^2,x)
```

```
[Out] a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*d
))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)
```

sympy [A] time = 1.96, size = 726, normalized size = 10.37

$$\begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2acdxx^n}{6n^3+11n^2+6n+1} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/x
 - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(x)
 + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (a*c
**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*
d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**
```

```
2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*
a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d*
*2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*
n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*
b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(
6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*d**2*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

3.287 $\int (a+bx^n)(c+dx^n)\,dx$

Optimal. Leaf size=40

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

```
[Out] a*c*x+(a*d+b*c)*x^(1+n)/(1+n)+b*d*x^(1+2*n)/(1+2*n)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)\,dx &= \int \left(ac+(bc+ad)x^n+bdx^{2n}\right)dx\\ &= acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.92

$$x\left(\frac{x^n(ad+bc)}{n+1}+ac+\frac{bdx^{2n}}{2n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

fricas [A] time = 0.98, size = 69, normalized size = 1.72

$$\frac{(bdn + bd)xx^{2n} + (bc + ad + 2(bc + ad)n)xx^{n} + \left(2acn^{2} + 3acn + ac\right)x}{2n^{2} + 3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2
 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

giac [B] time = 0.17, size = 83, normalized size = 2.08

$$\frac{2acn^{2}x + bdnxx^{2n} + 2bcnxx^{n} + 2adnxx^{n} + 3acnx + bdxx^{2n} + bcxx^{n} + adxx^{n} + acx}{2n^{2} + 3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
+ b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

maple [A] time = 0.04, size = 43, normalized size = 1.08

$$\frac{bdx\,e^{2n\ln(x)}}{2n+1} + acx + \frac{(ad+bc)\,x\,e^{n\ln(x)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n),x)
```

```
[Out] a*c*x+(a*d+b*c)/(n+1)*x*exp(n*ln(x))+b*d/(2*n+1)*x*exp(n*ln(x))^2
```

maxima [A] time = 0.51, size = 48, normalized size = 1.20

$$acx + \frac{bdx^{2n+1}}{2n+1} + \frac{bcx^{n+1}}{n+1} + \frac{adx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/(
n + 1)
```

mupad [B] time = 1.48, size = 38, normalized size = 0.95

$$a\,c\,x+\frac{x\,x^n\,\left(a\,d+b\,c\right)}{n+1}+\frac{b\,d\,x\,x^{2\,n}}{2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n),x)
```

```
[Out] a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)
```

sympy [A] time = 0.65, size = 236, normalized size = 5.90

$$\begin{cases} acx + ad\log{(x)} + bc\log{(x)} - \frac{bd}{x} & \text{for}\: n = -1 \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd\log{(x)} & \text{for}\: n = -\frac{1}{2} \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n),x)
```

```
[Out] Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2*
a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*n*
*2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1) + 2
*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*
x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n
)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

3.288 $\int \frac{a+bx^n}{c+dx^n}\,dx$

Optimal. Leaf size=43

$$\frac{bx}{d}-\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

```
[Out] b*x/d-(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 245}

$$\frac{bx}{d}-\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^n}\,dx}{d}$$
$$= \frac{bx}{d} - \frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.93

$$\frac{x\left((ad-bc)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bc\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n),x]
[Out] (x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)]))/(c*d)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n+a}{dx^n+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral((b*x^n + a)/(d*x^n + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)/(d*x^n + c), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{b\,x^n+a}{d\,x^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)/(c+d*x^n),x)
```

```
[Out] int((b*x^n+a)/(c+d*x^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-(bc-ad)\int \frac{1}{d^2x^n+cd}\,dx+\frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,x^n}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n), x)
```

sympy [C] time = 3.33, size = 73, normalized size = 1.70

$$\frac{ax\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}-\frac{bx\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^2\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n),x)
```

```
[Out] a*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*n**2*gamma(1 +
 1/n)) - b*x*lerchphi(c*x**(-n)*exp_polar(I*pi)/d, 1, exp_polar(I*pi)/n)*ga
mma(1/n)/(d*n**2*gamma(1 + 1/n))
```

3.289 $\int \frac{a+bx^n}{(c+dx^n)^2}\,dx$

Optimal. Leaf size=73

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

```
[Out] -(-a*d+b*c)*x/c/d/n/(c+d*x^n)+(b*c-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],
-d*x^n/c)/c^2/d/n
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = -\frac{(bc-ad)x}{cdn\,(c+dx^n)} + \frac{(bc-ad(1-n))\int \frac{1}{c+dx^n}\,dx}{cdn}$$
$$= -\frac{(bc-ad)x}{cdn\,(c+dx^n)} + \frac{(bc-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.77

$$\frac{x\left(\frac{b}{c+dx^n}-\frac{(ad(n-1)+bc)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}\right)}{d-dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] (x*(b/(c + d*x^n) - ((b*c + a*d*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((d*x^n)/c)])/c^2))/(d - d*n)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n+a}{d^2x^{2n}+2\,cdx^n+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{b\,x^n+a}{(d\,x^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)/(d*x^n+c)^2,x)
```

```
[Out] int((b*x^n+a)/(d*x^n+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(ad(n-1)+bc)\int \frac{1}{cd^2nx^n+c^2dn}\,dx - \frac{(bc-ad)x}{cd^2nx^n+c^2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*x
/(c*d^2*n*x^n + c^2*d*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^n}{(c+d\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^2, x)
```

sympy [C] time = 8.69, size = 592, normalized size = 8.11

$$a\left(\frac{nx\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c\left(cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{c\left(cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c\left(cn^3\Gamma\left(1+\frac{1}{n}\right)+dn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] a*(n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*(c*n**3*gam
ma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(c*(c*n**3*gamm
a(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1/n)*gamma(1/n)/(c*(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 +
1/n))) + d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(
c**2*(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) - d*x*x**n*lerch
phi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c**2*(c*n**3*gamma(1 + 1/
n) + d*n**3*x**n*gamma(1 + 1/n)))) + b*(n**2*x*x**n*gamma(1 + 1/n)/(c*(c*n*
```

```
*3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))) - n*x*x**n*lerchphi(d*x**n
*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 + 1/n) +
d*n**3*x**n*gamma(2 + 1/n))) + n*x*x**n*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 +
 1/n) + d*n**3*x**n*gamma(2 + 1/n))) - x*x**n*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*ga
mma(2 + 1/n))) - d*n*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1
/n)*gamma(1 + 1/n)/(c**2*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n
))) - d*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 +
 1/n)/(c**2*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))))
```

3.290 $\int \frac{a+bx^n}{(c+dx^n)^3}\,dx$

Optimal. Leaf size=78

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\left(c+dx^n\right)^2}$$

```
[Out] -1/2*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^2+1/2*(b*c-a*d*(1-2*n))*x*hypergeom([2, 1
/n],[1+1/n],-d*x^n/c)/c^3/d/n
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\left(c+dx^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hyperge
ometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad(1-2n))\int \frac{1}{(c+dx^n)^2}\,dx}{2cdn}$$
$$= -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad(1-2n))x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.74

$$\frac{x\left(\frac{b}{(c+dx^n)^2}-\frac{(ad(2n-1)+bc)\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3}\right)}{d-2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] (x*(b/(c + d*x^n)^2 - ((b*c + a*d*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^3))/(d - 2*d*n)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n+a}{d^3x^{3n}+3\,cd^2x^{2n}+3\,c^2dx^n+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{b\,x^{n}+a}{\left(d\,x^{n}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)/(d*x^n+c)^3,x)
```

```
[Out] int((b*x^n+a)/(d*x^n+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(\left(2\,n^{2}-3\,n+1\right)ad+bc(n-1)\right)\int \frac{1}{2\left(c^{2}d^{2}n^{2}x^{n}+c^{3}dn^{2}\right)}\,dx+\frac{\left(ad^{2}(2\,n-1)+bcd\right)xx^{n}+\left(acd(3\,n-1)-bc^{2}(n-1\right.}{2\left(c^{2}d^{3}n^{2}x^{2\,n}+2\,c^{3}d^{2}n^{2}x^{n}+c^{4}dn^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*x^n + c^3*
d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*c*d*(3*n - 1) - b*c^
2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^3, x)
```

sympy [C] time = 64.99, size = 3706, normalized size = 47.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] a*(2*c*n**2*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(2*c**4
*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4
*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 3*c*n**
2*x*gamma(1/n)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1
```

```
/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gam
ma(1 + 1/n)) - 3*c*n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n
)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*
d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n))
- c*n*x*gamma(1/n)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1
 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)
*gamma(1 + 1/n)) + c*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n
)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*
d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n))
+ 6*d*n**2*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(2*
c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*
n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 5*d
*n**2*x*x**n*gamma(1/n)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*ga
mma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**
(3*n)*gamma(1 + 1/n)) - 9*d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1,
1/n)*gamma(1/n)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 +
1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*ga
mma(1 + 1/n)) - 2*d*n*x*x**n*gamma(1/n)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**
3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2
*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n)) + 3*d*x*x**n*lerchphi(d*x**n*exp_pola
r(I*pi)/c, 1, 1/n)*gamma(1/n)/(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x
**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n*
*4*x**(3*n)*gamma(1 + 1/n)) + 6*d**2*n**2*x*x**(2*n)*lerchphi(d*x**n*exp_po
lar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n
**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d*
*3*n**4*x**(3*n)*gamma(1 + 1/n))) + 2*d**2*n**2*x*x**(2*n)*gamma(1/n)/(c*(2
*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2
*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n))) - 9
*d**2*n*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c
*(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d
**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n)))
- d**2*n*x*x**(2*n)*gamma(1/n)/(c*(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n*
*4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d**
3*n**4*x**(3*n)*gamma(1 + 1/n))) + 3*d**2*x*x**(2*n)*lerchphi(d*x**n*exp_po
lar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*(2*c**4*n**4*gamma(1 + 1/n) + 6*c**3*d*n
**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n) + 2*c*d*
*3*n**4*x**(3*n)*gamma(1 + 1/n))) + 2*d**3*n**2*x*x**(3*n)*lerchphi(d*x**n*
exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c**2*(2*c**4*n**4*gamma(1 + 1/n) + 6
*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1 + 1/n)
 + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n))) - 3*d**3*n*x*x**(3*n)*lerchphi(d
*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c**2*(2*c**4*n**4*gamma(1 + 1/
n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(1
+ 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n))) + d**3*x*x**(3*n)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c**2*(2*c**4*n**4*gamma(1 +
1/n) + 6*c**3*d*n**4*x**n*gamma(1 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(
```

1 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(1 + 1/n)))) + b*(2*c*n**3*x*x**n*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma (2 + 1/n)) - c*n**2*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/ n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) + c*n**2*x*x**n*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) - c*n*x*x**n*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) + c*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) + 3*d*n**3*x*x**(2*n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) - 3*d*n**2*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) + d*n**2*x*x**(2*n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) - 2*d*n*x*x**(2*n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) + 3*d*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n)) + d**2*n**3*x*x**(3*n)*gamma(1 + 1/n)/(c*(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n))) - 3*d**2*n**2*x*x**(3*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n))) - d**2*n*x*x**(3*n)*gamma(1 + 1/n)/(c*(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n))) + 3*d**2*x*x**(3*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n))) - d**3*n**2*x*x**(4*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c**2*(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n))) + d**3*x*x**(4*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c**2*(2*c**4*n**4*gamma(2 + 1/n) + 6*c**3*d*n**4*x**n*gamma(2 + 1/n) + 6*c**2*d**2*n**4*x**(2*n)*gamma(2 + 1/n) + 2*c*d**3*n**4*x**(3*n)*gamma(2 + 1/n))))

3.291 $\int \frac{a+bx^n}{(c+dx^n)^4}\,dx$

Optimal. Leaf size=78

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/d/n/(c+d*x^n)^3+1/3*(b*c-a*d*(1-3*n))*x*hypergeom([3, 1
/n],[1+1/n],-d*x^n/c)/c^4/d/n
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hyperge
ometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3} + \frac{(bc-ad(1-3n))\int \frac{1}{(c+dx^n)^3}\,dx}{3cdn}$$
$$= -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3} + \frac{(bc-ad(1-3n))x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}$$

Mathematica [**A**] time = 0.07, size = 58, normalized size = 0.74

$$\frac{x\left(\frac{b}{(c+dx^n)^3}-\frac{(ad(3n-1)+bc)\,{}_2F_1\left(4,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^4}\right)}{d-3dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] (x*(b/(c + d*x^n)^3 - ((b*c + a*d*(-1 + 3*n))*Hypergeometric2F1[4, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^4))/(d - 3*d*n)
```

fricas [**F**] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n+a}{d^4x^{4n}+4\,cd^3x^{3n}+6\,c^2d^2x^{2n}+4\,c^3dx^n+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4
*c^3*d*x^n + c^4), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{b\,x^{n}+a}{\left(d\,x^{n}+c\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)/(d*x^n+c)^4,x)
```

```
[Out] int((b*x^n+a)/(d*x^n+c)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(\left(2\,n^{2}-3\,n+1\right)bc+\left(6\,n^{3}-11\,n^{2}+6\,n-1\right)ad\right)\int \frac{1}{6\left(c^{3}d^{2}n^{3}x^{n}+c^{4}dn^{3}\right)}\,dx+\frac{\left(\left(6\,n^{2}-5\,n+1\right)ad^{3}+bcd^{2}(2\,n-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrate(1/6/(c^3
*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*(((6*n^2 - 5*n + 1)*a*d^3 + b*c*d^2*(2*
n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*d^2 + b*c^2*d*(5*n - 2))*x*x^n
 - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n^3*x
^(3*n) + 3*c^4*d^3*n^3*x^(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^4,x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

```
[Out] Timed out
```

3.292 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

Optimal. Leaf size=158

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}$$

```
[Out] a^2*d^3*x+a*d^2*(3*a*e+2*b*d)*x^(1+n)/(1+n)+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)
*x^(1+2*n)/(1+2*n)+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)*x^(1+3*n)/(1+3*n)+b*e^2*
(2*a*e+3*b*d)*x^(1+4*n)/(1+4*n)+b^2*e^3*x^(1+5*n)/(1+5*n)
```

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*b
*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4*n)
 + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2 (d+ex^n)^3 \, dx &= \int \left(a^2d^3+ad^2(2bd+3ae)x^n+d\left(b^2d^2+6abde+3a^2e^2\right)x^{2n}+e\left(3b^2d^2+6abde+a\right.\right.\\ &= a^2d^3x+\frac{ad^2(2bd+3ae)x^{1+n}}{1+n}+\frac{d\left(b^2d^2+6abde+3a^2e^2\right)x^{1+2n}}{1+2n}+\frac{e\left(3b^2d^2+6abde\right.}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.94

$$x\left(\frac{dx^{2n}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3+\frac{ad^2x^n(3ae+2bd)}{n+1}+\frac{be^2x^{4n}(2ae+3bd)}{4n+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
+ 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^(3
*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^(5*
n))/(1 + 5*n))
```

fricas [B] time = 1.00, size = 667, normalized size = 4.22

$$\frac{\left(24\,b^2e^3n^4+50\,b^2e^3n^3+35\,b^2e^3n^2+10\,b^2e^3n+b^2e^3\right)xx^{5\,n}+\left(3\,b^2de^2+2\,abe^3+30\left(3\,b^2de^2+2\,abe^3\right)n^4+61\,(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^3
)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4 +
 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 11*(
3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^
3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 + 12*(3
*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*d^2*e +
 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(b^2*d^3
+ 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)
*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2*a*b*d^3 +
 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d
^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)
*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d
^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 +
 15*n + 1)
```

giac [B] time = 0.24, size = 947, normalized size = 5.99

$$\frac{120\,a^2d^3n^5x+60\,b^2d^3n^4xx^{2\,n}+240\,abd^3n^4xx^n+120\,b^2d^2n^4xx^{3\,n}e+360\,abd^2n^4xx^{2\,n}e+360\,a^2d^2n^4xx^ne+274\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] (120*a^2*d^3*n^5*x + 60*b^2*d^3*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*x^n + 120
*b^2*d^2*n^4*x*x^(3*n)*e + 360*a*b*d^2*n^4*x*x^(2*n)*e + 360*a^2*d^2*n^4*x*
x^n*e + 274*a^2*d^3*n^4*x + 107*b^2*d^3*n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x
^n + 90*b^2*d*n^4*x*x^(4*n)*e^2 + 240*a*b*d*n^4*x*x^(3*n)*e^2 + 180*a^2*d*n
```

```
^4*x*x^(2*n)*e^2 + 234*b^2*d^2*n^3*x*x^(3*n)*e + 642*a*b*d^2*n^3*x*x^(2*n)*
e + 462*a^2*d^2*n^3*x*x^n*e + 225*a^2*d^3*n^3*x + 59*b^2*d^3*n^2*x*x^(2*n)
+ 142*a*b*d^3*n^2*x*x^n + 24*b^2*n^4*x*x^(5*n)*e^3 + 60*a*b*n^4*x*x^(4*n)*e
^3 + 40*a^2*n^4*x*x^(3*n)*e^3 + 183*b^2*d*n^3*x*x^(4*n)*e^2 + 468*a*b*d*n^3
*x*x^(3*n)*e^2 + 321*a^2*d*n^3*x*x^(2*n)*e^2 + 147*b^2*d^2*n^2*x*x^(3*n)*e
+ 354*a*b*d^2*n^2*x*x^(2*n)*e + 213*a^2*d^2*n^2*x*x^n*e + 85*a^2*d^3*n^2*x
+ 13*b^2*d^3*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n + 50*b^2*n^3*x*x^(5*n)*e^3 +
122*a*b*n^3*x*x^(4*n)*e^3 + 78*a^2*n^3*x*x^(3*n)*e^3 + 123*b^2*d*n^2*x*x^(4
*n)*e^2 + 294*a*b*d*n^2*x*x^(3*n)*e^2 + 177*a^2*d*n^2*x*x^(2*n)*e^2 + 36*b^
2*d^2*n*x*x^(3*n)*e + 78*a*b*d^2*n*x*x^(2*n)*e + 42*a^2*d^2*n*x*x^n*e + 15*
a^2*d^3*n*x + b^2*d^3*x*x^(2*n) + 2*a*b*d^3*x*x^n + 35*b^2*n^2*x*x^(5*n)*e^
3 + 82*a*b*n^2*x*x^(4*n)*e^3 + 49*a^2*n^2*x*x^(3*n)*e^3 + 33*b^2*d*n*x*x^(4
*n)*e^2 + 72*a*b*d*n*x*x^(3*n)*e^2 + 39*a^2*d*n*x*x^(2*n)*e^2 + 3*b^2*d^2*x
*x^(3*n)*e + 6*a*b*d^2*x*x^(2*n)*e + 3*a^2*d^2*x*x^n*e + a^2*d^3*x + 10*b^2
*n*x*x^(5*n)*e^3 + 22*a*b*n*x*x^(4*n)*e^3 + 12*a^2*n*x*x^(3*n)*e^3 + 3*b^2*
d*x*x^(4*n)*e^2 + 6*a*b*d*x*x^(3*n)*e^2 + 3*a^2*d*x*x^(2*n)*e^2 + b^2*x*x^(
5*n)*e^3 + 2*a*b*x*x^(4*n)*e^3 + a^2*x*x^(3*n)*e^3)/(120*n^5 + 274*n^4 + 22
5*n^3 + 85*n^2 + 15*n + 1)
```

maple [A] time = 0.06, size = 164, normalized size = 1.04

$$\frac{b^2e^3x\,\mathrm{e}^{5n\ln(x)}}{5n+1}+a^2d^3x+\frac{(3ae+2bd)\,a\,d^2x\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{(2ae+3bd)\,b\,e^2x\,\mathrm{e}^{4n\ln(x)}}{4n+1}+\frac{\left(3a^2e^2+6abde+b^2d^2\right)dx\,\mathrm{e}^{2n\ln(x)}}{2n+1}+\frac{(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d+e*x^n)^3,x)
```

```
[Out] a^2*d^3*x+b^2*e^3/(5*n+1)*x*exp(n*ln(x))^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/
(2*n+1)*x*exp(n*ln(x))^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(3*n+1)*x*exp(n*ln
(x))^3+a*d^2*(3*a*e+2*b*d)/(n+1)*x*exp(n*ln(x))+b*e^2*(2*a*e+3*b*d)/(4*n+1)
*x*exp(n*ln(x))^4
```

maxima [A] time = 0.62, size = 242, normalized size = 1.53

$$a^2d^3x+\frac{b^2e^3x^{5n+1}}{5n+1}+\frac{3b^2de^2x^{4n+1}}{4n+1}+\frac{2abe^3x^{4n+1}}{4n+1}+\frac{3b^2d^2ex^{3n+1}}{3n+1}+\frac{6abde^2x^{3n+1}}{3n+1}+\frac{a^2e^3x^{3n+1}}{3n+1}+\frac{b^2d^3x^{2n+1}}{2n+1}+\frac{6abd^2ex^{2n}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^2*d^3*x + b^2*e^3*x^(5*n + 1)/(5*n + 1) + 3*b^2*d*e^2*x^(4*n + 1)/(4*n +
1) + 2*a*b*e^3*x^(4*n + 1)/(4*n + 1) + 3*b^2*d^2*e*x^(3*n + 1)/(3*n + 1) +
6*a*b*d*e^2*x^(3*n + 1)/(3*n + 1) + a^2*e^3*x^(3*n + 1)/(3*n + 1) + b^2*d^3
*x^(2*n + 1)/(2*n + 1) + 6*a*b*d^2*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*d*e^2*x^
```

```
(2*n + 1)/(2*n + 1) + 2*a*b*d^3*x^(n + 1)/(n + 1) + 3*a^2*d^2*e*x^(n + 1)/(
n + 1)
```

mupad [B] time = 1.71, size = 157, normalized size = 0.99

$$a^2\,d^3\,x+\frac{x\,x^{2\,n}\,\left(3\,a^2\,d\,e^2+6\,a\,b\,d^2\,e+b^2\,d^3\right)}{2\,n+1}+\frac{x\,x^{3\,n}\,\left(a^2\,e^3+6\,a\,b\,d\,e^2+3\,b^2\,d^2\,e\right)}{3\,n+1}+\frac{b^2\,e^3\,x\,x^{5\,n}}{5\,n+1}+\frac{a\,d^2\,x\,x^n\,\left(3\,a\,e+\right.}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^3,x)
```

```
[Out] a^2*d^3*x + (x*x^(2*n)*(b^2*d^3 + 3*a^2*d*e^2 + 6*a*b*d^2*e))/(2*n + 1) + (
x*x^(3*n)*(a^2*e^3 + 3*b^2*d^2*e + 6*a*b*d*e^2))/(3*n + 1) + (b^2*e^3*x*x^(
5*n))/(5*n + 1) + (a*d^2*x*x^n*(3*a*e + 2*b*d))/(n + 1) + (b*e^2*x*x^(4*n)*
(2*a*e + 3*b*d))/(4*n + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

3.293 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

Optimal. Leaf size=112

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

```
[Out] a^2*d^2*x+2*a*d*(a*e+b*d)*x^(1+n)/(1+n)+(a^2*e^2+4*a*b*d*e+b^2*d^2)*x^(1+2*
n)/(1+2*n)+2*b*e*(a*e+b*d)*x^(1+3*n)/(1+3*n)+b^2*e^2*x^(1+4*n)/(1+4*n)
```

Rubi [A] time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3*n
) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = \int \left(a^2d^2+2ad(bd+ae)x^n+\left(b^2d^2+4abde+a^2e^2\right)x^{2n}+2be(bd+ae)x^{3n}+b^2e^2x^{4n}\right) dx$$
$$= a^2d^2x+\frac{2ad(bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d^2+4abde+a^2e^2\right)x^{1+2n}}{1+2n}+\frac{2be(bd+ae)x^{1+3n}}{1+3n}+\frac{b^2e^2x^{1+4n}}{1+4n}$$

Mathematica [A] time = 0.20, size = 105, normalized size = 0.94

$$x\left(\frac{x^{2n}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2+\frac{2bex^{3n}(ae+bd)}{3n+1}+\frac{2adx^n(ae+bd)}{n+1}+\frac{b^2e^2x^{4n}}{4n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2*
e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^2*
x^(4*n))/(1 + 4*n))
```

fricas [B] time = 0.86, size = 370, normalized size = 3.30

$$(6 b^2e^2n^3 + 11 b^2e^2n^2 + 6 b^2e^2n + b^2e^2)xx^{4n} + 2 (b^2de + abe^2 + 8 (b^2de + abe^2)n^3 + 14 (b^2de + abe^2)n^2 + 7 (b^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*n) + 2*(b^
2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d*e + a*b*e^2)*n^2 +
7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2 + 12*(b
^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 + 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^2
+ 8*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*e + 2
4*(a*b*d^2 + a^2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 + a^2*d
*e)*n)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 10*a^2*d
^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

giac [B] time = 0.22, size = 539, normalized size = 4.81

$$24 a^2d^2n^4x + 12 b^2d^2n^3xx^{2n} + 48 abd^2n^3xx^n + 16 b^2dn^3xx^{3n}e + 48 abdn^3xx^{2n}e + 48 a^2dn^3xx^ne + 50 a^2d^2n^3x +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] (24*a^2*d^2*n^4*x + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 16*b^
2*d*n^3*x*x^(3*n)*e + 48*a*b*d*n^3*x*x^(2*n)*e + 48*a^2*d*n^3*x*x^n*e + 50*
a^2*d^2*n^3*x + 19*b^2*d^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 6*b^2*n^3
*x*x^(4*n)*e^2 + 16*a*b*n^3*x*x^(3*n)*e^2 + 12*a^2*n^3*x*x^(2*n)*e^2 + 28*b
^2*d*n^2*x*x^(3*n)*e + 76*a*b*d*n^2*x*x^(2*n)*e + 52*a^2*d*n^2*x*x^n*e + 35
*a^2*d^2*n^2*x + 8*b^2*d^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 11*b^2*n^2*x*
x^(4*n)*e^2 + 28*a*b*n^2*x*x^(3*n)*e^2 + 19*a^2*n^2*x*x^(2*n)*e^2 + 14*b^2*
d*n*x*x^(3*n)*e + 32*a*b*d*n*x*x^(2*n)*e + 18*a^2*d*n*x*x^n*e + 10*a^2*d^2*
n*x + b^2*d^2*x*x^(2*n) + 2*a*b*d^2*x*x^n + 6*b^2*n*x*x^(4*n)*e^2 + 14*a*b*
n*x*x^(3*n)*e^2 + 8*a^2*n*x*x^(2*n)*e^2 + 2*b^2*d*x*x^(3*n)*e + 4*a*b*d*x*x
^(2*n)*e + 2*a^2*d*x*x^n*e + a^2*d^2*x + b^2*x*x^(4*n)*e^2 + 2*a*b*x*x^(3*n
)*e^2 + a^2*x*x^(2*n)*e^2)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

maple [A] time = 0.06, size = 117, normalized size = 1.04

$$\frac{b^{2}e^{2}x\,\mathrm{e}^{4n\ln(x)}}{4n+1}+a^{2}d^{2}x+\frac{2\left(ae+bd\right)adx\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{2\left(ae+bd\right)bex\,\mathrm{e}^{3n\ln(x)}}{3n+1}+\frac{\left(a^{2}e^{2}+4abde+b^{2}d^{2}\right)x\,\mathrm{e}^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d+e*x^n)^2,x)
```

```
[Out] a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(2*n+1)*x*exp(n*ln(x))^2+b^2*e^2/(4*n
+1)*x*exp(n*ln(x))^4+2*a*d*(a*e+b*d)/(n+1)*x*exp(n*ln(x))+2*b*e*(a*e+b*d)/(
3*n+1)*x*exp(n*ln(x))^3
```

maxima [A] time = 0.60, size = 168, normalized size = 1.50

$$a^{2}d^{2}x+\frac{b^{2}e^{2}x^{4\,n+1}}{4\,n+1}+\frac{2\,b^{2}dex^{3\,n+1}}{3\,n+1}+\frac{2\,abe^{2}x^{3\,n+1}}{3\,n+1}+\frac{b^{2}d^{2}x^{2\,n+1}}{2\,n+1}+\frac{4\,abdex^{2\,n+1}}{2\,n+1}+\frac{a^{2}e^{2}x^{2\,n+1}}{2\,n+1}+\frac{2\,abd^{2}x^{n+1}}{n+1}+\frac{2\,a^{2}dex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*d^2*x + b^2*e^2*x^(4*n + 1)/(4*n + 1) + 2*b^2*d*e*x^(3*n + 1)/(3*n + 1)
 + 2*a*b*e^2*x^(3*n + 1)/(3*n + 1) + b^2*d^2*x^(2*n + 1)/(2*n + 1) + 4*a*b*
d*e*x^(2*n + 1)/(2*n + 1) + a^2*e^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^2*x^(n
+ 1)/(n + 1) + 2*a^2*d*e*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.57, size = 108, normalized size = 0.96

$$a^{2}\,d^{2}\,x+\frac{x\,x^{2\,n}\,\left(a^{2}\,e^{2}+4\,a\,b\,d\,e+b^{2}\,d^{2}\right)}{2\,n+1}+\frac{b^{2}\,e^{2}\,x\,x^{4\,n}}{4\,n+1}+\frac{2\,b\,e\,x\,x^{3\,n}\,\left(a\,e+b\,d\right)}{3\,n+1}+\frac{2\,a\,d\,x\,x^{n}\,\left(a\,e+b\,d\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

```
[Out] a^2*d^2*x + (x*x^(2*n)*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/(2*n + 1) + (b^2*e^
2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d*x
*x^n*(a*e + b*d))/(n + 1)
```

sympy [A] time = 77.47, size = 1765, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(x
) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2/(
3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x) +
 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2*log
(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3*a**2
*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d*e*x**
(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x) - 3*b*
*2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e + b*d)/3
 - 4*b**2*e**2*log(x**(-1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - sqrt(x)*(-4*a*
*2*e**2 - 16*a*b*d*e - 4*b**2*d**2)/2, Eq(n, -1/4)), (24*a**2*d**2*n**4*x/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 10*a**2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + a**2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d*e*n**3*
x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2*x*x**n/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + a**2*e**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*
d**2*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a*b*d**2*n**
2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*d**2*n*x*x**n/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*d**2*x*x**n/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + 48*a*b*d*e*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 +
35*n**2 + 10*n + 1) + 76*a*b*d*e*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 32*a*b*d*e*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 4*a*b*d*e*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16
*a*b*e**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*a*b
*e**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*b*e**
2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*e**2*x*x**(
3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b**2*d**2*n**3*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*d**2*n**2*x*x**(2*n)/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*d**2*n*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*d**2*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 16*b**2*d*e*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35
*n**2 + 10*n + 1) + 28*b**2*d*e*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 14*b**2*d*e*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 2*b**2*d*e*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6
*b**2*e**2*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*
*2*e**2*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*e
**2*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*e**2*x*x**
(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1), True))
```

3.294 $\int (a+bx^n)^2 (c+dx^n)\,dx$

Optimal. Leaf size=70

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

```
[Out] a^2*c*x+a*(a*d+2*b*c)*x^(1+n)/(1+n)+b*(2*a*d+b*c)*x^(1+2*n)/(1+2*n)+b^2*d*x
^(1+3*n)/(1+3*n)
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2 (c+dx^n)\,dx &= \int \left(a^2c+a(2bc+ad)x^n+b(bc+2ad)x^{2n}+b^2dx^{3n}\right)dx\\ &= a^2cx+\frac{a(2bc+ad)x^{1+n}}{1+n}+\frac{b(bc+2ad)x^{1+2n}}{1+2n}+\frac{b^2dx^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.11, size = 70, normalized size = 1.00

$$\frac{dx(a+bx^n)^3-x\left(a^2+\frac{2abx^n}{n+1}+\frac{b^2x^{2n}}{2n+1}\right)(ad-b(3cn+c))}{3bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) + (
b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

fricas [B] time = 1.08, size = 175, normalized size = 2.50

$$\frac{\left(2\, b^{2} d n^{2} + 3\, b^{2} d n + b^{2} d\right) x x^{3\, n} + \left(b^{2} c + 2\, a b d + 3\, \left(b^{2} c + 2\, a b d\right) n^{2} + 4\, \left(b^{2} c + 2\, a b d\right) n\right) x x^{2\, n} + \left(2\, a b c + a^{2} d + 6\, \left(2\right.\right.}{6\, n^{3} + 11\, n^{2} + 6\, n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
+ 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*(2
*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*
c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [B] time = 0.27, size = 232, normalized size = 3.31

$$\frac{6\, a^{2} c n^{3} x + 2\, b^{2} d n^{2} x x^{3\, n} + 3\, b^{2} c n^{2} x x^{2\, n} + 6\, a b d n^{2} x x^{2\, n} + 12\, a b c n^{2} x x^{n} + 6\, a^{2} d n^{2} x x^{n} + 11\, a^{2} c n^{2} x + 3\, b^{2} d n x x^{3\, n} + }{6\, n^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n^
2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3*b
^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*n*x
*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n) +
2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

maple [A] time = 0.05, size = 74, normalized size = 1.06

$$\frac{b^{2} d x\, {\mathrm e}^{3 n \ln \left(x \right)}}{3 n +1}+a^{2} c x +\frac{\left(a d +2 b c \right) a x\, {\mathrm e}^{n \ln \left(x \right)}}{n +1}+\frac{\left(2 a d +b c \right) b x\, {\mathrm e}^{2 n \ln \left(x \right)}}{2 n +1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d*x^n+c),x)
```

```
[Out] a^2*c*x+a*(a*d+2*b*c)/(n+1)*x*exp(n*ln(x))+b*(2*a*d+b*c)/(2*n+1)*x*exp(n*ln
(x))^2+b^2*d/(3*n+1)*x*exp(n*ln(x))^3
```

maxima [A] time = 0.63, size = 94, normalized size = 1.34

$$a^2cx + \frac{b^2dx^{3n+1}}{3n+1} + \frac{b^2cx^{2n+1}}{2n+1} + \frac{2\,abdx^{2n+1}}{2n+1} + \frac{2\,abcx^{n+1}}{n+1} + \frac{a^2dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*c*x + b^2*d*x^(3*n + 1)/(3*n + 1) + b^2*c*x^(2*n + 1)/(2*n + 1) + 2*a*b
*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^(n + 1)/(n +
 1)
```

mupad [B] time = 1.53, size = 71, normalized size = 1.01

$$a^2\,c\,x + \frac{x\,x^{2\,n}\,\left(c\,b^2 + 2\,a\,d\,b\right)}{2\,n+1} + \frac{x\,x^n\,\left(d\,a^2 + 2\,b\,c\,a\right)}{n+1} + \frac{b^2\,d\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(c + d*x^n),x)
```

```
[Out] a^2*c*x + (x*x^(2*n)*(b^2*c + 2*a*b*d))/(2*n + 1) + (x*x^n*(a^2*d + 2*a*b*c
))/(n + 1) + (b^2*d*x*x^(3*n))/(3*n + 1)
```

sympy [A] time = 2.74, size = 726, normalized size = 10.37

$$\begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/x
 - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt
(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (a**
2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*
c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**
```

```
2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a
**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*
c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11
*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n*
*2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 +
 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b
**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/
(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*d*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

3.295 $\int \frac{(a+bx^n)^2}{c+dx^n} dx$

Optimal. Leaf size=84

$$\frac{x(bc-ad)^2 {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx\left(a+bx^n\right)}{d(n+1)}$$

```
[Out] -b*(b*c*(1+n)-a*d*(1+2*n))*x/d^2/(1+n)+b*x*(a+b*x^n)/d/(1+n)+(-a*d+b*c)^2*x
*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d^2
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {416, 388, 245}

$$\frac{x(bc-ad)^2 {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx\left(a+bx^n\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a + b*x^n))/(d
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^2}{c+dx^n}\,dx &= \frac{bx\,(a+bx^n)}{d(1+n)}+\frac{\int \frac{-a(bc-ad(1+n))-b(bc(1+n)-ad(1+2n))x^n}{c+dx^n}\,dx}{d(1+n)}\\ &= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)}+\frac{bx\,(a+bx^n)}{d(1+n)}+\frac{(bc-ad)^2\int \frac{1}{c+dx^n}\,dx}{d^2}\\ &= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)}+\frac{bx\,(a+bx^n)}{d(1+n)}+\frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.98

$$\frac{a^2x}{c}+\frac{x(ad-bc)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{x(bc-ad)^2}{cd^2}+\frac{b^2x^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] (a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n)) + ((-(b
*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*d
^2)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2x^{2n}+2\,abx^n+a^2}{dx^n+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{n}+a)^{2}}{d\,x^{n}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int((b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\int \frac{1}{d^{3}x^{n}+cd^{2}}\,dx+\frac{b^{2}dxx^{n}-\left(b^{2}c(n+1)-2\,abd(n+1)\right)x}{d^{2}(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x) + (b^2*d*
x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^{n})^{2}}{c+d\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n), x)
```

sympy [C] time = 6.83, size = 170, normalized size = 2.02

$$\frac{a^{2}x\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^{2}\Gamma\left(1+\frac{1}{n}\right)}-\frac{2abx\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^{2}\Gamma\left(1+\frac{1}{n}\right)}+\frac{2b^{2}xx^{2n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{cn\Gamma\left(3+\frac{1}{n}\right)}+\frac{b^{2}xx^{2n}\Phi\left(\frac{dx^{n}e^{i\pi}}{c},1,2\right.}{cn^{2}\Gamma\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] a**2*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*n**2*gamma(
1 + 1/n)) - 2*a*b*x*lerchphi(c*x**(-n)*exp_polar(I*pi)/d, 1, exp_polar(I*pi
)/n)*gamma(1/n)/(d*n**2*gamma(1 + 1/n)) + 2*b**2*x*x**(2*n)*lerchphi(d*x**n
*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(c*n*gamma(3 + 1/n)) + b**2*
x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(c
*n**2*gamma(3 + 1/n))
```

3.296 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$

Optimal. Leaf size=115

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\,(a+bx^n)}{cdn\,(c+dx^n)}$$

```
[Out] -b*(a*d-b*c*(1+n))*x/c/d^2/n-(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)+(-a*d+b
*c)*(a*d*(1-n)-b*c*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^2/d^2/n
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 388, 245}

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\,(a+bx^n)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^n))/(c*d*n
*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d^2*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = -\frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{\int \frac{a(bc-ad(1-n))-b(ad-bc(1+n))x^n}{c+dx^n}\,dx}{cdn}$$

$$= -\frac{b(ad-bc(1+n))x}{cd^2n} - \frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{((bc-ad)(ad(1-n)-bc(1+n)))\int \frac{1}{c+dx^n}\,dx}{cd^2n}$$

$$= -\frac{b(ad-bc(1+n))x}{cd^2n} - \frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{(bc-ad)(ad(1-n)-bc(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\right.}{c^2d^2n}$$

Mathematica [A] time = 0.16, size = 95, normalized size = 0.83

$$\frac{x\left(\frac{c\left(a^2d^2-2abcd+b^2c(cn+c+dnx^n)\right)}{c+dx^n}-(bc-ad)(ad(n-1)+bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^2d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] (x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (b
*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((d*x^n)/c)]))/(c^2*d^2*n)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^{2n}+2abx^n+a^2}{d^2x^{2n}+2cdx^n+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^2}{(d\,x^n + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2/(d*x^n+c)^2,x)
[Out] int((b*x^n+a)^2/(d*x^n+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(b^2c^2(n+1) - a^2d^2(n-1) - 2\,abcd\right)\int \frac{1}{cd^3nx^n + c^2d^2n}\, dx + \frac{b^2cdnxx^n + \left(b^2c^2(n+1) - 2\,abcd + a^2d^2\right)x}{cd^3nx^n + c^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
[Out] -(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d^3*n*x^n +
 c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) - 2*a*b*c*d + a^2*d^2
)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x^n)^2}{(c + d\,x^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^2,x)
[Out] int((a + b*x^n)^2/(c + d*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] Integral((a + b*x**n)**2/(c + d*x**n)**2, x)
```

3.297 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1}{2c^2d^2n^2\left(c+dx^n\right)}$$

```
[Out] -1/2*(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)^2+1/2*(-a*d+b*c)*(a*d*(1-2*n)-b
*c*(1+n))*x/c^2/d^2/n^2/(c+d*x^n)-1/2*(2*a*b*c*d*(1-n)-b^2*c^2*(1+n)-a^2*d^
2*(2*n^2-3*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/d^2/n^2
```

Rubi [A] time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 245}

$$-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1}{2c^2d^2n^2\left(c+dx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*(1
 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 - n)
 - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d^2*n^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)x\,(a+bx^n)}{2cdn\,(c+dx^n)^2} + \frac{\int \frac{a(bc-ad(1-2n))-b(ad(1-n)-bc(1+n))x^n}{(c+dx^n)^2}\,dx}{2cdn}$$

$$= -\frac{(bc-ad)x\,(a+bx^n)}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)} - \frac{\left(2abcd(1-n)-b^2c^2(1+n)-\right.}{2c^2}$$

$$= -\frac{(bc-ad)x\,(a+bx^n)}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)} - \frac{\left(2abcd(1-n)-b^2c^2(1+n)-\right.}{}$$

Mathematica [A] time = 0.19, size = 133, normalized size = 0.83

$$\frac{x\left(\left(a^2d^2\left(2n^2-3n+1\right)+2abcd(n-1)+b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{c^2n(bc-ad)^2}{(c+dx^n)^2}-\frac{c(bc-ad)(ad(2n-1)+b(2cn}{c+dx^n}\right.}{2c^3d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] (x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1 + 2*n) +
b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2*c^2*(1 + n) + a^2*d
^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)
]))/(2*c^3*d^2*n^2)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^{2\,n}+2\,abx^n+a^2}{d^3x^{3\,n}+3\,cd^2x^{2\,n}+3\,c^2dx^n+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3
*c^2*d*x^n + c^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^2}{(d\,x^n + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2/(d*x^n+c)^3,x)
```

```
[Out] int((b*x^n+a)^2/(d*x^n+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(\left(2\,n^2 - 3\,n + 1\right)a^2d^2 + b^2c^2(n+1) + 2\,abcd(n-1)\right)\int \frac{1}{2\left(c^2d^3n^2x^n + c^3d^2n^2\right)}\, dx - \frac{\left(b^2c^2d(2\,n+1) - a^2d^3(2\,n-1)\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))*integrate
(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^2*d*(2*n + 1) - a^2*
d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d^2*(3*n - 1) - b^2*c^3*(n + 1)
 - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c^4*d
^2*n^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,x^n)^2}{(c + d\,x^n)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

3.298 $\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$

Optimal. Leaf size=310

$$-\frac{dx\,(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-\right.}{}$$

```
[Out] -d*(a^3*d^3*(6*n^3+11*n^2+6*n+1)-b^3*c^3*(24*n^3+18*n^2+7*n+1)-a^2*b*c*d^2*
(24*n^3+38*n^2+19*n+3)+a*b^2*c^2*d*(36*n^3+45*n^2+20*n+3))*x/b^4/(6*n^3+11*
n^2+6*n+1)-d*(2*a*b*c*d*(1+3*n)^2-a^2*d^2*(6*n^2+5*n+1)-b^2*c^2*(18*n^2+7*n
+1))*x*(c+d*x^n)/b^3/(6*n^3+11*n^2+6*n+1)-d*(a*d*(1+3*n)-b*(6*c*n+c))*x*(c+
d*x^n)^2/b^2/(6*n^2+5*n+1)+d*x*(c+d*x^n)^3/b/(1+3*n)+(-a*d+b*c)^4*x*hyperge
om([1, 1/n],[1+1/n],-b*x^n/a)/a/b^4
```

Rubi [A] time = 0.50, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {416, 528, 388, 245}

$$-\frac{dx\,(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx\left(-a^2bcd^2\left(24n^3+38n^2+19n\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 45
*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1 + 3*n))) - (d*(2*a*b*c*d*(1 +
3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x
^n))/(b^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))
*x*(c + d*x^n)^2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n
)) + ((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a*b^4)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \frac{dx\,(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)^2(-c(ad-b(c+3cn))-d(ad(1+3n)-b(c+6cn))x^n)}{a+bx^n}\,dx}{b(1+3n)}$$

$$= -\frac{d(ad(1+3n)-b(c+6cn))x\,(c+dx^n)^2}{b^2\left(1+5n+6n^2\right)} + \frac{dx\,(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)\left(c\left(a^2d^2(1+3n)-2abcd(1+4n)+b^2\right.\right.}{}}{}$$

$$= -\frac{d\left(2abcd(1+3n)^2-a^2d^2\left(1+5n+6n^2\right)-b^2c^2\left(1+7n+18n^2\right)\right)x\,(c+dx^n)}{b^3(1+n)\left(1+5n+6n^2\right)} - \frac{d(ad(1+3n}{b}$$

$$= -\frac{d\left(a^3d^3\left(1+6n+11n^2+6n^3\right)-b^3c^3\left(1+7n+18n^2+24n^3\right)-a^2bcd^2\left(3+19n+38n^2+2\right.\right.}{b^4(1+n)\left(1+5n+6n^2\right)}$$

$$= -\frac{d\left(a^3d^3\left(1+6n+11n^2+6n^3\right)-b^3c^3\left(1+7n+18n^2+24n^3\right)-a^2bcd^2\left(3+19n+38n^2+2\right.\right.}{b^4(1+n)\left(1+5n+6n^2\right)}$$

Mathematica [C] time = 4.12, size = 133, normalized size = 0.43

$$\frac{x\left(c^4\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)+4c^3dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+6c^2d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+4cd^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+d^4x^{4n}\Phi\left(-\frac{bx^n}{a},1,4+\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] (x*(4*c^3*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 6*c^2*d^2*x^
(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + 4*c*d^3*x^(3*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + d^4*x^(4*n)*HurwitzLerchPhi[-((b*x
^n)/a), 1, 4 + n^(-1)] + c^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]))/(a*
n)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4x^{4\,n}+4\,cd^3x^{3\,n}+6\,c^2d^2x^{2\,n}+4\,c^3dx^n+c^4}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx^n+c)^4}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int\frac{(d\,x^n+c)^4}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^4/(b*x^n+a),x)
```

```
[Out] int((d*x^n+c)^4/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)\int\frac{1}{b^{5}x^{n}+ab^{4}}\,dx+\frac{\left(2\,n^{2}+3\,n+1\right)b^{3}d^{4}xx^{3\,n}+\left(4\left(3\,n^{2}+4\,n+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*int
egrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (4*
(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (6*(
6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2 + 5
*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6*(6*n
^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a^2*b*c
*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n + 1)*b
^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(c+d\,x^n\right)}^4}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^4/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)^4/(a + b*x^n), x)
```

sympy [C] time = 9.73, size = 369, normalized size = 1.19

$$-\frac{4c^{3}dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^{2}\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^{4}x\Phi\left(\frac{bx^{n}e^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^{2}\Gamma\left(1+\frac{1}{n}\right)}+\frac{12c^{2}d^{2}xx^{2n}\Phi\left(\frac{bx^{n}e^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{6c^{2}d^{2}xx^{2n}\Phi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] -4*c**3*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gam
ma(1/n)/(b*n**2*gamma(1 + 1/n)) + c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 12*c**2*d**2*x*x**(2*n)*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n
)) + 6*c**2*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*
```

```
gamma(2 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 12*c*d**3*x*x**(3*n)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + 4*c
*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1
/n)/(a*n**2*gamma(4 + 1/n)) + 4*d**4*x*x**(4*n)*lerchphi(b*x**n*exp_polar(I
*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n*gamma(5 + 1/n)) + d**4*x*x**(4*n)*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n**2*gamma(
5 + 1/n))
```

3.299 $\int \frac{(c+dx^n)^3}{a+bx^n} dx$

Optimal. Leaf size=173

$$\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b^3(n+1)(2n+1)}+\frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}-\frac{dx\,(}{}$$

```
[Out] d*(a^2*d^2*(2*n^2+3*n+1)+b^2*c^2*(6*n^2+4*n+1)-a*b*c*d*(6*n^2+7*n+2))*x/b^3
/(2*n^2+3*n+1)-d*(a*d*(1+2*n)-b*(4*c*n+c))*x*(c+d*x^n)/b^2/(2*n^2+3*n+1)+d*
x*(c+d*x^n)^2/b/(1+2*n)+(-a*d+b*c)^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)
/a/b^3
```

Rubi [A] time = 0.27, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {416, 528, 388, 245}

$$\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b^3(n+1)(2n+1)}+\frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}-\frac{dx\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*
n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))
*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n))
 + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{dx\,(c+dx^n)^2}{b(1+2n)} + \frac{\int \frac{(c+dx^n)(-c(ad-b(c+2cn))-d(ad(1+2n)-b(c+4cn))x^n)}{a+bx^n}\,dx}{b(1+2n)}$$

$$= -\frac{d(ad(1+2n)-b(c+4cn))x\,(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx\,(c+dx^n)^2}{b(1+2n)} + \frac{\int \frac{c\left(a^2d^2(1+2n)-abcd(2+5n)+b^2c^2\left(1+3n+2n^2 \right.\right.}{}}{b}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)+b^2c^2\left(1+4n+6n^2\right)-abcd\left(2+7n+6n^2\right)\right)x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+}{b^2(1+n)(}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)+b^2c^2\left(1+4n+6n^2\right)-abcd\left(2+7n+6n^2\right)\right)x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+}{b^2(1+n)(}$$

Mathematica [C] time = 1.59, size = 104, normalized size = 0.60

$$\frac{x\left(c^3\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)+3c^2dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+3cd^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+d^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (x*(3*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 3*c*d^2*x^(2
*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + d^3*x^(3*n)*HurwitzLerch
Phi[-((b*x^n)/a), 1, 3 + n^(-1)] + c^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(
-1)]))/(a*n)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3x^{3n} + 3\,cd^2x^{2n} + 3\,c^2dx^n + c^3}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x^n + a), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^3}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n + c)^3}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^3/(b*x^n+a),x)
```

```
[Out] int((d*x^n+c)^3/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\int \frac{1}{b^4x^n + ab^3}\,dx + \frac{b^2d^3(n+1)xx^{2n} + \left(3\,b^2cd^2(2\,n+1) - abd^3(2\,n+1)\right)xx^n}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n +
a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^3*
(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*a*b
*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x^n)^3}{a+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^3/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)^3/(a + b*x^n), x)
```

sympy [C] time = 13.01, size = 269, normalized size = 1.55

$$-\frac{3c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{6cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{3cd^2xx^{2n}\Phi\left(\frac{bx^n}{a}\right.}{an^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**3/(a+b*x**n),x)
```

```
[Out] -3*c**2*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gam
ma(1/n)/(b*n**2*gamma(1 + 1/n)) + c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 6*c*d**2*x*x**(2*n)*lerchphi(
b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) +
 3*c*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2
 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 3*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + d**3*x*x**(3*
n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n**2*ga
mma(4 + 1/n))
```

3.300 $\int \frac{(c+dx^n)^2}{a+bx^n}\,dx$

Optimal. Leaf size=84

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx\,(c+dx^n)}{b(n+1)}$$

```
[Out] -d*(a*d*(1+n)-b*(2*c*n+c))*x/b^2/(1+n)+d*x*(c+d*x^n)/b/(1+n)+(-a*d+b*c)^2*x
*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^2
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {416, 388, 245}

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx\,(c+dx^n)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c + d*x^n))/(b
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*b^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
```

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^n)^2}{a+bx^n}\,dx &= \frac{dx\,(c+dx^n)}{b(1+n)}+\frac{\int \frac{-c(ad-bc(1+n))-d(ad(1+n)-b(c+2cn))x^n}{a+bx^n}\,dx}{b(1+n)}\\ &= -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx\,(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2\int \frac{1}{a+bx^n}\,dx}{b^2}\\ &= -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)}+\frac{dx\,(c+dx^n)}{b(1+n)}+\frac{(bc-ad)^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}\end{aligned}$$

Mathematica [C] time = 0.36, size = 75, normalized size = 0.89

$$\frac{x\left(c^2\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)+2cdx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] (x*(2*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + d^2*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + c^2*HurwitzLerchPhi[-((b*x^n)/
a), 1, n^(-1)]))/(a*n)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{d^2x^{2\,n}+2\,cdx^n+c^2}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^n + c)^2/(b*x^n + a), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^{n}+c)^{2}}{b\,x^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^2/(b*x^n+a),x)
[Out] int((d*x^n+c)^2/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\int \frac{1}{b^{3}x^{n}+ab^{2}}\,dx+\frac{bd^{2}xx^{n}+\left(2\,bcd(n+1)-ad^{2}(n+1)\right)x}{b^{2}(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x) + (b*d^2*
x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x^{n})^{2}}{a+b\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^2/(a + b*x^n),x)
[Out] int((c + d*x^n)^2/(a + b*x^n), x)
```

sympy [C] time = 12.79, size = 170, normalized size = 2.02

$$-\frac{2cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^{2}\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^{2}x\Phi\left(\frac{bx^{n}e^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^{2}\Gamma\left(1+\frac{1}{n}\right)}+\frac{2d^{2}xx^{2n}\Phi\left(\frac{bx^{n}e^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{d^{2}xx^{2n}\Phi\left(\frac{bx^{n}e^{i\pi}}{a},\right.}{an^{2}\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**2/(a+b*x**n),x)
```

```
[Out] -2*c*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(
1/n)/(b*n**2*gamma(1 + 1/n)) + c**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 2*d**2*x*x**(2*n)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) + d**2
*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(
a*n**2*gamma(3 + 1/n))
```

3.301 $\int \frac{c+dx^n}{a+bx^n}\,dx$

Optimal. Leaf size=42

$$\frac{x(bc-ad)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

```
[Out] d*x/b+(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 245}

$$\frac{x(bc-ad)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^n}\,dx}{b}$$
$$= \frac{dx}{b} + \frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.95

$$\frac{x\left((bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)/(a + b*x^n),x]
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
]))/(a*b)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^n+c}{bx^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
[Out] integral((d*x^n + c)/(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^n+c}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate((d*x^n + c)/(b*x^n + a), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{d\,x^n+c}{b\,x^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)/(b*x^n+a),x)
```

```
[Out] int((d*x^n+c)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(bc - ad)\int \frac{1}{b^2x^n + ab}\,dx + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c + d\,x^n}{a + b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)/(a + b*x^n), x)
```

sympy [C] time = 4.11, size = 73, normalized size = 1.74

$$-\frac{dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{cx\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)/(a+b*x**n),x)
```

```
[Out] -d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(1/n)
/(b*n**2*gamma(1 + 1/n)) + c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(a*n**2*gamma(1 + 1/n))
```

3.302 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=72

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/n
],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 245}

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \frac{b\int \frac{1}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^n}\,dx}{bc-ad}$$
$$= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.89

$$\frac{x\left(ad\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-bc\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{ac(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{bdx^{2n}+ac+(bc+ad)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n+a)\,(d\,x^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] int(1/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\, x^n)\, (c + d\, x^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^n)\,(c + dx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

3.303 $\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$

Optimal. Leaf size=123

$$\frac{b^2 x\, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{dx(bc(1-2n)-ad(1-n))\, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\left(c+dx^n\right)}$$

```
[Out] -d*x/c/(-a*d+b*c)/n/(c+d*x^n)+b^2*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/
(-a*d+b*c)^2+d*(b*c*(1-2*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/
c)/c^2/(-a*d+b*c)^2/n
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 245}

$$\frac{b^2 x\, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{dx(bc(1-2n)-ad(1-n))\, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\left(c+dx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1
 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c
- a*d)^2*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx &= -\frac{dx}{c(bc-ad)n(c+dx^n)}+\frac{\int \frac{bcn+a(d-dn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{c(bc-ad)n}\\ &= -\frac{dx}{c(bc-ad)n(c+dx^n)}+\frac{b^2\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^2}-\frac{(d(ad(1-n)-b(c-2cn)))\int \frac{1}{c+dx^n}\,dx}{c(bc-ad)^2n}\\ &= -\frac{dx}{c(bc-ad)n(c+dx^n)}+\frac{b^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{d(bc(1-2n)-ad(1-n))x\,{}_2F}{c^2(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.22, size = 121, normalized size = 0.98

$$\frac{x\left(b^2c^2n(c+dx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\left((c+dx^n)(ad(n-1)+b(c-2cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+c(ad-b\right.\right.}{ac^2n(bc-ad)^2(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] (x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n)*
Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(a*c^2*(b*c - a*d
)^2*n*(c + d*x^n))
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bd^2x^{3\,n}+ac^2+\left(2\,bcd+ad^2\right)x^{2\,n}+\left(bc^2+2\,acd\right)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (b*c^2 + 2*
a*c*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,(d\,x^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)/(d*x^n+c)^2,x)
[Out] int(1/(b*x^n+a)/(d*x^n+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 \int \frac{1}{ab^2c^2 - 2\,a^2bcd + a^3d^2 + \left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)x^n}\,dx - \left(bcd(2\,n-1) - ad^2(n-1)\right)\int \frac{}{b^2c^4n - 2\,abc^3dn +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
[Out] b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n - 1))*integrate(1/(b^2
*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n + a
^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*n)*x
^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\,x^n)\,(c + d\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^2),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.304 $\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$

Optimal. Leaf size=210

$$-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{a(bc-}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/n/(c+d*x^n)^2-1/2*d*(a*d*(1-2*n)-b*(-4*c*n+c))*x/c^2/
(-a*d+b*c)^2/n^2/(c+d*x^n)+b^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a
*d+b*c)^3-1/2*d*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(3*n^2-4*n+1)+b^2*c^2*(6*n
^2-5*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/(-a*d+b*c)^3/n^2
```

Rubi [A] time = 0.33, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 245}

$$-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{a(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) - (d*(a*d*(1 - 2*n) - b*(c - 4*c*n
))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*n
+ 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hyper
geometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n^2
)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = -\frac{dx}{2c(bc-ad)n(c+dx^n)^2}+\frac{\int \frac{2bcn+a(d-2dn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{2c(bc-ad)n}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2}+\frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)}+\frac{\int \frac{2b^2c^2n^2+a^2d^2\left(1-3n+2n^2\right)-ab}{}}{2}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2}+\frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)}+\frac{b^3\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^3}-\frac{\left(d\left(a^2d^2\right.\right.}{}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2}+\frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3}$$

Mathematica [A] time = 0.26, size = 210, normalized size = 1.00

$$\frac{x\left(-ad(c+dx^n)^2\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+2b\right.}{2ac^3n^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] (x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c -
 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d
*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2*(c +
 d*x^n)^2)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bd^3x^{4\,n} + ac^3 + \left(3\,bcd^2 + ad^3\right)x^{3\,n} + 3\left(bc^2d + acd^2\right)x^{2\,n} + \left(bc^3 + 3\,ac^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2*
d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,(d\,x^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)/(d*x^n+c)^3,x)
```

```
[Out] int(1/(b*x^n+a)/(d*x^n+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b^3 \int -\frac{1}{ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3 + \left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)x^n}\,dx + \left(\left(6\,n^2 - 5\,n + 1\right)b^2c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] -b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 +
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^2
- 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)*a^
2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^2*n^
2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c^3*d^
3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(2*n -
1))*x*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2 - 2*a*b
*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2 + a^2*c
^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c^3*d^3*n^
2)*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b\,x^n)\,(c + d\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^3),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^3), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.305 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2} dx$

Optimal. Leaf size=341

$$-\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^4n}-\frac{dx\,(c+dx^n)\left(a^2d^2\left(6n^2+5n+1\right)-2abcd\left(5n^2+4\right.\right.}{ab^3n(n+1)(2n+1)}$$

```
[Out] -d*(b^3*c^3*(2*n^2+3*n+1)-a^3*d^3*(6*n^3+11*n^2+6*n+1)-a*b^2*c^2*d*(12*n^3+
17*n^2+12*n+3)+a^2*b*c*d^2*(16*n^3+26*n^2+15*n+3))*x/a/b^4/n/(2*n^2+3*n+1)-
d*(b^2*c^2*(2*n^2+3*n+1)-2*a*b*c*d*(5*n^2+4*n+1)+a^2*d^2*(6*n^2+5*n+1))*x*(
c+d*x^n)/a/b^3/n/(2*n^2+3*n+1)-d*(-3*a*d*n+2*b*c*n-a*d+b*c)*x*(c+d*x^n)^2/a
/b^2/n/(1+2*n)+(-a*d+b*c)*x*(c+d*x^n)^3/a/b/n/(a+b*x^n)-(-a*d+b*c)^3*(b*c*(
1-n)-a*d*(1+3*n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^4/n
```

Rubi [A] time = 0.55, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {413, 528, 388, 245}

$$-\frac{dx\,(c+dx^n)\left(a^2d^2\left(6n^2+5n+1\right)-2abcd\left(5n^2+4n+1\right)+b^2c^2\left(2n^2+3n+1\right)\right)}{ab^3n(n+1)(2n+1)}-\frac{dx\left(a^2bcd^2\left(16n^3+26n^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - a*b^
2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 + 15*n + 26*n^2 + 16*
n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b^2*c^2*(1 + 3*n + 2*n^2) - 2*a
*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3
*n*(1 + n)*(1 + 2*n)) + (d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)
/(a*b^2*n*(1 + 2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) -
((b*c - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x\,(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)^2(c(ad-bc(1-n))+d(ad(1+3n)-b(c+2cn))x^n)}{a+bx^n}\,dx}{abn}$$

$$= \frac{d(ad(1+3n)-b(c+2cn))x\,(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x\,(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)\left(c\left(2abcd(1+2n)-a^2d^2(1+3\right.\right.}{}}{}$$

$$= -\frac{d\left(b^2c^2\left(1+3n+2n^2\right)-2abcd\left(1+4n+5n^2\right)+a^2d^2\left(1+5n+6n^2\right)\right)x\,(c+dx^n)}{ab^3n(1+n)(1+2n)} + \frac{d(ad(1}{}$$

$$= -\frac{d\left(b^3c^3\left(1+3n+2n^2\right)-a^3d^3\left(1+6n+11n^2+6n^3\right)-ab^2c^2d\left(3+12n+17n^2+12n^3\right)+a^2}{ab^4n(1+n)(1+2n)}$$

$$= -\frac{d\left(b^3c^3\left(1+3n+2n^2\right)-a^3d^3\left(1+6n+11n^2+6n^3\right)-ab^2c^2d\left(3+12n+17n^2+12n^3\right)+a^2}{ab^4n(1+n)(1+2n)}$$

Mathematica [A] time = 5.26, size = 217, normalized size = 0.64

$$\frac{x\left(\frac{(bc-ad)^3(ad(3n+1)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{(ad-bc)^3(ad(3n+1)+bc(n-1))}{a^2n}+\frac{-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d+b^4c^4(n-1)}{a^2n}+\frac{2bd^3}{}\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] (x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4 + b^4*c^4*
(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n)))/(a^2*
n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b^2*d^4*x^(2*n))/(1 + 2*n) + (b
*c - a*d)^4/(a*n*(a + b*x^n)) + ((b*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n
))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b^4
```

fricas [F] time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4x^{4\,n}+4\,cd^3x^{3\,n}+6\,c^2d^2x^{2\,n}+4\,c^3dx^n+c^4}{b^2x^{2\,n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n+c)^4}{(b\,x^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^4/(b*x^n+a)^2,x)
```

```
[Out] int((d*x^n+c)^4/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(a^4d^4(3\,n+1)-4\,a^3bcd^3(2\,n+1)+6\,a^2b^2c^2d^2(n+1)-b^4c^4(n-1)-4\,ab^3c^3d\right)\int\frac{1}{ab^5nx^n+a^2b^4n}\,dx+\frac{\left(n^2+n\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2*(n + 1) -
 b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*x^n + a^2*b^4*n), x)
 + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2 + n)*a*b^3*c*d^3 - (3*n^2 + n
)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*n^3
+ 4*n^2 + n)*a^2*b^2*c*d^3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((2*n^2
 + 3*n + 1)*b^4*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*n^2 +
4*n + 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 + (6*n^3
 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*x^n + (2*n^3 +
3*n^2 + n)*a^2*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(c+d\,x^n\right)^4}{\left(a+b\,x^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^4/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^4/(a + b*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c+dx^n\right)^4}{\left(a+bx^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**4/(a + b*x**n)**2, x)
```

3.306 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=200

$$-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^3n}-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b\right.}{ab^3n(n+1)}$$

```
[Out] -d*(b^2*c^2*(1+n)+a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(3*n^2+4*n+2))*x/a/b^3/n/(1
+n)-d*(b*c*(1+n)-a*d*(1+2*n))*x*(c+d*x^n)/a/b^2/n/(1+n)+(-a*d+b*c)*x*(c+d*x
^n)^2/a/b/n/(a+b*x^n)-(-a*d+b*c)^2*(b*c*(1-n)-a*d*(1+2*n))*x*hypergeom([1,
1/n],[1+1/n],-b*x^n/a)/a^2/b^3/n
```

Rubi [A] time = 0.26, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {413, 528, 388, 245}

$$-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{ab^3n(n+1)}-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1\right.}{a^2b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^
2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 + 2*n))*x*(c + d*x^n))
/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) - ((
b*c - a*d)^2*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((b*x^n)/a)])/(a^2*b^3*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x\,(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)(c(ad-bc(1-n))-d(bc(1+n)-ad(1+2n))x^n)}{a+bx^n}\,dx}{abn}$$

$$= -\frac{d(bc(1+n)-ad(1+2n))x\,(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x\,(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{c\left(2abcd(1+n)-a^2d^2(1+2n)-b^2c^2(1\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{d\left(b^2c^2(1+n)+a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+4n+3n^2\right)\right)x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x}{ab^2n(1+n)}\cdots$$

$$= -\frac{d\left(b^2c^2(1+n)+a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+4n+3n^2\right)\right)x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x}{ab^2n(1+n)}\cdots$$

Mathematica [A] time = 5.38, size = 167, normalized size = 0.84

$$\frac{x\left(\frac{a\left(-a^3d^3\left(2n^2+3n+1\right)+a^2bd^2\left(3c(n+1)^2-dn(2n+1)x^n\right)+ab^2d\left(-3c^2(n+1)+3cdn(n+1)x^n+d^2nx^{2n}\right)+b^3c^3(n+1)\right)}{(n+1)(a+bx^n)}+(bc-ad)^2(ad(2n+1)+bc\cdots\right.}{a^2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] (x*((a*(b^3*c^3*(1 + n) - a^3*d^3*(1 + 3*n + 2*n^2) + a^2*b*d^2*(3*c*(1 + n
)^2 - d*n*(1 + 2*n)*x^n) + a*b^2*d*(-3*c^2*(1 + n) + 3*c*d*n*(1 + n)*x^n +
d^2*n*x^(2*n))))/((1 + n)*(a + b*x^n)) + (b*c - a*d)^2*(b*c*(-1 + n) + a*d*
(1 + 2*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(a^2*b^
3*n)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3x^{3n} + 3\,cd^2x^{2n} + 3\,c^2dx^n + c^3}{b^2x^{2n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n) +
 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^3}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n + c)^3}{(b\,x^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^3/(b*x^n+a)^2,x)
```

```
[Out] int((d*x^n+c)^3/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(a^3d^3(2\,n+1) - 3\,a^2bcd^2(n+1) + b^3c^3(n-1) + 3\,ab^2c^2d\right)\int \frac{1}{ab^4nx^n + a^2b^3n}\,dx + \frac{ab^2d^3nxx^{2n} + \left(3\left(n^2+n\right)ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*a*b^2*c^2*
d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d^3*n*x*x^(2*n) + (3*
(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3)*x*x^n + (3*(n^2 + 2*n + 1)*a
^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*d*(n
 + 1))*x)/((n^2 + n)*a*b^4*x^n + (n^2 + n)*a^2*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x^n)^3}{(a + b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^3/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^3/(a + b*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^n)^3}{(a + bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**3/(a + b*x**n)**2, x)
```

3.307 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2} \, dx$

Optimal. Leaf size=115

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\,(c+dx^n)}{abn\,(a+bx^n)}$$

```
[Out] -d*(b*c-a*d*(1+n))*x/a/b^2/n+(-a*d+b*c)*x*(c+d*x^n)/a/b/n/(a+b*x^n)-(-a*d+b
*c)*(b*c*(1-n)-a*d*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^2/n
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 388, 245}

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\,(c+dx^n)}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^n))/(a*b*n
*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^2*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x\,(c+dx^n)}{abn\,(a+bx^n)} + \frac{\int \frac{c(ad-bc(1-n))-d(bc-ad(1+n))x^n}{a+bx^n}\,dx}{abn}$$

$$= -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x\,(c+dx^n)}{abn\,(a+bx^n)} - \frac{((bc-ad)(bc(1-n)-ad(1+n)))\int \frac{1}{a+bx^n}\,dx}{ab^2n}$$

$$= -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x\,(c+dx^n)}{abn\,(a+bx^n)} - \frac{(bc-ad)(bc(1-n)-ad(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n}\right.}{a^2b^2n}$$

Mathematica [C] time = 2.58, size = 666, normalized size = 5.79

$$x\left(-2bc^2n^6x^n\,{}_4F_3\left(2,2,2,1+\frac{1}{n};1,1,4+\frac{1}{n};-\frac{bx^n}{a}\right) - 4bcdn^6x^{2n}\,{}_4F_3\left(2,2,2,1+\frac{1}{n};1,1,4+\frac{1}{n};-\frac{bx^n}{a}\right) - 2bd^2n^6x^{3n}\,{}_4F\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] (x*(-2*a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + n)^3 + 2*c*d*(1 + 3*n + 4*n^2
 + n^3)*x^n + d^2*(1 + n)^3*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n
^(-1)] + a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + 2*n)^3 + 2*c*d*(1 + 2*n)^3*
x^n + d^2*(1 + 6*n + 10*n^2 + 6*n^3)*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a),
 1, 2 + n^(-1)] + a*c^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*c^2*
n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 9*a*c^2*n^2*HurwitzLerchPhi[-(
(b*x^n)/a), 1, n^(-1)] - 4*a*c^2*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 10*a*c^2*n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 10*a*c^2*n^5*H
urwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c^2*n^6*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] + 2*a*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 + 12*a*c*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 22*a*c*d*n^2*x
^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c*d*n^3*x^n*HurwitzLerch
Phi[-((b*x^n)/a), 1, n^(-1)] + a*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 6*a*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 1
1*a*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*d^2*n^3*
x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 2*b*c^2*n^6*x^n*Hypergeo
```

```
metricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)] - 4*b*c*
d*n^6*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)},
-((b*x^n)/a)] - 2*b*d^2*n^6*x^(3*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}
, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)]))/(2*a^3*n^4*(1 + 6*n + 11*n^2 + 6*n^3)
)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2x^{2n}+2\,cdx^n+c^2}{b^2x^{2n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n+c)^2}{(b\,x^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^2/(b*x^n+a)^2,x)
```

```
[Out] int((d*x^n+c)^2/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(a^2d^2(n+1)-b^2c^2(n-1)-2\,abcd\right)\int \frac{1}{ab^3nx^n+a^2b^2n}\,dx+\frac{abd^2nxx^n+\left(a^2d^2(n+1)+b^2c^2-2\,abcd\right)x}{ab^3nx^n+a^2b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b^3*n*x^n +
 a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) + b^2*c^2 - 2*a*b*c*d
)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x^n)^2}{(a + b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^2/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^2/(a + b*x^n)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

3.308 $\int \frac{c+dx^n}{(a+bx^n)^2} \, dx$

Optimal. Leaf size=72

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

```
[Out] (-a*d+b*c)*x/a/b/n/(a+b*x^n)+(a*d-b*c*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-
b*x^n/a)/a^2/b/n
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x}{abn\,(a+bx^n)} + \frac{(ad-bc(1-n))\int \frac{1}{a+bx^n}\,dx}{abn}$$
$$= \frac{(bc-ad)x}{abn\,(a+bx^n)} + \frac{(ad-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.78

$$\frac{x\left(\frac{d}{a+bx^n}-\frac{(ad+bc(n-1))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)/(a + b*x^n)^2,x]
[Out] (x*(d/(a + b*x^n) - ((a*d + b*c*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^n+c}{b^2x^{2n}+2\,abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{d\,x^n+c}{(b\,x^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)/(b*x^n+a)^2,x)
```

```
[Out] int((d*x^n+c)/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(bc(n-1)+ad)\int \frac{1}{ab^2nx^n+a^2bn}\,dx+\frac{(bc-ad)x}{ab^2nx^n+a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*x
/(a*b^2*n*x^n + a^2*b*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x^n}{{\left(a+b\,x^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)/(a + b*x^n)^2, x)
```

sympy [C] time = 8.77, size = 592, normalized size = 8.22

$$c\left(\frac{nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}+\frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}-\frac{x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

```
[Out] c*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gam
ma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamm
a(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 +
1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/
n) + b*n**3*x**n*gamma(1 + 1/n)))) + d*(n**2*x*x**n*gamma(1 + 1/n)/(a*(a*n*
```

```
*3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))) - n*x*x**n*lerchphi(b*x**n
*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 + 1/n) +
b*n**3*x**n*gamma(2 + 1/n))) + n*x*x**n*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 +
 1/n) + b*n**3*x**n*gamma(2 + 1/n))) - x*x**n*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*ga
mma(2 + 1/n))) - b*n*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1
/n)*gamma(1 + 1/n)/(a**2*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n
))) - b*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 +
 1/n)/(a**2*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))))
```

3.309 $\int \frac{1}{(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=122

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n]
,[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d*x
^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 245}

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^2n}$$

$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,{}_2F_1}{}$$

Mathematica [A] time = 0.20, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(1-2n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n} + \frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b^2dx^{3n} + a^2c + \left(b^2c + 2\,abd\right)x^{2n} + \left(2\,abc + a^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^2 \,(d\,x^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int(1/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \int \frac{1}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\, dx - \left(abd(2\,n - 1) - b^2c(n - 1)\right) \int \frac{\quad}{a^2b^2c^2n - 2\,a^3bcdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a + b\,x^n\right)}^2\,\left(c + d\,x^n\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.310 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$

Optimal. Leaf size=193

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}-\frac{d^2x(bc(1-3n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{}{an(bc-}$$

```
[Out] d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)+b*x/a/(-a*d+b*c)/n/(a+b*x^n)/(c+
d*x^n)+b^2*(a*d*(1-3*n)-b*(-c*n+c))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/
a^2/(-a*d+b*c)^3/n-d^2*(b*c*(1-3*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n]
,-d*x^n/c)/c^2/(-a*d+b*c)^3/n
```

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 245}

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}-\frac{d^2x(bc(1-3n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{}{an(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)*
n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^3*n) - (d^2
*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((d*x^n)/c)])/(c^2*(b*c - a*d)^3*n)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{a(bc-ad)n}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} - \frac{\int \frac{n\left(b^2c^2(1-n)+a^2d^2(1-n)+2\right.}{(a+bx^n)}}{ac(bc-}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} + \frac{\left(d^2(ad(1-n)-b(c-3\right.}{c(bc-ad)}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} + \frac{b^2(ad(1-3n)-b(c-c}{a^2(bc}$$

Mathematica [A] time = 0.28, size = 147, normalized size = 0.76

$$\frac{x\left(\frac{b^2(ad(1-3n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}+\frac{b^2(bc-ad)}{a(a+bx^n)}+\frac{d^2(bc(3n-1)-ad(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}+\frac{d^2(bc-ad)}{c(c+dx^n)}\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c + d*x^n)) +
 (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n)) + b*c*(-1 + 3*n))*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2d^2x^{4\,n} + a^2c^2 + 2\left(b^2cd + abd^2\right)x^{3\,n} + \left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{2\,n} + 2\left(abc^2 + a^2cd\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(3*n) + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^2\,(d\,x^n + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^2/(d*x^n+c)^2,x)
```

```
[Out] int(1/(b*x^n+a)^2/(d*x^n+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(ab^2d(3\,n-1) - b^3c(n-1)\right)\int -\frac{1}{a^2b^3c^3n - 3\,a^3b^2c^2dn + 3\,a^4bcd^2n - a^5d^3n + \left(ab^4c^3n - 3\,a^2b^3c^2dn + 3\,a^3b^2cd^2n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n - 3*a^3*b^2
*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d*n +
 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - (b*c*d^2*(3*n - 1) - a*d^3*(n
- 1))*integrate(-1/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a^3*c
^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*c*d^4
*n)*x^n), x) + ((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2
*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^2*c^2*d
^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d*n - a^3*b*c^2*
d^2*n + a^4*c*d^3*n)*x^n)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x^n\right)}^2\,{\left(c+d\,x^n\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^2),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^2), x)
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.311 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

Optimal. Leaf size=299

$$\frac{b^3x(ad(1-4n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4}-\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n\right)}{2ac^2n^2(bc-ad)^3\,(c+dx^n)}+\frac{d^2x\left(a^2d^2\left(2n^2\right.\right.}{}$$

```
[Out] 1/2*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)^2+b*x/a/(-a*d+b*c)/n/(a+b*
x^n)/(c+d*x^n)^2-1/2*d*(a*b*c*d*(1-6*n)-a^2*d^2*(1-2*n)-2*b^2*c^2*n)*x/a/c^
2/(-a*d+b*c)^3/n^2/(c+d*x^n)+b^3*(a*d*(1-4*n)-b*c*(1-n))*x*hypergeom([1, 1/
n],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^4/n+1/2*d^2*(a^2*d^2*(2*n^2-3*n+1)-2*a*
b*c*d*(4*n^2-5*n+1)+b^2*c^2*(12*n^2-7*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c^3/(-a*d+b*c)^4/n^2
```

Rubi [A] time = 0.55, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 245}

$$\frac{d^2x\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4}-\frac{dx\left(-a^2d^2(1-\right.}{2ac^2n}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)/(a*(b*c -
 a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 - 2
*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*n^2*(c + d*x^n)) + (b^3*(a*d*(
1 - 4*n) - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*(b*c - a*d)^4*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*
(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{adn+b(c-cn)+bd(1-3n)x^n}{(a+bx^n)(c+dx^n)^3}\,dx}{a(bc-ad)n}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{n\left(a^2d^2(1-2n)+2b^2c^2(1\cdots}{(a\cdots}}{2a\cdots}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d\left(abcd(1-6n)-a^2\cdots\right.}{2ac^2(bc-a\cdots}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d\left(abcd(1-6n)-a^2\cdots\right.}{2ac^2(bc-a\cdots}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d\left(abcd(1-6n)-a^2\cdots\right.}{2ac^2(bc-a\cdots}$$

Mathematica [A] time = 0.44, size = 233, normalized size = 0.78

$$\frac{x\left(\frac{2b^3n(ad(1-4n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}+\frac{d^2\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3}+\frac{2b^3n(bc-}{a(a+bx}\right.}{2n^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)/(c*(c + d
*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c - 6*c*n)))/(c^2*(c +
d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n))*n*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*
b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2d^3x^{5n}+a^2c^3+\left(3\,b^2cd^2+2\,abd^3\right)x^{4n}+\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{3n}+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n) +
 (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n+a)^2\,(d\,x^n+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^2/(d*x^n+c)^3,x)
```

```
[Out] int(1/(b*x^n+a)^2/(d*x^n+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(\left(12\,n^{2}-7\,n+1\right)b^{2}c^{2}d^{2}-2\left(4\,n^{2}-5\,n+1\right)abcd^{3}+\left(2\,n^{2}-3\,n+1\right)a^{2}d^{4}\right)\int\frac{}{2\left(b^{4}c^{7}n^{2}-4\,ab^{3}c^{6}dn^{2}+6\,a^{2}b^{2}c^{5}d^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 + (2*n^2 -
3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2 + 6*a^2*b^
2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6*d*n^2 - 4*
a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c^2*d
^5*n^2)*x^n), x) - (a*b^3*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^2*b^4
*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n + a^6*d^
4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4*b^2*c*
d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1) - a^2*b*d^4*(2*
n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d^2*(7*n - 1) - a^3*d^4*(2
*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*n)*x*x^n + (a^2*b*c^2*d^2*(7*n - 1)
 - a^3*c*d^3*(3*n - 1) + 2*b^3*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^3*b^2*c^6*d
*n^2 + 3*a^4*b*c^5*d^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n^2 - 3*a^2*b
^3*c^4*d^3*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^(3*n) + (2*a*
b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2 + a^4*b*c^3*d
^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^2*b^3*c^6*d*n^2 - 3*
a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2*a^5*c^3*d^4*n^2)*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x^n\right)}^2\,{\left(c+d\,x^n\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^3),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{n}\right)^{2} \left(c + dx^{n}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)**3), x)
```

3.312 $\int (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal. Leaf size=81

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

```
[Out] x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1(1/n,-p,-q,1+1/n,-b*x^n/a,-d*x^n/c)/((1+b
*x^n/a)^p)/((1+d*x^n/c)^q)
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^
n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int \left(1+\frac{bx^n}{a}\right)^p (c+dx^n)^q \, dx$$
$$= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q}\right) \int \left(1+\frac{bx^n}{a}\right)^p \left(1+\frac{dx^n}{c}\right)^q dx$$
$$= x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right.$$

Mathematica [B] time = 0.57, size = 190, normalized size = 2.35

$$\frac{ac(n+1)x(a+bx^n)^p (c+dx^n)^q F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{bcnpx^nF_1\left(1+\frac{1}{n};1-p,-q;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+adnqx^nF_1\left(1+\frac{1}{n};-p,1-q;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(
-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
-q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1 + n^(-
1), -p, 1 - q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*Appell
F1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n+a)^p(dx^n+c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p(dx^n+c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int (b\,x^{n}+a)^{p}\,(d\,x^{n}+c)^{q}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^n+a)^p*(d*x^n+c)^q,x)

[Out] int((b*x^n+a)^p*(d*x^n+c)^q,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)^{p}(dx^{n}+c)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")

[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x^{n})^{p}\,(c+d\,x^{n})^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^n)^p*(c + d*x^n)^q,x)

[Out] int((a + b*x^n)^p*(c + d*x^n)^q, x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**n)**p*(c+d*x**n)**q,x)

[Out] Exception raised: HeuristicGCDFailed

3.313 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal. Leaf size=402

$$\frac{dx\,(a+bx^n)^{p+1}\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(n^2(p+7)+n(2p+9)+2\right)+b^2c^2\left(n^2\left(p^2+6p+11\right)+2n(p+3)-\right.\right.}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}$$

```
[Out] d*(a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(2+n^2*(7+p)+n*(9+2*p))+b^2*c^2*(1+2*n*(3+
p)+n^2*(p^2+6*p+11)))*x*(a+b*x^n)^(1+p)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p
))-d*(a*d*(1+2*n)-b*c*(1+n*(5+p)))*x*(a+b*x^n)^(1+p)*(c+d*x^n)/b^2/(1+n*(2+
p))/(1+n*(3+p))+d*x*(a+b*x^n)^(1+p)*(c+d*x^n)^2/b/(n*p+3*n+1)-(a^3*d^3*(2*n
^2+3*n+1)-3*a^2*b*c*d^2*(1+n)*(1+n*(3+p))+3*a*b^2*c^2*d*(1+n*(5+2*p)+n^2*(p
^2+5*p+6))-b^3*c^3*(1+3*n*(2+p)+n^2*(3*p^2+12*p+11)+n^3*(p^3+6*p^2+11*p+6))
)*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/b^3/(n*p+n+1)/(1+n*(2
+p))/(1+n*(3+p))/((1+b*x^n/a)^p)
```

Rubi [A] time = 0.58, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {416, 528, 388, 246, 245}

$$-\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-3a^2bcd^2(n+1)(n(p+3)+1)+a^3d^3\left(2n^2+3n+1\right)+3ab^2c^2d\left(n^2\left(p^2+5p+6\right)+n\right.\right.}{b^3(np+n+1)(n(p+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b
^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x^n)^(1 + p))/(b^
3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))) - (d*(a*d*(1 + 2*n) - b*(c
 + c*n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1
 + n*(3 + p))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))
) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3
*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2
+ p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3)))*x*(a + b*x^
n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(b^3*(1 + n +
 n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1 + (b*x^n)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} + \frac{\int (a+bx^n)^p (c+dx^n)\left(-c(ad-b(c+cn(3+p)))-d\right.}{b(1+n(3+p))}$$

$$= -\frac{d(ad(1+2n)-b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))} + \frac{dx(a+bx^n)^{1+p}(c+}{b(1+n(3+p))}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+n^2(7+p)+n(9+2p)\right)+b^2c^2\left(1+2n(3+p)+\right.\right.}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+n^2(7+p)+n(9+2p)\right)+b^2c^2\left(1+2n(3+p)+\right.\right.}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+n^2(7+p)+n(9+2p)\right)+b^2c^2\left(1+2n(3+p)+\right.\right.}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

Mathematica [A] time = 5.32, size = 168, normalized size = 0.42

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c^3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{3c^2dx^n\,{}_2F_1\left(1+\frac{1}{n},-p;2+\frac{1}{n};-\frac{bx^n}{a}\right)}{n+1}+\frac{3cd^2x^{2n}\,{}_2F_1\left(2+\right.}{2n}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1)
, -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1), -
p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^3*x^(3*n)*Hypergeometric2F1[3
+ n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)])/(1 + 3*n) + c^3*Hypergeometric2F1[
n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^3x^{3n}+3\,cd^2x^{2n}+3\,c^2dx^n+c^3\right)(bx^n+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p,
 x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,3,4,3,0]%%%}+%%%{6,[2,0,6
,4,2,2,4,3,0]%%%}+%%%{4,[2,0,6,4,2,1,4,3,0]%%%}+%%%{1,[2,0,6,4,2,0,4,3,0]%%
%}+%%%{3,[1,0,6,4,2,4,4,2,1]%%%}+%%%{12,[1,0,6,4,2,3,4,2,1]%%%}+%%%{18,[1,0
,6,4,2,2,4,2,1]%%%}+%%%{12,[1,0,6,4,2,1,4,2,1]%%%}+%%%{3,[1,0,6,4,2,0,4,2,1
]%%%}+%%%{-1,[1,0,6,4,1,4,5,3,0]%%%}+%%%{-4,[1,0,6,4,1,3,5,3,0]%%%}+%%%{-6,
[1,0,6,4,1,2,5,3,0]%%%}+%%%{-4,[1,0,6,4,1,1,5,3,0]%%%}+%%%{-1,[1,0,6,4,1,0,
5,3,0]%%%}+%%%{1,[0,0,6,4,3,4,3,0,3]%%%}+%%%{4,[0,0,6,4,3,3,3,0,3]%%%}+%%%{
6,[0,0,6,4,3,2,3,0,3]%%%}+%%%{4,[0,0,6,4,3,1,3,0,3]%%%}+%%%{1,[0,0,6,4,3,0,
3,0,3]%%%}+%%%{1,[0,0,6,3,3,3,3,0,3]%%%}+%%%{3,[0,0,6,3,3,2,3,0,3]%%%}+%%%{
3,[0,0,6,3,3,1,3,0,3]%%%}+%%%{1,[0,0,6,3,3,0,3,0,3]%%%}+%%%{-3,[0,0,6,3,2,3
,4,1,2]%%%}+%%%{-9,[0,0,6,3,2,2,4,1,2]%%%}+%%%{-9,[0,0,6,3,2,1,4,1,2]%%%}+%
%%{-3,[0,0,6,3,2,0,4,1,2]%%%}+%%%{3,[0,0,6,3,1,3,5,2,1]%%%}+%%%{9,[0,0,6,3,
1,2,5,2,1]%%%}+%%%{9,[0,0,6,3,1,1,5,2,1]%%%}+%%%{3,[0,0,6,3,1,0,5,2,1]%%%}+
%%%{-1,[0,0,6,3,0,3,6,3,0]%%%}+%%%{-3,[0,0,6,3,0,2,6,3,0]%%%}+%%%{-3,[0,0,6
,3,0,1,6,3,0]%%%}+%%%{-1,[0,0,6,3,0,0,6,3,0]%%%} / %%%{1,[0,0,7,4,3,4,4,0,0
]%%%}+%%%{4,[0,0,7,4,3,3,4,0,0]%%%}+%%%{6,[0,0,7,4,3,2,4,0,0]%%%}+%%%{4,[0,
0,7,4,3,1,4,0,0]%%%}+%%%{1,[0,0,7,4,3,0,4,0,0]%%%} Error: Bad Argument Valu
e
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \left(d\,x^{n}+c\right)^{3} \left(b\,x^{n}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p*(d*x^n+c)^3,x)
```

```
[Out] int((b*x^n+a)^p*(d*x^n+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^{n}+c)^{3}(bx^{n}+a)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x^n)^p\,(c + d\,x^n)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^3,x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n)^3, x)
```

sympy [C] time = 93.23, size = 199, normalized size = 0.50

$$\frac{a^p c^3 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{3a^p c^2 dx x^n \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{3a^p cd^2 x x^{2n} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| b \right.}{n\Gamma\left(3+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

```
[Out] a**p*c**3*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(1 + 1/n)) + 3*a**p*c**2*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 +
1/n), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**p*c*d
**2*x*x**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(3 + 1/n)) + a**p*d**3*x*x**(3*n)*gamma(3 + 1/n)*hype
r((-p, 3 + 1/n), (4 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(4 + 1/n))
```

3.314 $\int (a + bx^n)^p (c + dx^n)^2 \, dx$

Optimal. Leaf size=202

$$-\frac{x\,(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (bc(np+n+1)(ad-bc(n(p+2)+1)) - ad(ad(n+1)-bc(n(p+3)+1)))\,{}_2F_1\left(\frac{1}{n},-p;1\right.}{b^2(np+n+1)(n(p+2)+1)}$$

```
[Out] -d*(a*d*(1+n)-b*c*(1+n*(3+p)))*x*(a+b*x^n)^(1+p)/b^2/(n*p+n+1)/(1+n*(2+p))+
d*x*(a+b*x^n)^(1+p)*(c+d*x^n)/b/(n*p+2*n+1)-(b*c*(n*p+n+1)*(a*d-b*c*(1+n*(2
+p)))-a*d*(a*d*(1+n)-b*c*(1+n*(3+p))))*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1
+1/n],-b*x^n/a)/b^2/(n*p+n+1)/(1+n*(2+p))/((1+b*x^n/a)^p)
```

Rubi [A] time = 0.27, antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {416, 388, 246, 245}

$$-\frac{dx\,(a+bx^n)^{p+1}\,(ad(n+1)-b(cn(p+3)+c))}{b^2(np+n+1)(n(p+2)+1)}-\frac{x\,(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c(ad-b(cn(p+2)+c)) - \frac{ad(ad(n+1)-b(cn}{b(np+n+}\right.}{b(n(p+2)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b^2*(1 + n
 + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n
*(2 + p))) - ((c*(a*d - b*(c + c*n*(2 + p))) - (a*d*(a*d*(1 + n) - b*(c + c
*n*(3 + p))))/(b*(1 + n + n*p)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1),
-p, 1 + n^(-1), -((b*x^n)/a)])/(b*(1 + n*(2 + p))*(1 + (b*x^n)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^2\,dx = \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} + \frac{\int (a+bx^n)^p \left(-c(ad-b(c+cn(2+p)))-d(ad(1+n)\right.}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x\,(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(a\right.}{}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x\,(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(\left(c(\right.\right.}{}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x\,(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(a\right.}{}$$

Mathematica [A] time = 5.20, size = 140, normalized size = 0.69

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left((n+1)\left(c^2(2n+1)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+d^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)\right)+2cd\right.}{(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 +
 n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1),
```

```
 -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n)*Hypergeometric2F1[n^(-1), -p
, 1 + n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^{2n}+2\,cdx^n+c^2\right)(bx^n+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,2,3,2,0]%%%}+%%%{-3,[1,
0,4,3,1,1,3,2,0]%%%}+%%%{-1,[1,0,4,3,1,0,3,2,0]%%%}+%%%{-1,[0,0,4,3,2,3,2,0
,2]%%%}+%%%{-3,[0,0,4,3,2,2,2,0,2]%%%}+%%%{-3,[0,0,4,3,2,1,2,0,2]%%%}+%%%{-
1,[0,0,4,3,2,0,2,0,2]%%%}+%%%{-1,[0,0,4,2,2,2,2,0,2]%%%}+%%%{-2,[0,0,4,2,2,
1,2,0,2]%%%}+%%%{-1,[0,0,4,2,2,0,2,0,2]%%%}+%%%{2,[0,0,4,2,1,2,3,1,1]%%%}+%
%%{4,[0,0,4,2,1,1,3,1,1]%%%}+%%%{2,[0,0,4,2,1,0,3,1,1]%%%}+%%%{-1,[0,0,4,2,
0,2,4,2,0]%%%}+%%%{-2,[0,0,4,2,0,1,4,2,0]%%%}+%%%{-1,[0,0,4,2,0,0,4,2,0]%%%
} / %%%{-1,[0,0,5,3,2,3,3,0,0]%%%}+%%%{-3,[0,0,5,3,2,2,3,0,0]%%%}+%%%{-3,[0
,0,5,3,2,1,3,0,0]%%%}+%%%{-1,[0,0,5,3,2,0,3,0,0]%%%} Error: Bad Argument Va
lue
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left(d\,x^{n}+c\right)^{2} \left(b\,x^{n}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p*(d*x^n+c)^2,x)
```

```
[Out] int((b*x^n+a)^p*(d*x^n+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n+c)^2(bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,x^n)^p\,(c + d\,x^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n)^2,x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n)^2, x)
```

sympy [C] time = 42.75, size = 143, normalized size = 0.71

$$\frac{a^p c^2 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, -p \\ 1 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)} + \frac{2a^p c d x x^n \Gamma\left(1 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, 1 + \frac{1}{n} \\ 2 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(2 + \frac{1}{n}\right)} + \frac{a^p d^2 x x^{2n} \Gamma\left(2 + \frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, 2 + \frac{1}{n} \\ 3 + \frac{1}{n}\end{matrix}\middle| \frac{b x^n e^{i\pi}}{a}\right)}{n\Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] a**p*c**2*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(1 + 1/n)) + 2*a**p*c*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n
), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + a**p*d**2*x*x
**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(3 + 1/n))
```

3.315 $\int (a+bx^n)^p (c+dx^n) \, dx$

Optimal. Leaf size=98

$$\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(ad-bc(np+n+1))\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(np+n+1)}$$

```
[Out] d*x*(a+b*x^n)^(1+p)/b/(n*p+n+1)-(a*d-b*c*(n*p+n+1))*x*(a+b*x^n)^p*hypergeom
([-p, 1/n],[1+1/n],-b*x^n/a)/b/(n*p+n+1)/((1+b*x^n/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right){}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p)
)*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^p (c+dx^n)\,dx &= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(-c+\frac{ad}{b+bn+bnp}\right)\int (a+bx^n)^p\,dx\\ &= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(\left(-c+\frac{ad}{b+bn+bnp}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \left(1+\frac{bx^n}{a}\right)^p\,dx\\ &= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(c-\frac{ad}{b+bn+bnp}\right)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 0.96

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left((bc(np+n+1)-ad)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+d(a+bx^n)\left(\frac{bx^n}{a}+1\right)^p\right)}{b(np+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n),x]
[Out] (x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n +
 n*p)*(1 + (b*x^n)/a)^p)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left((dx^n+c)(bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
[Out] integral((d*x^n + c)*(b*x^n + a)^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%%{1,[0,0,2
,2,1,0,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,0,1,0,1]%%
%}+%%%{-1,[0,0,2,1,0,1,2,1,0]%%%}+%%%{-1,[0,0,2,1,0,0,2,1,0]%%%} / %%%{1,[0
,0,3,2,1,2,2,0,0]%%%}+%%%{2,[0,0,3,2,1,1,2,0,0]%%%}+%%%{1,[0,0,3,2,1,0,2,0,
0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(d\,x^{n}+c\right) \left(b\,x^{n}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p*(d*x^n+c),x)
```

```
[Out] int((b*x^n+a)^p*(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^{n} + c)(bx^{n} + a)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,x^n\right)}^p\,\left(c+d\,x^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(c + d*x^n),x)
```

```
[Out] int((a + b*x^n)^p*(c + d*x^n), x)
```

sympy [C] time = 6.15, size = 87, normalized size = 0.89

$$\frac{a^{p} c x \Gamma\left(\frac{1}{n}\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{n}, -p \\ 1 + \frac{1}{n} \end{matrix}\middle| {\frac{b x^{n} e^{i \pi}}{a}} \right)}}{n \Gamma\left(1 + \frac{1}{n}\right)} + \frac{a^{p} d x x^{n} \Gamma\left(1 + \frac{1}{n}\right) {{}_{2}F_{1}\left(\begin{matrix} - p, 1 + \frac{1}{n} \\ 2 + \frac{1}{n} \end{matrix}\middle| {\frac{b x^{n} e^{i \pi}}{a}} \right)}}{n \Gamma\left(2 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n),x)
```

```
[Out] a**p*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(n*gamma(1 + 1/n)) + a**p*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 +
 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

3.316 $\int (a+bx^n)^p \, dx$

Optimal. Leaf size=46

$$x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^p \, dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\left(1+\frac{bx^n}{a}\right)^p \, dx\\ &= x\left(a+bx^n\right)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.00

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p,x)
```

```
[Out] int((b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

mupad [B] time = 2.30, size = 47, normalized size = 1.02

$$\frac{x\,(a+b\,x^n)^p\;{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p,x)
```

```
[Out] (x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1)
^p
```

sympy [C] time = 1.90, size = 37, normalized size = 0.80

$$\frac{a^p x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p,x)
```

```
[Out] a**p*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n
*gamma(1 + 1/n))
```

3.317 $\int \frac{(a+bx^n)^p}{c+dx^n} \, dx$

Optimal. Leaf size=59

$$\frac{x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/((1+b*x^n/a)^p)

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^n)^p/(c + d*x^n),x]

[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c*(1 + (b*x^n)/a)^p)

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{c+dx^n}\,dx$$
$$= \frac{x\,(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

Mathematica [B] time = 0.31, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x\,(a+bx^n)^p\,F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,1;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^nF_1\left(1+\frac{1}{n};-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, 1,
 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1 + n^(-1), -
p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1)
, -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{dx^n+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{n}+a)^{p}}{d\,x^{n}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/(d*x^n+c),x)
[Out] int((b*x^n+a)^p/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^{n}+a)^{p}}{dx^{n}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^{n})^{p}}{c+d\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n),x)
[Out] int((a + b*x^n)^p/(c + d*x^n), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.318 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/((1+b*x^n/a)^p
)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^2}\,dx$$
$$= \frac{x\,(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

Mathematica [B] time = 0.34, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x\,(a+bx^n)^p F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^2\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^nF_1\left(1+\frac{1}{n};-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{d^2x^{2n}+2\,cdx^n+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{n}+a)^{p}}{(d\,x^{n}+c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/(d*x^n+c)^2,x)
[Out] int((b*x^n+a)^p/(d*x^n+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^{n}+a)^{p}}{(dx^{n}+c)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^2,x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.319 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

```
[Out] x*(a+b*x^n)^p*AppellF1(1/n,-p,3,1+1/n,-b*x^n/a,-d*x^n/c)/c^3/((1+b*x^n/a)^p
)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^3}\,dx$$
$$= \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

Mathematica [B] time = 0.65, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^3\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-3adnx^nF_1\left(1+\frac{1}{n};-p,4;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{d^3x^{3n}+3\,cd^2x^{2n}+3\,c^2dx^n+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^{n}+a)^{p}}{(d\,x^{n}+c)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/(d*x^n+c)^3,x)
[Out] int((b*x^n+a)^p/(d*x^n+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^{n}+a)^{p}}{(dx^{n}+c)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^3,x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^{n})^{p}}{(c+dx^{n})^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

3.320 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\, dx$

Optimal. Leaf size=93

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

```
[Out] x*(a+b*x^n)^p*(c+d*x^n)^(-1/n-p)*hypergeom([-p, 1/n],[1+1/n],-(-a*d+b*c)*x^
n/a/(c+d*x^n))/c/((c*(a+b*x^n)/a/(c+d*x^n))^p)
```

Rubi [A] time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {380}

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
+ n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c
+ d*x^n)))^p)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\, dx = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

Mathematica [A] time = 0.09, size = 94, normalized size = 1.01

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^{-\frac{np+1}{n}} \left(\frac{dx^n}{c}+1\right)^p {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1)
, ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*(1 + (b*x^n)/a)^p*(c + d*x^n)^(
(1 + n*p)/n))
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n + a)^p}{(dx^n + c)^{\frac{np+n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)^p\,(d\,x^n + c)^{-p-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p*(d*x^n+c)^(-1-1/n-p),x)
```

```
[Out] int((b*x^n+a)^p*(d*x^n+c)^(-1-1/n-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1),x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
[Out] Timed out
```

3.321 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx$

Optimal. Leaf size=178

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2\left(6n^2+5n+1\right)}+\frac{x(a+bx^n)^3(c+dx}{c(3n+1)}$$

```
[Out] x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/c/(1+3*n)+3*a*n*x*(a+b*x^n)^2*(c+d*x^n)^(-
2-1/n)/c^2/(6*n^2+5*n+1)+6*a^2*n^2*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(6*n^
3+11*n^2+6*n+1)+6*a^3*n^3*x/c^4/(6*n^3+11*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {378, 191}

$$\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2\left(6n^2+5n+1\right)}+\frac{x(a+bx^n)^3(c+dx}{c(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*x*(a + b
*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(
a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (
6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{(3an)\int (a+bx^n)^2(c+dx^n)^{-3-\frac{1}{n}}\,dx}{c(1+3n)}$$

$$= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{(6a^2n^2)\int (a+bx}{c^2(1+}$$

$$= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{6a^2n^2x(a+bx^n)}{c^3(1+n)(1+}$$

$$= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2(1+5n+6n^2)} + \frac{6a^2n^2x(a+bx^n)}{c^3(1+n)(1+}$$

Mathematica [A] time = 0.14, size = 218, normalized size = 1.22

$$\frac{x(c+dx^n)^{-\frac{1}{n}-3}\left(a^3\left(c^3\left(6n^3+11n^2+6n+1\right)+3c^2dn\left(6n^2+5n+1\right)x^n+6cd^2n^2(3n+1)x^{2n}+6d^3n^3x^{3n}\right)+3a^2b}{c^4(n+1)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*c
^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n + 6*
n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n + 11*
n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*x^(2
*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

fricas [B] time = 0.80, size = 478, normalized size = 2.69

$$\frac{\left(6\,a^3d^4n^3+b^3c^3d+\left(2\,b^3c^3d+3\,ab^2c^2d^2+6\,a^2bcd^3\right)n^2+3\left(b^3c^3d+ab^2c^2d^2\right)n\right)xx^{4\,n}+\left(24\,a^3cd^3n^3+b^3c^4+3\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 + b
^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + 3*
a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(3*n)
```

```
 + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12*a^2*
b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2*d^2)*
n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^2*b*c^4
 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*a^3*c^4*
n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 +
 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{81,[2,0,6,4,2,4,3,0]%%%}+%%%{108,[2,0,6,3,2,4,3,0]%%%}+%%%{54,[2,0,6
,2,2,4,3,0]%%%}+%%%{12,[2,0,6,1,2,4,3,0]%%%}+%%%{1,[2,0,6,0,2,4,3,0]%%%}+%%
%{243,[1,0,6,4,2,4,2,1]%%%}+%%%{-81,[1,0,6,4,1,5,3,0]%%%}+%%%{324,[1,0,6,3,
2,4,2,1]%%%}+%%%{-108,[1,0,6,3,1,5,3,0]%%%}+%%%{162,[1,0,6,2,2,4,2,1]%%%}+%
%%{-54,[1,0,6,2,1,5,3,0]%%%}+%%%{36,[1,0,6,1,2,4,2,1]%%%}+%%%{-12,[1,0,6,1,
1,5,3,0]%%%}+%%%{3,[1,0,6,0,2,4,2,1]%%%}+%%%{-1,[1,0,6,0,1,5,3,0]%%%}+%%%{8
1,[0,0,6,4,3,3,0,3]%%%}+%%%{81,[0,0,6,3,3,3,0,3]%%%}+%%%{81,[0,0,6,3,2,4,1,
2]%%%}+%%%{-81,[0,0,6,3,1,5,2,1]%%%}+%%%{27,[0,0,6,3,0,6,3,0]%%%}+%%%{27,[0
,0,6,2,3,3,0,3]%%%}+%%%{81,[0,0,6,2,2,4,1,2]%%%}+%%%{-81,[0,0,6,2,1,5,2,1]%
%%}+%%%{27,[0,0,6,2,0,6,3,0]%%%}+%%%{3,[0,0,6,1,3,3,0,3]%%%}+%%%{27,[0,0,6,
1,2,4,1,2]%%%}+%%%{-27,[0,0,6,1,1,5,2,1]%%%}+%%%{9,[0,0,6,1,0,6,3,0]%%%}+%%
%{3,[0,0,6,0,2,4,1,2]%%%}+%%%{-3,[0,0,6,0,1,5,2,1]%%%}+%%%{1,[0,0,6,0,0,6,3
,0]%%%} / %%%{81,[0,0,7,4,3,4,0,0]%%%}+%%%{108,[0,0,7,3,3,4,0,0]%%%}+%%%{54
,[0,0,7,2,3,4,0,0]%%%}+%%%{12,[0,0,7,1,3,4,0,0]%%%}+%%%{1,[0,0,7,0,3,4,0,0]
%%%} Error: Bad Argument Value
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \left(b\,x^{n}+a\right)^{3} \left(d\,x^{n}+c\right)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^3*(d*x^n+c)^(-1/n-4),x)
```

```
[Out] int((b*x^n+a)^3*(d*x^n+c)^(-1/n-4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)^{3}(dx^{n}+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^3}{(c+d\,x^n)^{\frac{1}{n}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4),x)
[Out] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
[Out] Timed out
```

3.322 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}} \, dx$

Optimal. Leaf size=116

$$\frac{2a^2n^2x\left(c+dx^n\right)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx\left(a+bx^n\right)\left(c+dx^n\right)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x\left(a+bx^n\right)^2\left(c+dx^n\right)^{-\frac{1}{n}-2}}{c(2n+1)}$$

```
[Out] x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c/(1+2*n)+2*a*n*x*(a+b*x^n)*(c+d*x^n)^(-1-
1/n)/c^2/(2*n^2+3*n+1)+2*a^2*n^2*x/c^3/(2*n^2+3*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {378, 191}

$$\frac{2a^2n^2x\left(c+dx^n\right)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx\left(a+bx^n\right)\left(c+dx^n\right)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x\left(a+bx^n\right)^2\left(c+dx^n\right)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*x*(a + b
*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c
^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{(2an)\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx}{c(1+2n)}$$
$$= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{\left(2a^2n^2\right)\int (c+dx^n}{c^2(1+n)(1+}$$
$$= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} + \frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)} + \frac{2a^2n^2x(c+dx^n)^{-}}{c^3(1+n)(1+2n}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.97

$$\frac{x(c+dx^n)^{-\frac{1}{n}-2}\left(a^2\left(c^2\left(2n^2+3n+1\right)+2cdn(2n+1)x^n+2d^2n^2x^{2n}\right)+2abcx^n(2cn+c+dnx^n)+b^2c^2(n+1)x^{2n}}{c^3(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2*
c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d^2
*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

fricas [A] time = 1.23, size = 231, normalized size = 1.99

$$\frac{\left(2\,a^2d^3n^2+b^2c^2d+\left(b^2c^2d+2\,abcd^2\right)n\right)xx^{3\,n}+\left(6\,a^2cd^2n^2+b^2c^3+2\,abc^2d+\left(b^2c^3+6\,abc^2d+2\,a^2cd^2\right)n\right)xx^{2\,n}}{\left(2\,c^3n^2+3\,c^3n+c^3\right)(dx^n+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*a
^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d^2
)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 + 5*
a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^3*n^
2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{8,[1,0,4,3,1,3,2,0]%%%}+%%%{12,[1,0,4,2,1,3,2,0]%%%}+%%%{6,[1,0,4,1,
1,3,2,0]%%%}+%%%{1,[1,0,4,0,1,3,2,0]%%%}+%%%{8,[0,0,4,3,2,2,0,2]%%%}+%%%{8,
[0,0,4,2,2,2,0,2]%%%}+%%%{8,[0,0,4,2,1,3,1,1]%%%}+%%%{-4,[0,0,4,2,0,4,2,0]%
%%}+%%%{2,[0,0,4,1,2,2,0,2]%%%}+%%%{8,[0,0,4,1,1,3,1,1]%%%}+%%%{-4,[0,0,4,1
,0,4,2,0]%%%}+%%%{2,[0,0,4,0,1,3,1,1]%%%}+%%%{-1,[0,0,4,0,0,4,2,0]%%%} / %%
%{8,[0,0,5,3,2,3,0,0]%%%}+%%%{12,[0,0,5,2,2,3,0,0]%%%}+%%%{6,[0,0,5,1,2,3,0
,0]%%%}+%%%{1,[0,0,5,0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \left(b\,x^{n}+a\right)^{2} \left(d\,x^{n}+c\right)^{-\frac{1}{n}-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-3),x)
```

```
[Out] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)^{2}(dx^{n}+c)^{-\frac{1}{n}-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x^{n}\right)}^{2}}{{\left(c+d\,x^{n}\right)}^{\frac{1}{n}+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

3.323 $\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=58

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

```
[Out] x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c/(1+n)+a*n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 191}

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n
)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{(an)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.14, size = 82, normalized size = 1.41

$$\frac{x\left(c+dx^n\right)^{-\frac{n+1}{n}}\left(a(n+1)\left(c+dx^n\right)\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bcx^n\right)}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F
1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n)^
((1 + n)/n))
```

fricas [A] time = 0.83, size = 85, normalized size = 1.47

$$\frac{\left(ad^2n+bcd\right)xx^{2\,n}+\left(2\,acdn+bc^2+acd\right)xx^n+\left(ac^2n+ac^2\right)x}{\left(c^2n+c^2\right)\left(dx^n+c\right)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] ((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*n
 + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,0,2,2,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,0,1]%%%}+%%%{1,[0,0,2,1,0
,2,1,0]%%%}+%%%{1,[0,0,2,0,0,2,1,0]%%%} / %%%{1,[0,0,3,2,1,2,0,0]%%%}+%%%{2
,[0,0,3,1,1,2,0,0]%%%}+%%%{1,[0,0,3,0,1,2,0,0]%%%} Error: Bad Argument Valu
e
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(b\,x^n+a\right)\left(d\,x^n+c\right)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(d*x^n+c)^(-1/n-2),x)
[Out] int((b*x^n+a)*(d*x^n+c)^(-1/n-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\, x^n}{(c + d\, x^n)^{\frac{1}{n}+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 2),x)
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
[Out] Timed out
```

3.324 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

Optimal. Leaf size=18

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

```
[Out] x/c/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {191}

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

fricas [A] time = 1.00, size = 31, normalized size = 1.72

$$\frac{dxx^n+cx}{(dx^n+c)^{\frac{n+1}{n}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
```

```
[Out] (d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

maple [B] time = 0.07, size = 53, normalized size = 2.94

$$\frac{dx\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(-\frac{1}{n}-1\right)\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}}{c}+x\,\mathrm{e}^{\left(-\frac{1}{n}-1\right)\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n-1),x)
```

```
[Out] x*exp((-1/n-1)*ln(c+d*exp(n*ln(x))))+1/c*d*x*exp(n*ln(x))*exp((-1/n-1)*ln(c
+d*exp(n*ln(x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

mupad [B] time = 1.76, size = 75, normalized size = 4.17

$$\frac{d\,x^{n+1}\left(\frac{c}{d\,x^n}-\left(\frac{c}{d\,x^n}+1\right)^{\frac{n+1}{n}}+1\right)}{c\,n\left(\frac{n+1}{n}-1\right)\left(c+d\,x^n\right)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^n)^(1/n + 1),x)
```

```
[Out] (d*x^(n + 1)*(c/(d*x^n) - (c/(d*x^n) + 1)^((n + 1)/n) + 1))/(c*n*((n + 1)/n
 - 1)*(c + d*x^n)^((n + 1)/n))
```

sympy **[A]** time = 33.05, size = 211, normalized size = 11.72

$$\begin{cases} -\frac{d^{-\frac{1}{n}}xx^{-n}(x^n)^{-\frac{1}{n}}}{dn} & \text{for } c = 0 \\ 0^{-1-\frac{1}{n}}x & \text{for } c = - \\ x\left(0^n\right)^{-1-\frac{1}{n}} & \text{for } c = 0 \\ \frac{c^2x}{c^3(c+dx^n)^{\frac{1}{n}}+2c^2dx^n(c+dx^n)^{\frac{1}{n}}+cd^2x^{2n}(c+dx^n)^{\frac{1}{n}}} + \frac{cdxx^n}{c^3(c+dx^n)^{\frac{1}{n}}+2c^2dx^n(c+dx^n)^{\frac{1}{n}}+cd^2x^{2n}(c+dx^n)^{\frac{1}{n}}} + \frac{dxx^n}{c^2(c+dx^n)^{\frac{1}{n}}+cdx^n(c+dx^n)^{\frac{1}{n}}} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n),x)
```

```
[Out] Piecewise((-d**(-1/n)*x*x**(-n)*(x**n)**(-1/n)/(d*n), Eq(c, 0)), (0**(-1 -
1/n)*x, Eq(c, -d*x**n)), (x*(0**n)**(-1 - 1/n), Eq(c, 0**n - d*x**n)), (c**
2*x/(c**3*(c + d*x**n)**(1/n) + 2*c**2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*
x**(2*n)*(c + d*x**n)**(1/n)) + c*d*x*x**n/(c**3*(c + d*x**n)**(1/n) + 2*c*
*2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*x**(2*n)*(c + d*x**n)**(1/n)) + d*x*
x**n/(c**2*(c + d*x**n)**(1/n) + c*d*x**n*(c + d*x**n)**(1/n)), True))
```

3.325 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(c+dx^n\right)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

```
[Out] x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/((c+d*x^n)^(1/n
))
```

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {379}

$$\frac{x\left(c+dx^n\right)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^n\right)^{-1/n}}{a+bx^n}\,dx=\frac{x\left(c+dx^n\right)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.98

$$\frac{x\left(c+dx^n\right)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*
x^n))])/(a*(c + d*x^n)^n^(-1))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(bx^n + a)(dx^n + c)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n + c)^{-\frac{1}{n}}}{b\,x^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)/((d*x^n+c)^(1/n)),x)
```

```
[Out] int(1/(b*x^n+a)/((d*x^n+c)^(1/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^(1/n)),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.326 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=54

$$\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

```
[Out] c*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/((c+d*x^n)^
(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {379}

$$\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d
*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx=\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.98

$$\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c +
d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{n-1}{n}}}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n + c)^{-\frac{1}{n}+1}}{(b\,x^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n+1)/(b*x^n+a)^2,x)
```

```
[Out] int((d*x^n+c)^(-1/n+1)/(b*x^n+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2,x)
```

```
[Out] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.327 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} \, dx$

Optimal. Leaf size=56

$$\frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

```
[Out] c^2*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/((c+d*x^n
)^(1/n))
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {379}

$$\frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
 d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} \, dx = \frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.98

$$\frac{c^2x(c+dx^n)^{-1/n} \, {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c
+ d*x^n))])/(a^3*(c + d*x^n)^n^(-1))
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{2n-1}{n}}}{b^3x^{3n} + 3ab^2x^{2n} + 3a^2bx^n + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b
*x^n + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n + c)^{-\frac{1}{n}+2}}{(b\,x^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n+2)/(b*x^n+a)^3,x)
```

```
[Out] int((d*x^n+c)^(-1/n+2)/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c + d\,x^n)^{2-\frac{1}{n}}}{(a + b\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3,x)
[Out] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.328 $\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx$

Optimal. Leaf size=193

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}(n(p+1)(bc-ad)+bc)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}\,{}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{acn(p+1)(bc-ad)}-\frac{bx(a+bx}{an(}$$

```
[Out] -b*x*(a+b*x^n)^(1+p)*(c+d*x^n)^(-1-1/n-p)/a/(-a*d+b*c)/n/(1+p)+(b*c+(-a*d+b
*c)*n*(1+p))*x*(a+b*x^n)^(1+p)*(c*(a+b*x^n)/a/(c+d*x^n))^(-1-p)*(c+d*x^n)^(
-1-1/n-p)*hypergeom([1/n, -1-p],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/c/(-
a*d+b*c)/n/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {382, 380}

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}\left(\frac{b}{n(p+1)(bc-ad)}+\frac{1}{c}\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}\,{}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}-\frac{bx(a+bx^n)^{p+1}}{an(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n*
(1 + p))) + ((c^(-1) + b/((b*c - a*d)*n*(1 + p)))*x*(a + b*x^n)^(1 + p)*((c
*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hyper
geometric2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)
))])/a
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
```

```
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)\int (a+bx^n)^{1+p}(c+dx}{a}$$

$$= -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)x(a+bx^n)^{1+p}\left(\frac{c(a+bx^n}{a(c+dx^n}\right.}{}$$

Mathematica [B] time = 51.21, size = 1414, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] (c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)^(3 + p)*(c + d*x^n)^(-2 - n^
(-1) - p)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Gamma[-p]*(Hypergeometric2F1[1,
 -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (d*n*x^n*((c*Hypergeo
metric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/(1 + n) +
((b*c - a*d)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p,
 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b*x^n)*Gam
ma[2 + n^(-1)]*Gamma[-p])))/c^2))/(-(c*d*(1 + 3*n)*(1 + n + n*p)*x^n*(a + b
*x^n)^2*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hype
rgeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x
^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[1
, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*
x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1),
((b*c - a*d)*x^n)/(c*(a + b*x^n))]))) + b*c*n*(1 + 3*n)*p*x^n*(a + b*x^n)*(
c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*H
ypergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*
n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F
1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 +
n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1
), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + c*(1 + 3*n)*(a + b*x^n)^2*(c + d*
x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hyperge
ometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*
(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[1, -
```

```
p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*x^n
*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))])) + n^2*x^n*(c + d*x^n)*(a*c^2*(-(b*c) + a*
d)*(1 + 2*n)*(1 + 3*n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeome
tric2F1[2, 1 - p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + c*d*(1 +
 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hy
pergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*
c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1
- p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) - d*(b*c - a*d)*x^n*(b
*c*(1 + n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hyperge
ometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*(1
+ n)*(1 + 3*n)*(a + b*x^n)^2*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2
F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + a*c*n*(1 + 3*
n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[2, 1 - p, 3
+ n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 2*a*(-(b*c) + a*d)*n*(1 + n)
*(-1 + p)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[3, 2 - p, 4
+ n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p}{(dx^n+c)^{\frac{np+2n+1}{n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \left(b\,x^{n}+a\right)^{p} \left(d\,x^{n}+c\right)^{-p-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p*(d*x^n+c)^(-2-1/n-p),x)
[Out] int((b*x^n+a)^p*(d*x^n+c)^(-2-1/n-p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (dx^n + c)^{-p-\frac{1}{n}-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\, x^n)^p}{(c + d\, x^n)^{p+\frac{1}{n}+2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2),x)
[Out] int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
[Out] Timed out
```

3.329 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}} (c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

```
[Out] x*(c+d*x^n)^(a*d/(-a*d+b*c)/n)/a/c/((a+b*x^n)^(b*c/(-a*d+b*c)/n))
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {381}

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}} (c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d -
 b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*d
)*n)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.96

$$\frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}} (c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^(
(a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a*
d*n)))
```

fricas [A] time = 1.27, size = 108, normalized size = 1.89

$$\frac{\left(bdxx^{2n}+acx+(bc+ad)xx^{n}\right)(dx^{n}+c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^{n}+a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d)
*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))*a
*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^{n}+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^{n}+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int (b\,x^{n}+a)^{\frac{adn-(n+1)bc}{(-ad+bc)n}}(d\,x^{n}+c)^{\frac{adn-bcn+ad}{-adn+bcn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^((a*d*n-b*c*(n+1))/(-a*d+b*c)/n)*(d*x^n+c)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

```
[Out] int((b*x^n+a)^((a*d*n-b*c*(n+1))/(-a*d+b*c)/n)*(d*x^n+c)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{bc(n+1) - adn}{(bc - ad)n}} (dx^n + c)^{\frac{bcn - adn - ad}{bcn - adn}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + b\,x^n)^{\frac{a\,d\,n - b\,c\,(n+1)}{n\,(a\,d - b\,c)}} \, (c + d\,x^n)^{\frac{a\,d + a\,d\,n - b\,c\,n}{a\,d\,n - b\,c\,n}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))),x)
```

```
[Out] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Timed out
```

3.330 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx$

Optimal. Leaf size=327

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+\cdots}{c^2\left(6n^2\cdots\right.}$$

```
[Out] -1/3*b*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/(-a*d+b*c)/n-1/3*(3*a*d*n-b*(3*c*
n+c))*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/a/c/(-a*d+b*c)/n/(1+3*n)-(3*a*d*n-b*
(3*c*n+c))*x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c^2/(-a*d+b*c)/(6*n^2+5*n+1)-2*
a*n*(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)*(c+d*x^n)^(-1-1/n)/c^3/(-a*d+b*c)/(6*
n^3+11*n^2+6*n+1)-2*a^2*n^2*(3*a*d*n-b*(3*c*n+c))*x/c^4/(-a*d+b*c)/(6*n^3+1
1*n^2+6*n+1)/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.18, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {382, 378, 191}

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2\left(6n^2+5n+1\right)(bc-ad)}-\frac{2anx(a+bx^n)(c+\cdots}{c^3(n+1)(2n\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] -(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*(b*c - a*d)*n) - ((3*a*
d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*c*(b*c
 - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^2*(c + d*x^
n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n - b
*(c + 3*c*n))*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(1
+ n)*(1 + 2*n)*(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*(b
*c - a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
```

```
&& GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)\int (a+bx^n)^3(c+dx^n)^{-4-\frac{1}{n}}\,dx}{3a}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{\left(n\left(3\right.\right.}{}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3\right.}{}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3\right.}{}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3\right.}{}$$

Mathematica [C] time = 0.46, size = 136, normalized size = 0.42

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left(b^2c^2\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-(bc-ad)\left((ad-bc)\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+2bc\right.\right.}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b^2*c^2*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
+ n^(-1), -((d*x^n)/c)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[3 + n^(-1),
```

```
n^(-1), 1 + n^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[4 + n^
(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*d^2*(c + d*x^n)^n^(-1))
```

fricas [A] time = 0.97, size = 400, normalized size = 1.22

$$\frac{\left(6\,a^2d^4n^3 + b^2c^2d^2n + \left(b^2c^2d^2 + 4\,abcd^3\right)n^2\right)xx^{4\,n} + \left(24\,a^2cd^3n^3 + b^2c^3d + 2\left(2\,b^2c^3d + 8\,abc^2d^2 + 3\,a^2cd^3\right)n^2 + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*n
) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^2*
c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^2*n
^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2
+ (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n
^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4
 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4*n +
a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1
)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{27,[1,0,4,3,1,3,2,0]%%%}+%%%{27,[1,0,4,2,1,3,2,0]%%%}+%%%{9,[1,0,4,1
,1,3,2,0]%%%}+%%%{1,[1,0,4,0,1,3,2,0]%%%}+%%%{27,[0,0,4,3,2,2,0,2]%%%}+%%%{
18,[0,0,4,2,2,2,0,2]%%%}+%%%{18,[0,0,4,2,1,3,1,1]%%%}+%%%{-9,[0,0,4,2,0,4,2
,0]%%%}+%%%{3,[0,0,4,1,2,2,0,2]%%%}+%%%{12,[0,0,4,1,1,3,1,1]%%%}+%%%{-6,[0,
0,4,1,0,4,2,0]%%%}+%%%{2,[0,0,4,0,1,3,1,1]%%%}+%%%{-1,[0,0,4,0,0,4,2,0]%%%}
 / %%%{27,[0,0,5,3,2,3,0,0]%%%}+%%%{27,[0,0,5,2,2,3,0,0]%%%}+%%%{9,[0,0,5,1
,2,3,0,0]%%%}+%%%{1,[0,0,5,0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \left(b\,x^n + a\right)^2 \left(d\,x^n + c\right)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-4),x)
```

```
[Out] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,x^n)^2}{(c + d\,x^n)^{\frac{1}{n}+4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

3.331 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal. Leaf size=127

$$\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

```
[Out] -(-a*d+b*c)*x*(c+d*x^n)^(-2-1/n)/c/d/(1+2*n)+(2*a*d*n+b*c)*x*(c+d*x^n)^(-1-
1/n)/c^2/d/(1+n)/(1+2*n)+n*(2*a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x^n)^(
1/n))
```

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {385, 192, 191}

$$\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}+\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a*
d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a
*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rubi steps

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)\int (c+dx^n)^{-2-\frac{1}{n}}\,dx}{cd(1+2n)}$$
$$= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{(n(bc+2adn))\int (c+\ldots}{c^2d(1+n)(1\ldots}$$
$$= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{n(bc+2adn)x(c+d\ldots}{c^3d(1+n)(1+2\ldots}$$

Mathematica [C] time = 0.17, size = 94, normalized size = 0.74

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left((ad-bc)\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bc\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b*c*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^
(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3 + n^(-1), n^(-1),
1 + n^(-1), -((d*x^n)/c)]))/(c^3*d*(c + d*x^n)^n^(-1))
```

fricas [A] time = 1.35, size = 173, normalized size = 1.36

$$\frac{\left(2\,ad^3n^2+bcd^2n\right)xx^{3\,n}+\left(6\,acd^2n^2+bc^2d+\left(3\,bc^2d+2\,acd^2\right)n\right)xx^{2\,n}+\left(6\,ac^2dn^2+bc^3+ac^2d+\left(2\,bc^3+5\,ac^2d\ldots\right.\right.}{\left(2\,c^3n^2+3\,c^3n+c^3\right)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2*
d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3 +
 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2 + 3
*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%%{4,[0,0,2,2,1,1,0,1]%%%}+%%%{2,[0,0,2,1,1,1,0,1]%%%}+%%%{2,[0,0,2,1,0,2,1,0]%%%}+%%%{1,[0,0,2,0,0,2,1,0]%%%} / %%%{4,[0,0,3,2,1,2,0,0]%%%}+%%%{4,[0,0,3,1,1,2,0,0]%%%}+%%%{1,[0,0,3,0,1,2,0,0]%%%} Error: Bad Argument Value

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \left(b\,x^{n}+a\right)\left(d\,x^{n}+c\right)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^n+a)*(d*x^n+c)^(-1/n-3),x)

[Out] int((b*x^n+a)*(d*x^n+c)^(-1/n-3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)(dx^{n}+c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")

[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^{n}}{{\left(c+d\,x^{n}\right)}^{\frac{1}{n}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 3),x)

[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
[Out] Timed out
```

3.332 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=50

$$\frac{nx\left(c+dx^n\right)^{-1/n}}{c^2(n+1)}+\frac{x\left(c+dx^n\right)^{-\frac{1}{n}-1}}{c(n+1)}$$

```
[Out] x*(c+d*x^n)^(-1-1/n)/c/(1+n)+n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {192, 191}

$$\frac{nx\left(c+dx^n\right)^{-1/n}}{c^2(n+1)}+\frac{x\left(c+dx^n\right)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^
n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x\left(c+dx^n\right)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{n\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x\left(c+dx^n\right)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx\left(c+dx^n\right)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 1.10

$$\frac{x\left(c+dx^n\right)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

fricas [A] time = 1.41, size = 68, normalized size = 1.36

$$\frac{d^2nxx^{2\,n}+(2\,cdn+cd)xx^n+\left(c^2n+c^2\right)x}{\left(c^2n+c^2\right)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] (d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)*
(d*x^n + c)^((2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int(d\,x^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n-2),x)
```

```
[Out] int((d*x^n+c)^(-1/n-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

mupad [B] time = 1.76, size = 64, normalized size = 1.28

$$-\frac{x^{1-2\,n}\,{\left(\frac{c}{d\,x^n}+1\right)}^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{c}{d\,x^n}\right)}{2\,d^2\,n\,{\left(c+d\,x^n\right)}^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^n)^(1/n + 2),x)
```

```
[Out] -(x^(1 - 2*n)*(c/(d*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -c/(d*x^n)))
/(2*d^2*n*(c + d*x^n)^(1/n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

3.333 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx$

Optimal. Leaf size=95

$$\frac{bx\,(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx\,(c+dx^n)^{-1/n}}{c(bc-ad)}$$

```
[Out] -d*x/c/(-a*d+b*c)/((c+d*x^n)^(1/n))+b*x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b
*c)*x^n/a/(c+d*x^n))/a/(-a*d+b*c)/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {382, 379}

$$\frac{bx\,(c+dx^n)^{-1/n}\ {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx\,(c+dx^n)^{-1/n}}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c +
 d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = -\frac{dx\,(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{b\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{bc-ad}$$
$$= -\frac{dx\,(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{bx\,(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}$$

Mathematica [C] time = 6.94, size = 153, normalized size = 1.61

$$\frac{x\,(c+dx^n)^{-\frac{n+1}{n}}\left(\frac{bnx^{2n}(ad-bc)\,{}_2F_1\left(2,2+\frac{1}{n};3+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2(2n+1)(c+dx^n)} + \frac{bx^n\Phi\left(\frac{(ad-bc)x^n}{a(dx^n+c)},1,1+\frac{1}{n}\right)}{a} + \frac{a(c+dx^n)}{c(a+bx^n)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] (x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(b*c) + a*d
)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c) + a*d)*n*x^(2*n)*Hyp
ergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^
n))])/(a^2*(1 + 2*n)*(c + d*x^n))))/(a*(c + d*x^n)^((1 + n)/n))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(bx^n+a)(dx^n+c)^{\frac{n+1}{n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^{n}+c)^{-\frac{1}{n}-1}}{b\,x^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n-1)/(b*x^n+a),x)
[Out] int((d*x^n+c)^(-1/n-1)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^{n}+c)^{-\frac{1}{n}-1}}{bx^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^{n})\,(c+d\,x^{n})^{\frac{1}{n}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.334 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=127

$$\frac{bx\left(c+dx^n\right)^{-\frac{1-n}{n}}}{an(bc-ad)\left(a+bx^n\right)}-\frac{x\left(c+dx^n\right)^{-1/n}\left(adn+bc(1-n)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)/((c+d*x^n)^((1-n)/n))-(b*c*(1-n)+a*d*n)*x*hype
rgeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/(-a*d+b*c)/n/((c+d*
x^n)^(1/n))
```

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {382, 379}

$$\frac{bx\left(c+dx^n\right)^{-\frac{1-n}{n}}}{an(bc-ad)\left(a+bx^n\right)}-\frac{x\left(c+dx^n\right)^{-1/n}\left(adn+bc(1-n)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n)
 + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a
*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \frac{bx\,(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc-(bc-ad)n)\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{a(bc-ad)n}$$

$$= \frac{bx\,(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)} - \frac{(bc(1-n)+adn)x\,(c+dx^n)^{-1/n}\;{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}$$

Mathematica [**B**] time = 63.26, size = 1070, normalized size = 8.43

$$-cd(1-n)(2n+1)(3n+1)\,(bx^n+a)^2\left(2(bc-ad)n\,(dx^n+c)\,\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(2,3;3+\frac{1}{n};\frac{(bc-ad)x^n}{c(bx^n+a)}\right)x^n + c\,(bx^n+a)\,(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (c^2*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Ga
mma[3 + n^(-1)]*((c*(c + c*n + d*n*x^n)*Hypergeometric2F1[1, 2, 2 + n^(-1),
 ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/Gamma[2 + n^(-1)] + (2*(b*c - a*d)*n*x
^n*(c + d*x^n)*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a
+ b*x^n))])/((a + b*x^n)*Gamma[3 + n^(-1)])))/((c + d*x^n)^n^(-1)*(-(c*d*(1
 - n)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*(c*(a + b*x^n)*(c + c*n + d*n*x
^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)
/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*Hyper
geometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) - 2*b*c
*n*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*(c*(a + b*x^n)*(c + c*n
+ d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*
d)*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)
]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) +
 c*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c*(a + b*x^n)*(c + c*n +
d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)
*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*
Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + n
^2*x^n*(c + d*x^n)*(c^2*d*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^3*Gamma[3 + n^(-1
)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] +
 2*c*d*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*Gamma[2 + n^(-1)]*
Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 2*
b*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*Gamma[2 + n
^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n)
)] + 2*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*Gamma[2
```

```
+ n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x
^n))] + 2*a*c*(b*c - a*d)*(1 + 3*n)*(a + b*x^n)*(c + c*n + d*n*x^n)*Gamma[3
 + n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] + 12*a*(b*c - a*d)^2*n*(1 + 2*n)*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*H
ypergeometric2F1[3, 4, 4 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)(dx^n + c)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="fricas")
[Out] integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{(d\,x^n + c)^{-\frac{1}{n}}}{(b\,x^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^2/((d*x^n+c)^(1/n)),x)
[Out] int(1/(b*x^n+a)^2/((d*x^n+c)^(1/n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^{1/n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.335 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=131

$$\frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)\,(a+bx^n)^2}-\frac{cx\,(c+dx^n)^{-1/n}\,(2adn+bc(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

```
[Out] 1/2*b*x*(c+d*x^n)^(2-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^2-1/2*c*(b*c*(1-2*n)+2*a
*d*n)*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/(-a*d+b
*c)/n/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {382, 379}

$$\frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)\,(a+bx^n)^2}-\frac{cx\,(c+dx^n)^{-1/n}\,(2adn+bc(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c*
(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a
*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} - \frac{(bc-2(bc-ad)n)\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx}{2a(bc-ad)n}$$

$$= \frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} - \frac{c(bc(1-2n)+2adn)x\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n}$$

Mathematica [B] time = 44.86, size = 1241, normalized size = 9.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] -((c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(c + d*x^n)^(2 - n^(-1))*Gamma[2 + n^(
-1)]*(Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))
] + (d*n*x^n*((c*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(
a + b*x^n))])/(1 + n) + (3*(b*c - a*d)*x^n*Gamma[1 + n^(-1)]*Hypergeometric
2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b
*x^n)*Gamma[2 + n^(-1)])))/c^2))/(c*d*(1 - 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2
*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1,
3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a
 + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d
)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hyper
geometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + 3*b*c
*n*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)
*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c
*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hyperge
ometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c -
a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*
c - a*d)*x^n)/(c*(a + b*x^n))])) - c*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c
^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3,
1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a +
b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x
^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeo
metric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + n^2*x^n*
(c + d*x^n)*(3*a*c^2*(-(b*c) + a*d)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)*Gamma[2
 + n^(-1)]*Hypergeometric2F1[2, 4, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] - c*d*(1 + 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(
```

```
-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]
 + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 +
n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + 3*d*(b*c - a*d)*x^n*(b*c*(1 +
 n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 +
 n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*(1 + n)*(1 + 3*n)*(a + b*x^
n)^2*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n
)/(c*(a + b*x^n))] - a*c*n*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeo
metric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 8*a*(-(b*c
) + a*d)*n*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[3, 5, 4 + n^(-1)
, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n+c)^{\frac{n-1}{n}}}{b^3x^{3n}+3\,ab^2x^{2n}+3\,a^2bx^n+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x
^n + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int\frac{(d\,x^n+c)^{-\frac{1}{n}+1}}{(b\,x^n+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n+1)/(b*x^n+a)^3,x)
```

```
[Out] int((d*x^n+c)^(-1/n+1)/(b*x^n+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x^n)^{1-\frac{1}{n}}}{(a + b\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3,x)
[Out] int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
[Out] Timed out
```

3.336 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx$

Optimal. Leaf size=133

$$\frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)\,(a+bx^n)^3}-\frac{c^2x\,(c+dx^n)^{-1/n}\,(3adn+bc(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

```
[Out] 1/3*b*x*(c+d*x^n)^(3-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^3-1/3*c^2*(b*c*(1-3*n)+3
*a*d*n)*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^4/(-a*d
+b*c)/n/((c+d*x^n)^(1/n))
```

Rubi [A] time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {382, 379}

$$\frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)\,(a+bx^n)^3}-\frac{c^2x\,(c+dx^n)^{-1/n}\,(3adn+bc(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b*
c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n\,(a+bx^n)^3} - \frac{(bc-3(bc-ad)n)\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx}{3a(bc-ad)n}$$
$$= \frac{bx\,(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n\,(a+bx^n)^3} - \frac{c^2(bc(1-3n)+3adn)x\,(c+dx^n)^{-1/n}\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
[Out] $Aborted
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(dx^n+c)^{\frac{2n-1}{n}}}{b^4x^{4n}+4ab^3x^{3n}+6a^2b^2x^{2n}+4a^3bx^n+a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="fricas")
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n) + 6*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="giac")
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{n}+c\right)^{-\frac{1}{n}+2}}{\left(b\,x^{n}+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n+2)/(b*x^n+a)^4,x)
```

```
[Out] int((d*x^n+c)^(-1/n+2)/(b*x^n+a)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{n}+c\right)^{-\frac{1}{n}+2}}{\left(bx^{n}+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c+d\,x^{n}\right)^{2-\frac{1}{n}}}{\left(a+b\,x^{n}\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4,x)
```

```
[Out] int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.337 $\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=152

$$\frac{(dx-c)^{7/2}(c+dx)^{7/2}\left(ad^2+3bc^2\right)}{7d^8}+\frac{c^2(dx-c)^{5/2}(c+dx)^{5/2}\left(2ad^2+3bc^2\right)}{5d^8}+\frac{c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^8}+\frac{b}{}$$

```
[Out] 1/3*c^4*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^8+1/5*c^2*(2*a*d^2+3*b*
c^2)*(d*x-c)^(5/2)*(d*x+c)^(5/2)/d^8+1/7*(a*d^2+3*b*c^2)*(d*x-c)^(7/2)*(d*x
+c)^(7/2)/d^8+1/9*b*(d*x-c)^(9/2)*(d*x+c)^(9/2)/d^8
```

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{21d^4}+\frac{4c^2x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{105d^6}+\frac{8c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2\right.}{315d^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(315*d^8) + (4
*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) +
((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(21*d^4) + (b*x^
6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(9*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^5\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx &= \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int x^5\sqrt{-c+dx}\,\sqrt{c+dx}\,dx \\
&= \frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+ \\
&= \frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+ \\
&= \frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(2bc^2+3ad^2\right)x^4(-c+dx}{21d^4} \\
&= \frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(2bc^2+3ad^2\right)x^4(-c+dx}{21d^4} \\
&= \frac{8c^4\left(2bc^2+3ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{315d^8}+\frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+d}{105d^6}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.72

$$
\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(d^2x^2-c^2\right)\left(3ad^2\left(8c^4+12c^2d^2x^2+15d^4x^4\right)+b\left(16c^6+24c^4d^2x^2+30c^2d^4x^4+35d^6x^6\right)\right)}{315d^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(3*a*d^2*(8*c^4 + 12*c^2*d^2
*x^2 + 15*d^4*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x
^6)))/(315*d^8)
```

fricas [A] time = 1.34, size = 114, normalized size = 0.75

$$\frac{\left(35\,bd^{8}x^{8}-16\,bc^{8}-24\,ac^{6}d^{2}-5\left(bc^{2}d^{6}-9\,ad^{8}\right)x^{6}-3\left(2\,bc^{4}d^{4}+3\,ac^{2}d^{6}\right)x^{4}-4\left(2\,bc^{6}d^{2}+3\,ac^{4}d^{4}\right)x^{2}\right)\sqrt{dx+c}\,\sqrt{}}{315\,d^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b*d^8*x^8 - 16*b*c^8 - 24*a*c^6*d^2 - 5*(b*c^2*d^6 - 9*a*d^8)*x^6
 - 3*(2*b*c^4*d^4 + 3*a*c^2*d^6)*x^4 - 4*(2*b*c^6*d^2 + 3*a*c^4*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^8
```

giac [B] time = 0.76, size = 621, normalized size = 4.09

$$168\left(\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{5\,(dx+c)}{d^{5}}-\frac{31\,c}{d^{5}}\right)+\frac{321\,c^{2}}{d^{5}}\right)-\frac{451\,c^{3}}{d^{5}}\right)(dx+c)+\frac{745\,c^{4}}{d^{5}}\right)(dx+c)-\frac{405\,c^{5}}{d^{5}}\right)\sqrt{dx+c}\,\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/40320*(168*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 32
1*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)
*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^5)*a*c + 3*(((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x + c)/d^7 - 57*
c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^4/d^7)*(d*x + c
) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23205*c^7/d^7)*sq
rt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)
))/d^7)*b*c + 24*(((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^
6) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 633
5*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7
*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*a*d + (((2*((4*(5*(2*(d*x +
c)*(7*(d*x + c)*(8*(d*x + c)/d^8 - 73*c/d^8) + 2073*c^2/d^8) - 9833*c^3/d^8
)*(d*x + c) + 75293*c^4/d^8)*(d*x + c) - 310203*c^5/d^8)*(d*x + c) + 216993
*c^6/d^8)*(d*x + c) - 205275*c^7/d^8)*(d*x + c) + 69615*c^8/d^8)*sqrt(d*x +
 c)*sqrt(d*x - c) + 22050*c^9*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^8)
*b*d)/d
```

maple [A] time = 0.04, size = 92, normalized size = 0.61

$$\frac{(dx+c)^{\frac{3}{2}}\left(35b\,x^{6}d^{6}+45a\,d^{6}x^{4}+30b\,c^{2}d^{4}x^{4}+36a\,c^{2}d^{4}x^{2}+24b\,c^{4}d^{2}x^{2}+24a\,c^{4}d^{2}+16b\,c^{6}\right)(dx-c)^{\frac{3}{2}}}{315d^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/315*(d*x+c)^(3/2)*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*d^4*x^4+36*a*c^2*d^
4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)*(d*x-c)^(3/2)/d^8
```

maxima [A] time = 0.50, size = 178, normalized size = 1.17

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^4}{21\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x^2}{105\,d^6}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x^2}{35\,d^4}+\frac{16\left(d^2x^2-\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^4/
d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2*x^2/d^4 + 16/315*(d^2*x^2 - c
^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^(3/2)*a*c^4/d^6
```

mupad [B] time = 1.76, size = 152, normalized size = 1.00

$$-\sqrt{d\,x-c}\,\left(\frac{\left(16\,b\,c^8+24\,a\,c^6\,d^2\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{b\,x^8\,\sqrt{c+d\,x}}{9}+\frac{x^4\,\left(6\,b\,c^4\,d^4+9\,a\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}+\frac{x^2\,\left(8\,b\,c^6\,d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((16*b*c^8 + 24*a*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8) - (
b*x^8*(c + d*x)^(1/2))/9 + (x^4*(9*a*c^2*d^6 + 6*b*c^4*d^4)*(c + d*x)^(1/2)
)/(315*d^8) + (x^2*(12*a*c^4*d^4 + 8*b*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8)
- (x^6*(45*a*d^8 - 5*b*c^2*d^6)*(c + d*x)^(1/2))/(315*d^8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.338 $\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=109

$$\frac{(dx-c)^{5/2}(c+dx)^{5/2}\left(ad^2+2bc^2\right)}{5d^6}+\frac{c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^6}+\frac{b(dx-c)^{7/2}(c+dx)^{7/2}}{7d^6}$$

```
[Out] 1/3*c^2*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/5*(a*d^2+2*b*c^2)*(
d*x-c)^(5/2)*(d*x+c)^(5/2)/d^6+1/7*b*(d*x-c)^(7/2)*(d*x+c)^(7/2)/d^6
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{35d^4}+\frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{105d^6}+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) + ((
4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(35*d^4) + (b*x^4*
(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx = \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} - \frac{1}{7}\left(-7a-\frac{4bc^2}{d^2}\right)\int x^3\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} +$$

$$= \frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} +$$

$$= \frac{2c^2\left(4bc^2+7ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/}}{35d^4}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.81

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(d^2x^2-c^2\right)\left(7ad^2\left(2c^2+3d^2x^2\right)+b\left(8c^4+12c^2d^2x^2+15d^4x^4\right)\right)}{105d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(7*a*d^2*(2*c^2 + 3*d^2*x^2)
 + b*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

fricas [A] time = 0.99, size = 90, normalized size = 0.83

$$\frac{\left(15\,bd^6x^6-8\,bc^6-14\,ac^4d^2-3\left(bc^2d^4-7\,ad^6\right)x^4-\left(4\,bc^4d^2+7\,ac^2d^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*d^6*x^6 - 8*b*c^6 - 14*a*c^4*d^2 - 3*(b*c^2*d^4 - 7*a*d^6)*x^4
- (4*b*c^4*d^2 + 7*a*c^2*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

giac [B] time = 0.75, size = 495, normalized size = 4.54

$$70\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)ac+7\left(\left(\left(2\left((d\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*(70*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d
^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x +
c) + sqrt(d*x - c)))/d^3)*a*c + 7*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)
/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*
x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d
*x + c) + sqrt(d*x - c)))/d^5)*b*c + 14*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x
 + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4
)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^4)*a*d + (((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6
) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335
*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*
log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*b*d)/d
```

maple [A] time = 0.05, size = 68, normalized size = 0.62

$$\frac{(dx+c)^{\frac{3}{2}}\left(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4\right)(dx-c)^{\frac{3}{2}}}{105d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/105*(d*x+c)^(3/2)*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*d^2*x^2+14*a*c^2*d^
2+8*b*c^4)*(d*x-c)^(3/2)/d^6
```

maxima [A] time = 0.50, size = 124, normalized size = 1.14

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^4}{7\,d^2}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^2}{35\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^2}{5\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4}{105\,d^6}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^2/
d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

mupad [B] time = 1.74, size = 118, normalized size = 1.08

$$-\sqrt{d\,x-c}\,\left(\frac{\left(8\,b\,c^6+14\,a\,c^4\,d^2\right)\,\sqrt{c+d\,x}}{105\,d^6}-\frac{b\,x^6\,\sqrt{c+d\,x}}{7}+\frac{x^2\,\left(4\,b\,c^4\,d^2+7\,a\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}-\frac{x^4\,\left(21\,a\,d^6-\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((8*b*c^6 + 14*a*c^4*d^2)*(c + d*x)^(1/2))/(105*d^6) - (b
*x^6*(c + d*x)^(1/2))/7 + (x^2*(7*a*c^2*d^4 + 4*b*c^4*d^2)*(c + d*x)^(1/2))
/(105*d^6) - (x^4*(21*a*d^6 - 3*b*c^2*d^4)*(c + d*x)^(1/2))/(105*d^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(a + bx^{2}\right) \sqrt{-c + dx} \sqrt{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.339 $\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx$

Optimal. Leaf size=67

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^4}+\frac{b(dx-c)^{5/2}(c+dx)^{5/2}}{5d^4}$$

```
[Out] 1/3*(a*d^2+b*c^2)*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/5*b*(d*x-c)^(5/2)*(d*x+
c)^(5/2)/d^4
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {460, 74}

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(5ad^2+2bc^2\right)}{15d^4}+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((2*b*c^2 + 5*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (b*x^2*(-
c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx=\frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}-\frac{1}{5}\left(-5a-\frac{2bc^2}{d^2}\right)\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$
$$=\frac{\left(2bc^2+5ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{15d^4}+\frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.93

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(d^2x^2-c^2\right)\left(5ad^2+2bc^2+3bd^2x^2\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2
*x^2))/(15*d^4)
```

fricas [A] time = 1.14, size = 66, normalized size = 0.99

$$\frac{\left(3\,bd^4x^4-2\,bc^4-5\,ac^2d^2-\left(bc^2d^2-5\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*sqrt
(d*x + c)*sqrt(d*x - c)/d^4
```

giac [B] time = 0.37, size = 361, normalized size = 5.39

$$5\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bc+20\left(\sqrt{dx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*(5*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3
) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c)
 + sqrt(d*x - c)))/d^3)*b*c + 20*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2
*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqr
```

```
t(d*x - c)))/d^2)*a*d + (((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c
/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*
sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*d -
60*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) - sqrt(d*x + c)*sqrt(d*x
 - c)*(d*x - 2*c))*a*c/d)/d
```

maple [A] time = 0.05, size = 44, normalized size = 0.66

$$\frac{(dx+c)^{\frac{3}{2}}\left(3b\,d^{2}x^{2}+5a\,d^{2}+2b\,c^{2}\right)(dx-c)^{\frac{3}{2}}}{15d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(3/2)*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)*(d*x-c)^(3/2)/d^4
```

maxima [A] time = 0.62, size = 70, normalized size = 1.04

$$\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bx^{2}}{5\,d^{2}}+\frac{2\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bc^{2}}{15\,d^{4}}+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}a}{3\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*b*c^2/d^4
+ 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

mupad [B] time = 1.64, size = 83, normalized size = 1.24

$$\sqrt{d\,x-c}\,\left(\frac{b\,x^4\,\sqrt{c+d\,x}}{5}-\frac{\left(2\,b\,c^4+5\,a\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}+\frac{x^2\,\left(5\,a\,d^4-b\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (d*x - c)^(1/2)*((b*x^4*(c + d*x)^(1/2))/5 - ((2*b*c^4 + 5*a*c^2*d^2)*(c +
d*x)^(1/2))/(15*d^4) + (x^2*(5*a*d^4 - b*c^2*d^2)*(c + d*x)^(1/2))/(15*d^4)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^{2}\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.340 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x}\,dx$

Optimal. Leaf size=80

$$a\sqrt{dx-c}\,\sqrt{c+dx}-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

```
[Out] 1/3*b*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-a*c*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2
)/c)+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {460, 101, 12, 92, 205}

$$a\sqrt{dx-c}\,\sqrt{c+dx}-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^
2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
```

```
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)}{x}\,dx = \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} + a\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x}\,dx$$

$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - a\int \frac{c^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}$$

$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2\right)\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c}}$$

$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2d\right)\text{Subst}\left(\int \frac{1}{c^2d+}\right.$$

$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c}}{c}\right.$$

Mathematica [A] time = 0.24, size = 85, normalized size = 1.06

$$\frac{1}{3}\sqrt{dx-c}\,\sqrt{c+dx}\left(-\frac{3ac\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)}{\sqrt{d^2x^2-c^2}} + 3a + b\left(x^2-\frac{c^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*a + b*(-(c^2/d^2) + x^2) - (3*a*c*ArcTan[S
qrt[-c^2 + d^2*x^2]/c])/Sqrt[-c^2 + d^2*x^2]))/3
```

fricas [A] time = 0.94, size = 80, normalized size = 1.00

$$-\frac{6\,acd^2\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-\left(bd^2x^2-bc^2+3\,ad^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*c*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) - (b*d^2*x^2
 - b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c))/d^2
```

giac [A] time = 0.32, size = 78, normalized size = 0.98

$$2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)+\frac{1}{3}\,\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^2}-\frac{2\,bc}{d^2}\right)+3\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/3*sqrt(d*x + c)*s
qrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^2 - 2*b*c/d^2) + 3*a)
```

maple [B] time = 0.10, size = 174, normalized size = 2.18

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3a\,c^2d^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\,d^2x^2+3\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2-\sqrt{-c^2}}{3\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(x^2*b*d^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
+3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+3*(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(d^2*x^2
-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

maxima [A] time = 1.30, size = 52, normalized size = 0.65

$$ac \arcsin\left(\frac{c}{d|x|}\right) + \sqrt{d^2x^2 - c^2}\, a + \frac{\left(d^2x^2 - c^2\right)^{\frac{3}{2}} b}{3\, d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*c*arcsin(c/(d*abs(x))) + sqrt(d^2*x^2 - c^2)*a + 1/3*(d^2*x^2 - c^2)^(3/2
)*b/d^2
```

mupad [B] time = 3.60, size = 248, normalized size = 3.10

$$a\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right) - a\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right) - \frac{b\left(c^2-d^2\,x^2\right)\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{3\,d^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x,x)
```

```
[Out] a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((
-c)^(1/2) - (d*x - c)^(1/2))) - (b*(c^2 - d^2*x^2)*(c + d*x)^(1/2)*(d*x - c
)^(1/2))/(3*d^2) - (8*a*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(
((-c)^(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*
x - c)^(1/2))^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)\sqrt{-c + dx}\,\sqrt{c + dx}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

3.341 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c}-\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2x^2}+b\sqrt{dx-c}\,\sqrt{c+dx}$$

```
[Out] -1/2*(-a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/
2)*(d*x+c)^(1/2)-1/2*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 101, 12, 92, 205}

$$\frac{1}{2}\sqrt{dx-c}\,\sqrt{c+dx}\left(2b-\frac{ad^2}{c^2}\right)-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((2*b - (a*d^2)/c^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2)*
(c + d*x)^(3/2))/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sq
rt[c + d*x])/c])/(2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
```

```
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^3}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$

$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(-2b+\frac{a}{c}\right.$$

$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(-2bc^2+\right.$$

$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{1}{2}\left(d\left(2bc^2-\right.\right.$$

$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{\left(2bc^2-ad^2\right.}{}$$

Mathematica [A] time = 0.06, size = 114, normalized size = 1.19

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(c\left(a-2bx^2\right)\sqrt{d^2x^2-c^2}+x^2\left(2bc^2-ad^2\right)\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)\right)}{2cx^2\sqrt{d^2x^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] -1/2*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(c*(a - 2*b*x^2)*Sqrt[-c^2 + d^2*x^2] +
(2*b*c^2 - a*d^2)*x^2*ArcTan[Sqrt[-c^2 + d^2*x^2]/c]))/(c*x^2*Sqrt[-c^2 + d
^2*x^2])
```

fricas [A] time = 1.07, size = 85, normalized size = 0.89

$$-\frac{2\left(2\,bc^2 - ad^2\right)x^2 \arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right) - \left(2\,bcx^2 - ac\right)\sqrt{dx+c}\,\sqrt{dx-c}}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b*c^2 - a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c
) - (2*b*c*x^2 - a*c)*sqrt(d*x + c)*sqrt(d*x - c))/(c*x^2)
```

giac [A] time = 0.42, size = 157, normalized size = 1.64

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\,bd + \frac{\left(2\,bc^2d-ad^3\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c} + \frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2*(sqrt(d*x
 + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6
- 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/((sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 4*c^2)^2)/d
```

maple [B] time = 0.07, size = 182, normalized size = 1.90

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(a\,d^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right) - 2b\,c^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right) - 2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\,x^2}{2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/
2))/x)*x^2*a*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*b*c^
2-2*x^2*b*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1/2)
```

maxima [A] time = 1.46, size = 98, normalized size = 1.02

$$bc\arcsin\left(\frac{c}{d|x|}\right)-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c}+\sqrt{d^2x^2-c^2}\,b-\frac{\sqrt{d^2x^2-c^2}\,ad^2}{2\,c^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] b*c*arcsin(c/(d*abs(x))) - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2
- c^2)*b - 1/2*sqrt(d^2*x^2 - c^2)*a*d^2/c^2 + 1/2*(d^2*x^2 - c^2)^(3/2)*a/
(c^2*x^2)
```

mupad [B] time = 6.89, size = 584, normalized size = 6.08

$$b\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\frac{\frac{a\sqrt{-c}\,d^2}{32\,c^{3/2}}+\frac{a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}-b\sqrt{-c}\,\sqrt{c}\,\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^3,x)
```

```
[Out] b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - ((a*(-c)^(1/2)*d^2)/(32*c^(3/2)) + (a*(-c)^(1/2)*d^2*((
c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
- (15*a*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/
2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x
 - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6)
- b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x -
 c)^(1/2))) + (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2)
 - (d*x - c)^(1/2))))/(2*c^(3/2)) - (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
- c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) - (a*(-c)^(
1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)
^(1/2))^2) - (8*b*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(((-c)^
(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*
x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

3.342 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^5}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^2x^2}+\frac{d^2\left(ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

```
[Out] 1/4*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^4+1/8*d^2*(a*d^2+4*b*c^2)*arctan((d
*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3-1/8*(a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^
(1/2)/c^2/x^2
```

Rubi [A] time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 94, 92, 205}

$$-\frac{\sqrt{dx-c}\,(c+dx)^{3/2}\left(ad^2+4bc^2\right)}{8c^3x^2}+\frac{d\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^3x}+\frac{d^2\left(ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d*(4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^3*x) - ((4*b*c^2 +
a*d^2)*Sqrt[-c + d*x]*(c + d*x)^(3/2))/(8*c^3*x^2) + (a*(-c + d*x)^(3/2)*(c
 + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/(8*c^3)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)}{x^5}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x^3}\,dx$$

$$= -\frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{8}\left(d\left(4b+\right.\right.$$

$$= \frac{d\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a}{}$$

$$= \frac{d\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a}{}$$

$$= \frac{d\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a}{}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 1.13

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(\left(c^2-d^2x^2\right)\left(2ac^2-ad^2x^2+4bc^2x^2\right)-d^2x^4\sqrt{1-\frac{d^2x^2}{c^2}}\left(ad^2+4bc^2\right)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right)\right)}{8c^2d^2x^6-8c^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*((c^2 - d^2*x^2)*(2*a*c^2 + 4*b*c^2*x^2 - a*d
^2*x^2) - d^2*(4*b*c^2 + a*d^2)*x^4*Sqrt[1 - (d^2*x^2)/c^2]*ArcTanh[Sqrt[1
- (d^2*x^2)/c^2]]))/(-8*c^4*x^4 + 8*c^2*d^2*x^6)
```

fricas [A] time = 1.09, size = 100, normalized size = 0.83

$$\frac{2\left(4\,bc^{2}d^{2}+ad^{4}\right)x^{4}\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-\left(2\,ac^{3}+\left(4\,bc^{3}-acd^{2}\right)x^{2}\right)\sqrt{dx+c}\,\sqrt{dx-c}}{8\,c^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c)
)/c) - (2*a*c^3 + (4*b*c^3 - a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^
3*x^4)
```

giac [B] time = 0.45, size = 324, normalized size = 2.68

$$-\frac{\frac{\left(4\,bc^{2}d^{3}+ad^{5}\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}}{2\,c}\right)}{c^{3}}-\frac{2\left(4\,bc^{2}d^{3}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}-ad^{5}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^{4}d^{3}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+28\,ac^{2}d^{5}\left(\sqrt{dx-\cdots}\right.\right.}{\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 - a*d^5*(sqrt(d*x
+ c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^10
+ 28*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x
+ c) - sqrt(d*x - c))^6 - 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 -
 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt(d*x +
 c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^2)
)/d
```

maple [B] time = 0.07, size = 226, normalized size = 1.87

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(a\,d^{4}x^{4}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)+4b\,c^{2}d^{2}x^{4}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,a\cdots\right.}{8\sqrt{d^{2}x^{2}-c^{2}}\,\sqrt{-c^{2}}\,c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)
^(1/2))/x)*x^4*a*d^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*
b*c^2*d^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^2+4*(-c^2)^(1/2)*(d^2*x^
2-c^2)^(1/2)*x^2*b*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*c^2)/(d^2*x^2-c
^2)^(1/2)/x^4/(-c^2)^(1/2)
```

maxima [A] time = 1.35, size = 162, normalized size = 1.34

$$-\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2c}-\frac{ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8c^3}-\frac{\sqrt{d^2x^2-c^2}\,bd^2}{2c^2}-\frac{\sqrt{d^2x^2-c^2}\,ad^4}{8c^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}b}{2c^2x^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ad^2}{8c^4x^2}+\frac{\left(d^2x^2-\right.}{4c^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c - 1/8*a*d^4*arcsin(c/(d*abs(x)))/c^3 - 1/
2*sqrt(d^2*x^2 - c^2)*b*d^2/c^2 - 1/8*sqrt(d^2*x^2 - c^2)*a*d^4/c^4 + 1/2*(
d^2*x^2 - c^2)^(3/2)*b/(c^2*x^2) + 1/8*(d^2*x^2 - c^2)^(3/2)*a*d^2/(c^4*x^2
) + 1/4*(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^4)
```

mupad [B] time = 15.56, size = 1004, normalized size = 8.30

$$\frac{\frac{a\sqrt{-c}\,d^4}{1024\,c^{7/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{128\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{11\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{512\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{7\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{256\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}-\frac{239\,a\sqrt{-c}\,d^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{1024\,c^{7/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{a\sqrt{-c}\,d^4}{256\,c^{7/2}\left(\right.}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{6\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^5,x)
```

```
[Out] ((a*(-c)^(1/2)*d^4)/(1024*c^(7/2)) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c
^(1/2))^2)/(128*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (11*a*(-c)^(1/2
)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(7/2)*((-c)^(1/2) - (d*x - c)^(
1/2))^4) + (7*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/(256*c^(7/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^6) - (239*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2)
- c^(1/2))^8)/(1024*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8) + (a*(-c)^(1/
2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(7/2)*((-c)^(1/2) - (d*x - c)
^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*x - c)^(1/2))^
4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (6
*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 + (4*((c +
 d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + ((c + d*x)^(
1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) - ((b*(-c)^(1/2)*d^2)
```

```
/(32*c^(3/2)) + (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2
)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(1/2)*d^2*((c + d*x)^(1/2)
 - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/((((c + d*x)^(
1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) -
c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^
6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(7/2)) + (b*(-c)^(1/2)*d
^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(3
/2)) - (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d
*x - c)^(1/2))^2 + 1))/(8*c^(7/2)) - (b*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
 - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) + (a*(-c)^
(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*c^(7/2)*((-c)^(1/2) - (d*x -
c)^(1/2))^2) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(1024*c^(7/
2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) - (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) -
 c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.343 $\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx$

Optimal. Leaf size=208

$$\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}-\frac{c^6\left(8ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}$$

```
[Out] 1/64*c^2*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^6+1/48*(8*a*d^2+
5*b*c^2)*x^3*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/8*b*x^5*(d*x-c)^(3/2)*(d*x+c
)^(3/2)/d^2-1/64*c^6*(8*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))
/d^7+1/128*c^4*(8*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

Rubi [A] time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {460, 100, 12, 90, 38, 63, 217, 206}

$$\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}+\frac{c^4x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(8ad^2+5bc^2\right)}{128d^6}+\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6) + (c^2*(
5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(64*d^6) + ((5*b*c^2
 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(48*d^4) + (b*x^5*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{1}{8}\left(-8a-\frac{5bc^2}{d^2}\right)\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{(5}{}$$

$$= \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{(c}{}$$

$$= \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(}{48d^4}$$

$$= \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(}{48d^4}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c}{64d^6}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c}{64d^6}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c}{64d^6}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c}{64d^6}$$

Mathematica [A] time = 0.31, size = 161, normalized size = 0.77

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(3\left(8ac^5d^2+5bc^7\right)\sin^{-1}\left(\frac{dx}{c}\right)-dx\sqrt{1-\frac{d^2x^2}{c^2}}\left(8ad^2\left(3c^4+2c^2d^2x^2-8d^4x^4\right)+b\left(15c^6+10c^4d^2x\right.\right.\right.}{384d^7\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(d*x*Sqrt[1 - (d^2*x^2)/c^2]*(8*a*d^2*(3*c^
4 + 2*c^2*d^2*x^2 - 8*d^4*x^4) + b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4
 - 48*d^6*x^6))) + 3*(5*b*c^7 + 8*a*c^5*d^2)*ArcSin[(d*x)/c]))/(384*d^7*Sqr
t[1 - (d^2*x^2)/c^2])
```

fricas [A] time = 1.32, size = 138, normalized size = 0.66

$$\frac{\left(48\,bd^7x^7 - 8\left(bc^2d^5 - 8\,ad^7\right)x^5 - 2\left(5\,bc^4d^3 + 8\,ac^2d^5\right)x^3 - 3\left(5\,bc^6d + 8\,ac^4d^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c} + 3\left(5\,bc^8 + \right.}{384\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*((48*b*d^7*x^7 - 8*(b*c^2*d^5 - 8*a*d^7)*x^5 - 2*(5*b*c^4*d^3 + 8*a*c
^2*d^5)*x^3 - 3*(5*b*c^6*d + 8*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) +
3*(5*b*c^8 + 8*a*c^6*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

giac [B] time = 0.63, size = 558, normalized size = 2.68

$$112\left(\left(\left(\left(2\,(dx+c)\left(3\,(dx+c)\left(\frac{4\,(dx+c)}{d^4} - \frac{21\,c}{d^4}\right) + \frac{133\,c^2}{d^4}\right) - \frac{295\,c^3}{d^4}\right)(dx+c) + \frac{195\,c^4}{d^4}\right)\sqrt{dx+c}\,\sqrt{dx-c} + \frac{90\,c^5\log\left(\left|-\sqrt{dx+c}\right.\right.}{d^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13440*(112*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133
*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x -
c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*a*c + 8*(((2*((4*
(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6) + 661*c^2/d^6) - 4551*c
^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335*c^5/d^6)*(d*x + c) + 283
5*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*log(abs(-sqrt(d*x + c) +
sqrt(d*x - c)))/d^6)*b*c + 56*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5
 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x +
c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x +
 c) + sqrt(d*x - c)))/d^5)*a*d + (((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x
 + c)/d^7 - 57*c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^
4/d^7)*(d*x + c) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23
205*c^7/d^7)*sqrt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c)
+ sqrt(d*x - c)))/d^7)*b*d)/d
```

maple [C] time = 0.10, size = 298, normalized size = 1.43

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(48\sqrt{d^2x^2-c^2}\,b\,d^7x^7\mathrm{csgn}(d) + 64\sqrt{d^2x^2-c^2}\,a\,d^7x^5\mathrm{csgn}(d) - 8\sqrt{d^2x^2-c^2}\,b\,c^2d^5x^5\mathrm{csgn}(d)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/384*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(48*csgn(d)*x^7*b*d^7*(d^2*x^2-c^2)^(1/2)
+64*csgn(d)*x^5*a*d^7*(d^2*x^2-c^2)^(1/2)-8*csgn(d)*x^5*b*c^2*d^5*(d^2*x^2-
c^2)^(1/2)-16*csgn(d)*x^3*a*c^2*d^5*(d^2*x^2-c^2)^(1/2)-10*csgn(d)*x^3*b*c^
4*d^3*(d^2*x^2-c^2)^(1/2)-24*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^4-15*csg
n(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^6-24*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*
csgn(d))*a*c^6*d^2-15*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*b*c^8)*
csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7
```

maxima [A] time = 0.55, size = 246, normalized size = 1.18

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^3}{48\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^3}{6\,d^2}-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{128\,d^7}-\frac{ac^6\log\left(2\,d^2x+2\,\cdots\right.}{16\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^3/
d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8*log(2*d^2*x + 2*sqr
t(d^2*x^2 - c^2)*d)/d^7 - 1/16*a*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)
/d^5 + 5/128*sqrt(d^2*x^2 - c^2)*b*c^6*x/d^6 + 1/16*sqrt(d^2*x^2 - c^2)*a*c
^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/2)*b*c^4*x/d^6 + 1/8*(d^2*x^2 - c^2)^(3/
2)*a*c^2*x/d^4
```

mupad [B] time = 39.15, size = 2314, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (a*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^5) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x
 - c)^(1/2))^7) + (41929*a*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2
) - (d*x - c)^(1/2))^9) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2)
)^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*a*c^6*((c + d*x)^(1/2)
 - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*a*c^6*((c + d
*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*a*c^6
*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (3
5*a*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^
21) - (a*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/
2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x -
```

c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24) - ((5*b*c^8*((c + d*x)^(1/2) - c^(1/2)))/(32*((-c)^(1/2) - (d*x - c)^(1/2))) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^3)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^5)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^7)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (848801*b*c^8*((c + d*x)^(1/2) - c^(1/2))^9)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (4181067*b*c^8*((c + d*x)^(1/2) - c^(1/2))^11)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^13)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^15)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^17)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^19)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (4181067*b*c^8*((c + d*x)^(1/2) - c^(1/2))^21)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^21) + (848801*b*c^8*((c + d*x)^(1/2) - c^(1/2))^23)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^23) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^25)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^25) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^27)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^27) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^29)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^29) + (5*b*c^8*((c + d*x)^(1/2) - c^(1/2))^31)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^31))/(d^7 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (120*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (560*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (1820*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (4368*d^7*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (8008*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (11440*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (12870*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (11440*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (8008*d^7*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (4368*d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (1820*d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24 - (560*d^7*((c + d*x)^(1/2) - c^(1/2))^26)/((-c)^(1/2) - (d*x - c)^(1/2))^26 + (120*d^7*((c + d*x)^(1/2) - c^(1/2))^28)/((-c)^(1/2) - (d*x - c)^(1/2))^28 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^30)/((-c)^(1/2) - (d*x - c)^(1/2))^

```
30 + (d^7*((c + d*x)^(1/2) - c^(1/2))^32)/((-c)^(1/2) - (d*x - c)^(1/2))^32
) + (a*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))
))/(4*d^5) + (5*b*c^8*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
- c)^(1/2))))/(32*d^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^2\right) \sqrt{-c + dx} \sqrt{c + dx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.344 $\int x^2\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx$

Optimal. Leaf size=159

$$\frac{c^2x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(d\ldots}{}$$

```
[Out] 1/8*(2*a*d^2+b*c^2)*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^4+1/6*b*x^3*(d*x-c)^(3/
2)*(d*x+c)^(3/2)/d^2-1/8*c^4*(2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^
(1/2))/d^5+1/16*c^2*(2*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A] time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 90, 12, 38, 63, 217, 206}

$$\frac{c^2x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(d\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) + ((b*c^2 +
 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) + (b*x^3*(-c + d*x)^(
3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(8*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx = \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2} + \frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\int x^2\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4} + \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2} + \frac{(b$$

$$= \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4} + \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2} + \frac{(c$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+d}{8d^4}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+d}{8d^4}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+d}{8d^4}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4} + \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+d}{8d^4}$$

Mathematica [A] time = 0.19, size = 135, normalized size = 0.85

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(3\left(2ac^3d^2+bc^5\right)\sin^{-1}\left(\frac{dx}{c}\right)+dx\sqrt{1-\frac{d^2x^2}{c^2}}\,\left(b\left(-3c^4-2c^2d^2x^2+8d^4x^4\right)-6ad^2\left(c^2-2d^2x^2\right)\right)\right.}{48d^5\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(d*x*Sqrt[1 - (d^2*x^2)/c^2]*(-6*a*d^2*(c^2 -
 2*d^2*x^2) + b*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4*x^4)) + 3*(b*c^5 + 2*a*c^3*
d^2)*ArcSin[(d*x)/c]))/(48*d^5*Sqrt[1 - (d^2*x^2)/c^2])
```

fricas [A] time = 1.50, size = 112, normalized size = 0.70

$$\frac{\left(8\,bd^5x^5-2\left(bc^2d^3-6\,ad^5\right)x^3-3\left(bc^4d+2\,ac^2d^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}\,+3\left(bc^6+2\,ac^4d^2\right)\log\left(-dx+\sqrt{dx+c}\right.}{48\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 - 2*(b*c^2*d^3 - 6*a*d^5)*x^3 - 3*(b*c^4*d + 2*a*c^2*d^3
)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^6 + 2*a*c^4*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^5
```

giac [B] time = 0.46, size = 432, normalized size = 2.72

$$40\left(\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)ac+2\left(\left(\left(2\,(dx+c)\left(3\,(dx+c)\left(\frac{4\,(dx+\right.\right.\right.\right.\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*(40*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2 - 7*c/d^
2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)*a*c +
 2*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^4)
- 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^
5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*c + 10*(((d*x + c)*(2*(d*
x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3) - 39*c^3/d^3)*sqrt(d*x +
c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^3)*a*d
 + (((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 321*c^2/d^5)
 - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)*sqrt(d*x
+ c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^5)*
b*d)/d
```

maple [C] time = 0.07, size = 240, normalized size = 1.51

$$\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\sqrt{d^2x^2-c^2}\,b\,d^5x^5\mathrm{csgn}(d)+12\sqrt{d^2x^2-c^2}\,a\,d^5x^3\mathrm{csgn}(d)-2\sqrt{d^2x^2-c^2}\,b\,c^2d^3x^3\mathrm{csgn}(d)-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*x^5*b*d^5*(d^2*x^2-c^2)^(1/2)+1
2*csgn(d)*x^3*a*d^5*(d^2*x^2-c^2)^(1/2)-2*csgn(d)*x^3*b*c^2*d^3*(d^2*x^2-c^
2)^(1/2)-6*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^2-3*csgn(d)*d*(d^2*x^2-c^2
)^(1/2)*x*b*c^4-6*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*a*c^4*d^2-3
*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*b*c^6)*csgn(d)/(d^2*x^2-c^2)
^(1/2)/d^5
```

maxima [A] time = 0.67, size = 192, normalized size = 1.21

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^3}{6\,d^2}-\frac{bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^5}-\frac{ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^3}+\frac{\sqrt{d^2x^2-c^2}\,bc^4x}{16\,d^4}+\frac{\sqrt{d^2x^2-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^5 - 1/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 1
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^4 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^2 +
 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/d^4 + 1/4*(d^2*x^2 - c^2)^(3/2)*a*x/d^2
```

mupad [B] time = 42.57, size = 1681, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^5) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x
 - c)^(1/2))^7) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2
) - (d*x - c)^(1/2))^9) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2)
)^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*b*c^6*((c + d*x)^(1/2)
 - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*b*c^6*((c + d
*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*b*c^6
*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (3
5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^
21) - (b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/
2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x -
c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x -
c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x -
c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x -
 c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x
 - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d
*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) -
(d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2)
- (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2)
 - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2
) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2)
- (d*x - c)^(1/2))^24) - ((a*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2
) - (d*x - c)^(1/2))) + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(
1/2) - (d*x - c)^(1/2))^3) + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^7
```

```
)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*a*c^4*((c + d*x)^(1/2) - c^(1
/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*a*c^4*((c + d*x)^(1/2)
- c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35*a*c^4*((c + d*x)
^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (a*c^4*((c +
d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^3 - (8*
d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d
^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^
3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^3
*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^3*
((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^3
*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^3
*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^3*(
(c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) + (a*c^4*
atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
+ (b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))
/(4*d^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right) \sqrt{-c + dx}\, \sqrt{c + dx}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.345 $\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=114

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

```
[Out] 1/4*b*x*(d*x-c)^(3/2)*(d*x+c)^(3/2)/d^2-1/4*c^2*(4*a*d^2+b*c^2)*arctanh((d*
x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/8*(4*a*d^2+b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(
1/2)/d^2
```

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {389, 38, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(4*d^3)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(-bc^2-4ad^2\right)\int \sqrt{-c+dx}\,\sqrt{c+dx}\,dx}{4d^2}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} + \frac{\left(c^2\left(-bc^2-\right.\right.}{}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(c^2\left(bc^2+\right.\right.}{}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(c^2\left(bc^2+\right.\right.}{}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2\left(bc^2+4\right.}{}$$

Mathematica [A] time = 0.28, size = 129, normalized size = 1.13

$$\frac{dx\left(c^2-d^2x^2\right)\left(b\left(c^2-2d^2x^2\right)-4ad^2\right)-2c^{5/2}\sqrt{dx-c}\,\sqrt{\frac{dx}{c}+1}\,\left(4ad^2+bc^2\right)\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\,\sqrt{c}}\right)}{8d^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*(c^2 - d^2*x^2)*(-4*a*d^2 + b*(c^2 - 2*d^2*x^2)) - 2*c^(5/2)*(b*c^2 +
4*a*d^2)*Sqrt[-c + d*x]*Sqrt[1 + (d*x)/c]*ArcSinh[Sqrt[-c + d*x]/(Sqrt[2]*S
qrt[c])])/(8*d^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.13, size = 88, normalized size = 0.77

$$\frac{\left(2\,bd^3x^3-\left(bc^2d-4\,ad^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}\,+\left(bc^4+4\,ac^2d^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\,\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 - (b*c^2*d - 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (b
*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^3
```

giac [B] time = 0.35, size = 288, normalized size = 2.53

$$24\left(2\,c\log\left(\left|-\sqrt{dx+c}\,+\sqrt{dx-c}\,\right|\right)+\sqrt{dx+c}\,\sqrt{dx-c}\,\right)ac+4\left(\sqrt{dx+c}\,\sqrt{dx-c}\,\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*(2*c*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) + sqrt(d*x + c)*sqrt
(d*x - c))*a*c + 4*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2
 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d
^2)*b*c + (((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^
3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c
) + sqrt(d*x - c)))/d^3)*b*d - 12*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x
- c))) - sqrt(d*x + c)*sqrt(d*x - c)*(d*x - 2*c))*a)/d
```

maple [C] time = 0.06, size = 182, normalized size = 1.60

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\sqrt{d^2x^2-c^2}\,b\,d^3x^3\mathrm{csgn}(d)-4a\,c^2d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-c}\right.}{8\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*x^3*b*d^3*(d^2*x^2-c^2)^(1/2)+4*
csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a-csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^2-4
```

```
*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*a*c^2*d^2-ln((d*x+(d^2*x^2-c
^2)^(1/2)*csgn(d))*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^3
```

maxima [A] time = 0.54, size = 137, normalized size = 1.20

$$-\frac{bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^3}-\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d}+\frac{1}{2}\,\sqrt{d^2x^2-c^2}\,ax+\frac{\sqrt{d^2x^2-c^2}\,bc^2x}{8\,d^2}+\frac{\left(d^2x^2-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 - 1/2*a*c^2*log(2*d^2
*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(d^
2*x^2 - c^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^(3/2)*b*x/d^2
```

mupad [B] time = 17.43, size = 734, normalized size = 6.44

$$\frac{a\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}-\frac{\frac{b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{35\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{273\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{715\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{715\,b\,c^4\left(\sqrt{c}\right.}{2\left(\sqrt{-c}-\right.}}{d^3-\frac{8\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^3\left(\right.}{\left(\sqrt{-c}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (a*x*(c + d*x)^(1/2)*(d*x - c)^(1/2))/2 - ((b*c^4*((c + d*x)^(1/2) - c^(1/2
)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (35*b*c^4*((c + d*x)^(1/2) - c^(1/
2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*b*c^4*((c + d*x)^(1/2) -
 c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*b*c^4*((c + d*x)^(
1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*b*c^4*((c +
d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*b*c^4*
((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13
) + (b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2)
)^15))/(d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(
1/2))^6 + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1
/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1
/2))^10 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(
1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(
1/2))^14 + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/
```

```
2))^16) - (a*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(1/2)))/(2*d) + (b*c^4
*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx^2)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.346 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{2}x\sqrt{dx-c}\,\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

```
[Out] a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x-(-2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/
(d*x+c)^(1/2))/d+1/2*(b-2*a*d^2/c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 38, 63, 217, 206}

$$\frac{1}{2}x\sqrt{dx-c}\,\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)}{x^2}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \left(b-\frac{2ad^2}{c^2}\right)\int \sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{1}{2}\left(-bc^2+\right.$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{\left(-bc^2+2\right.}{}$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{\left(-bc^2+2\right.}{}$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} - \frac{\left(bc^2-2ad\right.}{}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.97

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(cd\left(bx^2-2a\right)\sqrt{1-\frac{d^2x^2}{c^2}}+x\left(bc^2-2ad^2\right)\sin^{-1}\left(\frac{dx}{c}\right)\right)}{2cdx\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(c*d*(-2*a + b*x^2)*Sqrt[1 - (d^2*x^2)/c^2] +
 (b*c^2 - 2*a*d^2)*x*ArcSin[(d*x)/c]))/(2*c*d*x*Sqrt[1 - (d^2*x^2)/c^2])
```

fricas [A] time = 1.22, size = 83, normalized size = 0.80

$$-\frac{2\,ad^2x-\left(bc^2-2\,ad^2\right)x\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\,\right)-\left(bdx^2-2\,ad\right)\sqrt{dx+c}\,\sqrt{dx-c}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*x - (b*c^2 - 2*a*d^2)*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c
)) - (b*d*x^2 - 2*a*d)*sqrt(d*x + c)*sqrt(d*x - c))/(d*x)
```

giac [A] time = 0.40, size = 110, normalized size = 1.06

$$-\frac{\frac{32\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-2\left((dx+c)b-bc\right)\sqrt{dx+c}\,\sqrt{dx-c}\,-\left(bc^2-2\,ad^2\right)\log\left(\left(\sqrt{dx+c}\,-\sqrt{dx-c}\,\right)^4\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(32*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*x +
c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*log((sqrt(d*x +
 c) - sqrt(d*x - c))^4))/d
```

maple [C] time = 0.06, size = 153, normalized size = 1.47

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2a\,d^2x\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)-b\,c^2x\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d\right.\right.}{2\sqrt{d^2x^2-c^2}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(csgn(d)*x^2*b*d*(d^2*x^2-c^2)^(1/2)+2*ln((
d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*x*a*d^2-ln((d*x+(d^2*x^2-c^2)^(1/
2)*csgn(d))*csgn(d))*x*b*c^2-2*csgn(d)*d*(d^2*x^2-c^2)^(1/2)*a)*csgn(d)/(d^
2*x^2-c^2)^(1/2)/x/d
```

maxima [A] time = 1.50, size = 105, normalized size = 1.01

$$-\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d}+ad\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)+\frac{1}{2}\,\sqrt{d^2x^2-c^2}\,bx-\frac{\sqrt{d^2x^2-c^2}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + a*d*log(2*d^2*x + 2*s
qrt(d^2*x^2 - c^2)*d) + 1/2*sqrt(d^2*x^2 - c^2)*b*x - sqrt(d^2*x^2 - c^2)*a
/x
```

mupad [B] time = 3.49, size = 243, normalized size = 2.34

$$\frac{a\,d+\frac{5\,a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}}-4\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}\,-\sqrt{c}}{\sqrt{-c}\,-\sqrt{d\,x-c}}\right)+\frac{b\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}-\frac{b\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{\ }\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^2,x)
```

```
[Out] (a*d + (5*a*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*a*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) + (b*x*(c + d
*x)^(1/2)*(d*x - c)^(1/2))/2 - (b*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(
1/2)))/(2*d) + (a*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)
^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

3.347 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

```
[Out] 1/3*a*(d*x-c)^(3/2)*(d*x+c)^(3/2)/c^2/x^3+2*b*d*arctanh((d*x-c)^(1/2)/(d*x+
c)^(1/2))-b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {454, 97, 12, 63, 217, 206}

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)}{x^4}\,dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x^2}\,dx\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\int \frac{d^2}{\sqrt{-c+dx}\,\sqrt{c+d}}\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+\left(bd^2\right)\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c}}\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+(2bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+}}\right.\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+(2bd)\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+2bd\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right.
\end{aligned}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 1.25

$$-\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\,\left(a\left(c^2-d^2x^2\right)+3bc^2x^2\right)+3bcdx^3\sin^{-1}\left(\frac{dx}{c}\right)\right)}{3c^2x^3\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -1/3*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(Sqrt[1 - (d^2*x^2)/c^2]*(3*b*c^2*x^2 +
a*(c^2 - d^2*x^2)) + 3*b*c*d*x^3*ArcSin[(d*x)/c]))/(c^2*x^3*Sqrt[1 - (d^2*x
^2)/c^2])
```

fricas [A] time = 1.34, size = 100, normalized size = 1.19

$$-\frac{3\,bc^2dx^3\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)+\left(3\,bc^2d-ad^3\right)x^3+\left(ac^2+\left(3\,bc^2-ad^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*c^2*d*x^3*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) + (3*b*c^2*d -
a*d^3)*x^3 + (a*c^2 + (3*b*c^2 - a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(
c^2*x^3)
```

giac [B] time = 0.38, size = 171, normalized size = 2.04

$$-\frac{3\,bd^2\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)+\frac{16\left(3\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8-3\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d^2-16\cdots\right.}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d^2*log((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2*d^2*(sqr
t(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^8 +
 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^6*d^2 - 16*a*c^4*d
^4)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3)/d
```

maple [C] time = 0.07, size = 153, normalized size = 1.82

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3b\,c^2d\,x^3\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^2x^2-c^2}\,a\,d^2x^2\operatorname{csgn}(d)-3\sqrt{d^2x^2-c^2}\cdots\right.}{3\sqrt{d^2x^2-c^2}\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(3*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csg
n(d))*x^3*b*c^2*d+csgn(d)*x^2*a*d^2*(d^2*x^2-c^2)^(1/2)-3*csgn(d)*x^2*b*c^2
*(d^2*x^2-c^2)^(1/2)-csgn(d)*a*c^2*(d^2*x^2-c^2)^(1/2))*csgn(d)/(d^2*x^2-c^
2)^(1/2)/c^2/x^3
```

maxima [A] time = 1.47, size = 75, normalized size = 0.89

$$bd\log\left(2\,d^2x + 2\,\sqrt{d^2x^2-c^2}\,d\right) - \frac{\sqrt{d^2x^2-c^2}\,b}{x} + \frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] b*d*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d) - sqrt(d^2*x^2 - c^2)*b/x + 1/3*
(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^3)
```

mupad [B] time = 3.44, size = 236, normalized size = 2.81

$$\frac{b\,d+\frac{5\,b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}}-4\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)-\frac{\left(\frac{a\,\sqrt{c+d\,x}}{3}-\frac{a\,d^2\,x^2\,\sqrt{c+d\,x}}{3\,c^2}\right)\,\sqrt{d\,x-c}}{x^3}+\frac{b\,d\,\left(\sqrt{c+d\right.}{4\,\left(\sqrt{-c}-\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^4,x)
```

```
[Out] (b*d + (5*b*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*b*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) - (((a*(c + d
*x)^(1/2))/3 - (a*d^2*x^2*(c + d*x)^(1/2))/(3*c^2))*(d*x - c)^(1/2))/x^3 +
(b*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.348 $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=125

$$\frac{(6ac^2+5b)\cosh^{-1}(cx)}{16c^7}+\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,(6ac^2+5b)}{16c^6}+\frac{x^3\sqrt{cx-1}\,\sqrt{cx+1}\,(6ac^2+5b)}{24c^4}+\frac{bx^5\sqrt{cx-1}\,\sqrt{cx+1}}{6c^2}$$

```
[Out] 1/16*(6*a*c^2+5*b)*arccosh(c*x)/c^7+1/16*(6*a*c^2+5*b)*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^6+1/24*(6*a*c^2+5*b)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/6*b*
x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {460, 100, 12, 90, 52}

$$\frac{x^3\sqrt{cx-1}\,\sqrt{cx+1}\,(6ac^2+5b)}{24c^4}+\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,(6ac^2+5b)}{16c^6}+\frac{(6ac^2+5b)\cosh^{-1}(cx)}{16c^7}+\frac{bx^5\sqrt{cx-1}\,\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2
)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c*x])/(16*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2} - \frac{1}{6}\left(-6a-\frac{5b}{c^2}\right)\int \frac{x^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$

$$= \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2} + \frac{\left(5b+6ac^2\right)\int \frac{3x^2}{\sqrt{-1+cx}\,\cdots}}{24c^4}$$

$$= \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2} + \frac{\left(5b+6ac^2\right)\int \frac{x^2}{\sqrt{-1+cx}\,\cdots}}{8c^4}$$

$$= \frac{\left(5b+6ac^2\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6} + \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+c\cdots}}{6c^2\cdots}$$

$$= \frac{\left(5b+6ac^2\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6} + \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4} + \frac{bx^5\sqrt{-1+c\cdots}}{6c^2\cdots}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.94

$$\frac{3\sqrt{c^2x^2-1}\,\left(6ac^2+5b\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)+cx\left(c^2x^2-1\right)\left(6ac^2\left(2c^2x^2+3\right)+b\left(8c^4x^4+10c^2x^2+15\right)\right)}{48c^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*(-1 + c^2*x^2)*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^2*x^2 + 8*c^4*x
^4)) + 3*(5*b + 6*a*c^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2
]])/(48*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 1.25, size = 96, normalized size = 0.77

$$\frac{\left(8\,bc^5x^5+2\left(6\,ac^5+5\,bc^3\right)x^3+3\left(6\,ac^3+5\,bc\right)x\right)\sqrt{cx+1}\,\sqrt{cx-1}-3\left(6\,ac^2+5\,b\right)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{48\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*c^5*x^5 + 2*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*sqr
t(c*x + 1)*sqrt(c*x - 1) - 3*(6*a*c^2 + 5*b)*log(-c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)))/c^7
```

giac [A] time = 0.23, size = 172, normalized size = 1.38

$$\frac{\left(\left(2\left((cx+1)\left(4\,(cx+1)\left(\frac{(cx+1)b}{c^6}-\frac{5\,b}{c^6}\right)+\frac{3\left(2\,ac^{38}+15\,bc^{36}\right)}{c^{42}}\right)-\frac{18\,ac^{38}+55\,bc^{36}}{c^{42}}\right)(cx+1)+\frac{54\,ac^{38}+85\,bc^{36}}{c^{42}}\right)(cx+1)-\frac{3\left(10\,ac^{38}\right.}{c}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((c*x + 1)*(4*(c*x + 1)*((c*x + 1)*b/c^6 - 5*b/c^6) + 3*(2*a*c^38
 + 15*b*c^36)/c^42) - (18*a*c^38 + 55*b*c^36)/c^42)*(c*x + 1) + (54*a*c^38
+ 85*b*c^36)/c^42)*(c*x + 1) - 3*(10*a*c^38 + 11*b*c^36)/c^42)*sqrt(c*x + 1
)*sqrt(c*x - 1) - 6*(6*a*c^2 + 5*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^6)
/c
```

maple [C] time = 0.11, size = 191, normalized size = 1.53

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8\sqrt{c^2x^2-1}\,b\,c^5x^5\mathrm{csgn}(c)+12\sqrt{c^2x^2-1}\,a\,c^5x^3\mathrm{csgn}(c)+10\sqrt{c^2x^2-1}\,b\,c^3x^3\mathrm{csgn}(c)+18\sqrt{c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/48*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(8*csgn(c)*x^5*b*c^5*(c^2*x^2-1)^(1/2)+12*
csgn(c)*x^3*a*c^5*(c^2*x^2-1)^(1/2)+10*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x^3*b+
18*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x*a+15*(c^2*x^2-1)^(1/2)*csgn(c)*c*x*b+18*
ln(((c^2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+15*ln(((c^2*x^2-1)^(1/2)*
csgn(c)+c*x)*csgn(c))*b)*csgn(c)/c^7/(c^2*x^2-1)^(1/2)
```

maxima [A] time = 0.55, size = 153, normalized size = 1.22

$$\frac{\sqrt{c^2x^2-1}\,bx^5}{6\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax^3}{4\,c^2}+\frac{5\,\sqrt{c^2x^2-1}\,bx^3}{24\,c^4}+\frac{3\,\sqrt{c^2x^2-1}\,ax}{8\,c^4}+\frac{3\,a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{8\,c^5}+\frac{5\,\sqrt{c^2x^2-1}\,b}{16\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*sq
rt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5 + 5/16*sqrt(c^2*x^2 - 1)*b*x/c^6 + 5/16*b*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7
```

mupad [B] time = 32.63, size = 1154, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*a*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*a*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*a*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*a*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*a*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*a*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*a*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*a*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((311*b*((c*x - 1)^(1/2) - 1i)^5)/(4*((c*x + 1)^(1/2) - 1)^
5) - (175*b*((c*x - 1)^(1/2) - 1i)^3)/(12*((c*x + 1)^(1/2) - 1)^3) + (8361*
b*((c*x - 1)^(1/2) - 1i)^7)/(4*((c*x + 1)^(1/2) - 1)^7) + (42259*b*((c*x -
1)^(1/2) - 1i)^9)/(6*((c*x + 1)^(1/2) - 1)^9) + (25295*b*((c*x - 1)^(1/2) -
 1i)^11)/(2*((c*x + 1)^(1/2) - 1)^11) + (25295*b*((c*x - 1)^(1/2) - 1i)^13)
```

```
/(2*((c*x + 1)^(1/2) - 1)^13) + (42259*b*((c*x - 1)^(1/2) - 1i)^15)/(6*((c*
x + 1)^(1/2) - 1)^15) + (8361*b*((c*x - 1)^(1/2) - 1i)^17)/(4*((c*x + 1)^(1
/2) - 1)^17) + (311*b*((c*x - 1)^(1/2) - 1i)^19)/(4*((c*x + 1)^(1/2) - 1)^1
9) - (175*b*((c*x - 1)^(1/2) - 1i)^21)/(12*((c*x + 1)^(1/2) - 1)^21) + (5*b
*((c*x - 1)^(1/2) - 1i)^23)/(4*((c*x + 1)^(1/2) - 1)^23) + (5*b*((c*x - 1)^
(1/2) - 1i))/(4*((c*x + 1)^(1/2) - 1)))/(c^7 - (12*c^7*((c*x - 1)^(1/2) - 1
i)^2)/((c*x + 1)^(1/2) - 1)^2 + (66*c^7*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1
)^(1/2) - 1)^4 - (220*c^7*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6
 + (495*c^7*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (792*c^7*((
c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (924*c^7*((c*x - 1)^(1/
2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (792*c^7*((c*x - 1)^(1/2) - 1i)^14)
/((c*x + 1)^(1/2) - 1)^14 + (495*c^7*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^
(1/2) - 1)^16 - (220*c^7*((c*x - 1)^(1/2) - 1i)^18)/((c*x + 1)^(1/2) - 1)^1
8 + (66*c^7*((c*x - 1)^(1/2) - 1i)^20)/((c*x + 1)^(1/2) - 1)^20 - (12*c^7*(
(c*x - 1)^(1/2) - 1i)^22)/((c*x + 1)^(1/2) - 1)^22 + (c^7*((c*x - 1)^(1/2)
- 1i)^24)/((c*x + 1)^(1/2) - 1)^24) + (3*a*atanh(((c*x - 1)^(1/2) - 1i)/((c
*x + 1)^(1/2) - 1)))/(2*c^5) + (5*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)
^(1/2) - 1)))/(4*c^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.349 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{bx^4\sqrt{cx-1}\,\sqrt{cx+1}}{5c^2}$$

```
[Out] 2/15*(5*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6+1/15*(5*a*c^2+4*b)*x^2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/5*b*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{bx^4\sqrt{cx-1}\,\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b + 5*a*c^2
)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}-\frac{1}{5}\left(-5a-\frac{4b}{c^2}\right)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}+\frac{\left(4b+5ac^2\right)\int \frac{2x}{\sqrt{-1+cx}\sqrt{1}}}{15c^4}\\ &= \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}+\frac{\left(2\left(4b+5ac^2\right)\right)\int \frac{}{\sqrt{-1+c}}}{15c^4}\\ &= \frac{2\left(4b+5ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6}+\frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}}{5c^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.68

$$\frac{\left(c^2x^2-1\right)\left(5ac^2\left(c^2x^2+2\right)+b\left(3c^4x^4+4c^2x^2+8\right)\right)}{15c^6\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c^2*x^2 + 3*c^4*x^4)))/(1
5*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 1.13, size = 55, normalized size = 0.53

$$\frac{\left(3\,bc^4x^4+10\,ac^2+\left(5\,ac^4+4\,bc^2\right)x^2+8\,b\right)\sqrt{cx+1}\,\sqrt{cx-1}}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*c^4*x^4 + 10*a*c^2 + (5*a*c^4 + 4*b*c^2)*x^2 + 8*b)*sqrt(c*x + 1)
*sqrt(c*x - 1)/c^6
```

giac [A] time = 0.22, size = 108, normalized size = 1.05

$$\frac{\left(\left((cx+1)\left(3\,(cx+1)\left(\frac{(cx+1)b}{c^5}-\frac{4\,b}{c^5}\right)+\frac{5\,ac^{27}+22\,bc^{25}}{c^{30}}\right)-\frac{10\left(ac^{27}+2\,bc^{25}\right)}{c^{30}}\right)(cx+1)+\frac{15\left(ac^{27}+bc^{25}\right)}{c^{30}}\right)\sqrt{cx+1}\,\sqrt{cx-1}}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((c*x + 1)*(3*(c*x + 1)*((c*x + 1)*b/c^5 - 4*b/c^5) + (5*a*c^27 + 22*
b*c^25)/c^30) - 10*(a*c^27 + 2*b*c^25)/c^30)*(c*x + 1) + 15*(a*c^27 + b*c^2
5)/c^30)*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

maple [A] time = 0.05, size = 57, normalized size = 0.55

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10a\,c^2+8b\right)}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/15*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*
c^2+8*b)/c^6
```

maxima [A] time = 0.63, size = 95, normalized size = 0.92

$$\frac{\sqrt{c^2x^2-1}\,bx^4}{5\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax^2}{3\,c^2}+\frac{4\,\sqrt{c^2x^2-1}\,bx^2}{15\,c^4}+\frac{2\,\sqrt{c^2x^2-1}\,a}{3\,c^4}+\frac{8\,\sqrt{c^2x^2-1}\,b}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2 + 4/15*sq
rt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4 + 8/15*sqrt(c^2*x^2
 - 1)*b/c^6
```

mupad [B] time = 2.44, size = 108, normalized size = 1.05

$$\frac{\sqrt{c\,x-1}\,\left(\frac{10\,a\,c^2+8\,b}{15\,c^6}+\frac{b\,x^5}{5\,c}+\frac{b\,x^4}{5\,c^2}+\frac{x^2\,\left(5\,a\,c^4+4\,b\,c^2\right)}{15\,c^6}+\frac{x^3\,\left(5\,a\,c^5+4\,b\,c^3\right)}{15\,c^6}+\frac{x\,\left(10\,a\,c^3+8\,b\,c\right)}{15\,c^6}\right)}{\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((8*b + 10*a*c^2)/(15*c^6) + (b*x^5)/(5*c) + (b*x^4)/(5*c^
2) + (x^2*(5*a*c^4 + 4*b*c^2))/(15*c^6) + (x^3*(5*a*c^5 + 4*b*c^3))/(15*c^6
) + (x*(8*b*c + 10*a*c^3))/(15*c^6)))/(c*x + 1)^(1/2)
```

sympy [C] time = 62.84, size = 216, normalized size = 2.10

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} & \\ -\frac{5}{2},- \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijerg(((-2, -7/4, -3/2,
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(
c**2*x**2))/(4*pi**(3/2)*c**4) + b*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)
), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**6)
 + I*b*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -
5/2, -5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

3.350 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=87

$$\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

```
[Out] 1/8*(4*a*c^2+3*b)*arccosh(c*x)/c^5+1/8*(4*a*c^2+3*b)*x*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^4+1/4*b*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {460, 90, 52}

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh[c*x])/(8*c^5)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
```

```
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2} - \frac{1}{4}\left(-4a-\frac{3b}{c^2}\right)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= \frac{\left(3b+4ac^2\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2} + \frac{\left(3b+4ac^2\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+}}}{8c^4}$$
$$= \frac{\left(3b+4ac^2\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^2} + \frac{\left(3b+4ac^2\right)\cosh^{-1}(cx)}{8c^5}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 1.13

$$\frac{cx\left(c^2x^2-1\right)\left(4ac^2+b\left(2c^2x^2+3\right)\right)+\sqrt{c^2x^2-1}\,\left(4ac^2+3b\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{8c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*(-1 + c^2*x^2)*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (3*b + 4*a*c^2)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(8*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

fricas [A] time = 1.01, size = 77, normalized size = 0.89

$$\frac{\left(2\,bc^3x^3+\left(4\,ac^3+3\,bc\right)x\right)\sqrt{cx+1}\,\sqrt{cx-1}-\left(4\,ac^2+3\,b\right)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*c^3*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - (4*a
*c^2 + 3*b)*log(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/c^5
```

giac [A] time = 0.37, size = 121, normalized size = 1.39

$$\frac{\left((cx+1)\left(2\,(cx+1)\left(\frac{(cx+1)b}{c^4}-\frac{3\,b}{c^4}\right)+\frac{4\,ac^{18}+9\,bc^{16}}{c^{20}}\right)-\frac{4\,ac^{18}+5\,bc^{16}}{c^{20}}\right)\sqrt{cx+1}\,\sqrt{cx-1}-\frac{2\left(4\,ac^2+3\,b\right)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((c*x + 1)*(2*(c*x + 1)*((c*x + 1)*b/c^4 - 3*b/c^4) + (4*a*c^18 + 9*b*
c^16)/c^20) - (4*a*c^18 + 5*b*c^16)/c^20)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(
4*a*c^2 + 3*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^4)/c
```

maple [C] time = 0.08, size = 147, normalized size = 1.69

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\sqrt{c^2x^2-1}\,b\,c^3x^3\mathrm{csgn}(c)+4\sqrt{c^2x^2-1}\,a\,c^3x\,\mathrm{csgn}(c)+4a\,c^2\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)c\right.\right.}{8\sqrt{c^2x^2-1}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*(c^2*x^2-1)^(1/2)*b*c^3*x^3*csgn(c)+4*(c
^2*x^2-1)^(1/2)*a*c^3*x*csgn(c)+3*(c^2*x^2-1)^(1/2)*b*c*x*csgn(c)+4*a*c^2*l
n((c*x+(c^2*x^2-1)^(1/2)*csgn(c))*csgn(c))+3*b*ln((c*x+(c^2*x^2-1)^(1/2)*cs
gn(c))*csgn(c)))*csgn(c)/c^5/(c^2*x^2-1)^(1/2)
```

maxima [A] time = 0.54, size = 113, normalized size = 1.30

$$\frac{\sqrt{c^2x^2-1}\,bx^3}{4\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax}{2\,c^2}+\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{2\,c^3}+\frac{3\,\sqrt{c^2x^2-1}\,bx}{8\,c^4}+\frac{3\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 + 1/2*a*log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 3/8*sqrt(c^2*x^2 - 1)*b*x/c^4 + 3/8
*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5
```

mupad [B] time = 22.50, size = 720, normalized size = 8.28

$$\frac{\frac{23\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{2\left(\sqrt{c\,x+1}-1\right)^3}+\frac{333\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{2\left(\sqrt{c\,x+1}-1\right)^5}+\frac{671\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{2\left(\sqrt{c\,x+1}-1\right)^7}+\frac{671\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^9}{2\left(\sqrt{c\,x+1}-1\right)^9}+\frac{333\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{c\,x+1}-1\right)^{11}}+\frac{23\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{13}}{2\left(\sqrt{c\,x+1}-1\right)^{13}}-\frac{3\,b\left(\sqrt{c}\right.}{2\left(\sqrt{c}\right.}}{c^5-\frac{8\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{28\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{56\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{70\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}-\frac{56\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{c\,x+1}-1\right)^{10}}+\frac{28\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{c\,x+1}-1\right)^{12}}-\frac{8\,c^5\left(\right.}{\left(\sqrt{c}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*b*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*b*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*b*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*b*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*b*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*b*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*b*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*b*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((14*a*((c*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 +
(14*a*((c*x - 1)^(1/2) - 1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*a*((c*x - 1)^(
1/2) - 1i)^7)/((c*x + 1)^(1/2) - 1)^7 + (2*a*((c*x - 1)^(1/2) - 1i))/((c*x
+ 1)^(1/2) - 1))/(c^3 - (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) -
 1)^2 + (6*c^3*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*(
(c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) -
1i)^8)/((c*x + 1)^(1/2) - 1)^8) + (2*a*atanh(((c*x - 1)^(1/2) - 1i)/((c*x +
 1)^(1/2) - 1)))/c^3 + (3*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))/(2*c^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.351 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{3c^2}$$

```
[Out] 1/3*(3*a*c^2+2*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4+1/3*b*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {460, 74}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(3*c^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2} - \frac{1}{3}\left(-3a-\frac{2b}{c^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{(2b+3ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4} + \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.80

$$\frac{(c^2x^2-1)(3ac^2+b(c^2x^2+2))}{3c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
[Out] ((-1 + c^2*x^2)*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

fricas [A] time = 1.04, size = 37, normalized size = 0.57

$$\frac{(bc^2x^2+3\,ac^2+2\,b)\sqrt{cx+1}\,\sqrt{cx-1}}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b*c^2*x^2 + 3*a*c^2 + 2*b)*sqrt(c*x + 1)*sqrt(c*x - 1)/c^4
```

giac [A] time = 0.22, size = 59, normalized size = 0.91

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\left((cx+1)\left(\frac{(cx+1)b}{c^3}-\frac{2\,b}{c^3}\right)+\frac{3\,(ac^{11}+bc^9)}{c^{12}}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*((c*x + 1)*b/c^3 - 2*b/c^3) + 3*
(a*c^11 + b*c^9)/c^12)/c
```

maple [A] time = 0.04, size = 38, normalized size = 0.58

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(b\,c^{2}x^{2}+3a\,c^{2}+2b\right)}{3c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

maxima [A] time = 0.47, size = 54, normalized size = 0.83

$$\frac{\sqrt{c^{2}x^{2}-1}\,bx^{2}}{3\,c^{2}}+\frac{\sqrt{c^{2}x^{2}-1}\,a}{c^{2}}+\frac{2\,\sqrt{c^{2}x^{2}-1}\,b}{3\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*sqrt(c^2*x^
2 - 1)*b/c^4
```

mupad [B] time = 2.36, size = 66, normalized size = 1.02

$$\frac{\sqrt{c\,x-1}\,\left(\frac{3\,a\,c^{2}+2\,b}{3\,c^{4}}+\frac{b\,x^{3}}{3\,c}+\frac{b\,x^{2}}{3\,c^{2}}+\frac{x\left(3\,a\,c^{3}+2\,b\,c\right)}{3\,c^{4}}\right)}{\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((2*b + 3*a*c^2)/(3*c^4) + (b*x^3)/(3*c) + (b*x^2)/(3*c^2)
 + (x*(2*b*c + 3*a*c^3))/(3*c^4)))/(c*x + 1)^(1/2)
```

sympy [C] time = 41.91, size = 202, normalized size = 3.11

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|{\frac{1}{c^{2}x^{2}}}\right)}{4\pi^{\frac{3}{2}}c^{2}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|{\frac{e^{2i\pi}}{c^{2}x^{2}}}\right)}{4\pi^{\frac{3}{2}}c^{2}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4} \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/
2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*m
eijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2,
 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

3.352 $\int \frac{a+bx^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{2c^2}$$

```
[Out] 1/2*(2*a*c^2+b)*arccosh(c*x)/c^3+1/2*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {389, 52}

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/(
2*c^3)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^2} - \frac{\left(-b-2ac^2\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2c^2}$$
$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{2c^2} + \frac{\left(b+2ac^2\right)\cosh^{-1}(cx)}{2c^3}$$

Mathematica [B] time = 0.21, size = 101, normalized size = 2.15

$$\frac{4\left(ac^2+b\right)\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)+\frac{b\left(cx\sqrt{-(cx-1)^2}\,\sqrt{cx+1}-2\sqrt{cx-1}\,\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{\sqrt{1-cx}}}{2c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((b*(c*x*Sqrt[-(-1 + c*x)^2]*Sqrt[1 + c*x] - 2*Sqrt[-1 + c*x]*ArcSin[Sqrt[1
 - c*x]/Sqrt[2]]))/Sqrt[1 - c*x] + 4*(b + a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1
 + c*x)]])/(2*c^3)
```

fricas [A] time = 1.10, size = 55, normalized size = 1.17

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,bcx-\left(2\,ac^2+b\right)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - (2*a*c^2 + b)*log(-c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)))/c^3
```

giac [A] time = 0.18, size = 69, normalized size = 1.47

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\left(\frac{(cx+1)b}{c^2}-\frac{b}{c^2}\right)-\frac{2\left(2\,ac^2+b\right)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*b/c^2 - b/c^2) - 2*(2*a*c^2 + b
)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^2)/c
```

maple [C] time = 0.07, size = 103, normalized size = 2.19

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2a\,c^2\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\operatorname{csgn}(c)\right)\operatorname{csgn}(c)\right)+\sqrt{c^2x^2-1}\,bcx\operatorname{csgn}(c)+b\ln\left(\left(cx+\sqrt{c^2x^2-1}\right.\right.\right.}{2\sqrt{c^2x^2-1}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*((c^2*x^2-1)^(1/2)*b*c*x*csgn(c)+2*a*c^2*ln
((c*x+(c^2*x^2-1)^(1/2)*csgn(c))*csgn(c))+b*ln((c*x+(c^2*x^2-1)^(1/2)*csgn(
c))*csgn(c)))*csgn(c)/c^3/(c^2*x^2-1)^(1/2)
```

maxima [A] time = 0.50, size = 74, normalized size = 1.57

$$\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c}+\frac{\sqrt{c^2x^2-1}\,bx}{2\,c^2}+\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + 1/2*sqrt(c^2*x^2 - 1)*b*x/c^2 +
1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
```

mupad [B] time = 12.69, size = 293, normalized size = 6.23

$$-\frac{\frac{14\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{c\,x+1}-1\right)^3}+\frac{14\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{c\,x+1}-1\right)^5}+\frac{2\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{c\,x+1}-1\right)^7}+\frac{2\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\sqrt{c\,x+1}-1}}{c^3-\frac{4\,c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{6\,c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{4\,c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{c^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{c\left(\sqrt{c\,x-1}-\right.}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{}}\right.}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))/c^3 - ((14*b*((c*
x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 + (14*b*((c*x - 1)^(1/2) - 1i
)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*b*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)^(1
/2) - 1)^7 + (2*b*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1))/(c^3 - (4*
c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x - 1)^(
1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)^6)/((
c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1
```

```
)^8) - (4*a*atan((c*((c*x - 1)^(1/2) - 1i))/(((c*x + 1)^(1/2) - 1)*(-c^2)^(
1/2))))/(-c^2)^(1/2)
```

sympy **[C]** time = 45.51, size = 182, normalized size = 3.87

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(c**2*x**2))/(4*pi**(3/2)*c) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2)*c) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2,
-1/4, 0, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**3) - I*b*meijerg(((-3/2, -
5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(
2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**3)
```

3.353 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=46

$$a\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^2}$$

```
[Out] a*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {460, 92, 205}

$$a\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^2}+a\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^2}+(ac)\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\,\sqrt{1+cx}\right)\\ &= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^2}+a\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.43

$$\frac{ac^2\sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+b\left(c^2x^2-1\right)}{c^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*(-1 + c^2*x^2) + a*c^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(c
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.98, size = 48, normalized size = 1.04

$$\frac{2\,ac^2\arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\sqrt{cx+1}\,\sqrt{cx-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*c^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(c*x + 1)*sqrt(c*
x - 1)*b)/c^2
```

giac [A] time = 0.22, size = 45, normalized size = 0.98

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)*sqrt(c*x
 - 1)*b/c^2
```

maple [A] time = 0.07, size = 62, normalized size = 1.35

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-a\,c^{2}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)+\sqrt{c^{2}x^{2}-1}\,b\right)}{\sqrt{c^{2}x^{2}-1}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(-arctan(1/(c^2*x^2-1)^(1/2))*a*c^2+(c^2*x^2-1)
^(1/2)*b)/(c^2*x^2-1)^(1/2)/c^2
```

maxima [A] time = 1.15, size = 29, normalized size = 0.63

$$-a\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^{2}x^{2}-1}\,b}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(1/(c*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

mupad [B] time = 3.86, size = 77, normalized size = 1.67

$$\frac{b\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c^{2}}-a\,\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{2}}{\left(\sqrt{c\,x+1}-1\right)^{2}}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (b*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c^2 - a*(log(((c*x - 1)^(1/2) - 1i)^2/(
(c*x + 1)^(1/2) - 1)^2 + 1) - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))*1i
```

sympy [C] time = 40.14, size = 162, normalized size = 3.52

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2, -1/4
, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*
x**2))/(4*pi**(3/2)*c**2)
```

3.354 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=33

$$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

```
[Out] b*arccosh(c*x)/c+a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {454, 52}

$$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x} + b\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x} + \frac{b\cosh^{-1}(cx)}{c}$$

Mathematica [B] time = 0.03, size = 73, normalized size = 2.21

$$\frac{\sqrt{c^2x^2-1}\left(\frac{a\sqrt{c^2x^2-1}}{x}+\frac{b\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c^2*x^2]*((a*Sqrt[-1 + c^2*x^2])/x + (b*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/c))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 1.18, size = 56, normalized size = 1.70

$$\frac{ac^2x+\sqrt{cx+1}\,\sqrt{cx-1}\,ac-bx\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*c^2*x + sqrt(c*x + 1)*sqrt(c*x - 1)*a*c - b*x*log(-c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)))/(c*x)
```

giac [A] time = 0.22, size = 58, normalized size = 1.76

$$\frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4}-b\log\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*log((sqrt(c*x + 1
) - sqrt(c*x - 1))^4))/c
```

maple [C] time = 0.07, size = 77, normalized size = 2.33

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\sqrt{c^2x^2-1}\,ac\,\mathrm{csgn}(c)+bx\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)\right)\mathrm{csgn}(c)}{\sqrt{c^2x^2-1}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(csgn(c)*c*(c^2*x^2-1)^(1/2)*a+ln((c*x+(c^2*x^2
-1)^(1/2)*csgn(c))*csgn(c))*x*b)*csgn(c)/(c^2*x^2-1)^(1/2)/c/x
```

maxima [A] time = 1.18, size = 44, normalized size = 1.33

$$\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c}+\frac{\sqrt{c^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + sqrt(c^2*x^2 - 1)*a/x
```

mupad [B] time = 2.59, size = 61, normalized size = 1.85

$$\frac{a\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{x}-\frac{4\,b\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (a*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/x - (4*b*atan((c*((c*x - 1)^(1/2) - 1i)
)/(((c*x + 1)^(1/2) - 1)*(-c^2)^(1/2))))/(-c^2)^(1/2)
```

sympy [C] time = 35.17, size = 148, normalized size = 4.48

$$-\frac{acG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iacG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3
/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/
(4*pi**(3/2)*c)
```

3.355 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=60

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^2}$$

```
[Out] 1/2*(a*c^2+2*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*a*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {454, 92, 205}

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 +
c*x]*Sqrt[1 + c*x]])/2
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
```

```
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(c\left(2b+ac^2\right)\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\sqrt{1+cx}\right)\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\end{aligned}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 1.28

$$\frac{x^2\sqrt{c^2x^2-1}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+a\left(c^2x^2-1\right)}{2x^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*(-1 + c^2*x^2) + (2*b + a*c^2)*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c
^2*x^2]])/(2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 1.07, size = 57, normalized size = 0.95

$$\frac{2\left(ac^2+2\,b\right)x^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b)*x^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(
c*x + 1)*sqrt(c*x - 1)*a)/x^2
```

giac [B] time = 0.21, size = 114, normalized size = 1.90

$$-\frac{\left(ac^3+2\,bc\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4\,ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2*(a*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))
^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2)/c
```

maple [A] time = 0.08, size = 84, normalized size = 1.40

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a\,c^2x^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)+2b\,x^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)-\sqrt{c^2x^2-1}\,a\right)}{2\sqrt{c^2x^2-1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*x^2*a*c^2+2*a
rctan(1/(c^2*x^2-1)^(1/2))*x^2*b-(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^2
```

maxima [A] time = 1.23, size = 45, normalized size = 0.75

$$-\frac{1}{2}\,ac^2\arcsin\left(\frac{1}{c|x|}\right)-b\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*arcsin(1/(c*abs(x))) - b*arcsin(1/(c*abs(x))) + 1/2*sqrt(c^2*x^2
 - 1)*a/x^2
```

mupad [B] time = 8.67, size = 297, normalized size = 4.95

$$\frac{\frac{a\,c^2\,1\mathrm{i}}{32}+\frac{a\,c^2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\left(\sqrt{c\,x+1}-1\right)^2}-\frac{a\,c^2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\left(\sqrt{c\,x+1}-1\right)^4}}{\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{2\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}+\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}}-b\left(\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\right)1\mathrm{i}-\frac{a\,c^2\ln\left(\frac{(\sqrt{c\,x-1}}{(\sqrt{c\,x+1}}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((a*c^2*1i)/32 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (a*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
```

```
/(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - b*(log(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))*1i - (a*c^2*log(((c*x - 1)
^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^2*log(((c*x - 1)^(
1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1
i)/(32*((c*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 63.62, size = 141, normalized size = 2.35

$$-\frac{ac^2 G_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2} \\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iac^2 G_{6,6}^{2,6}\left(\begin{matrix} 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ib\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*
pi**(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/
2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1
, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2))
```

3.356 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{3x^3}$$

```
[Out] 1/3*a*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^3+1/3*(2*a*c^2+3*b)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/x
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {454, 95}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*x)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3} + \frac{1}{3}\left(3b+2ac^2\right)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3} + \frac{\left(3b+2ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{3x}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.82

$$\frac{\left(c^2x^2-1\right)\left(2ac^2x^2+a+3bx^2\right)}{3x^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

fricas [A] time = 0.70, size = 52, normalized size = 0.84

$$\frac{\left(2\,ac^3+3\,bc\right)x^3+\left(\left(2\,ac^2+3\,b\right)x^2+a\right)\sqrt{cx+1}\,\sqrt{cx-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*c^3 + 3*b*c)*x^3 + ((2*a*c^2 + 3*b)*x^2 + a)*sqrt(c*x + 1)*sqrt(c
*x - 1))/x^3
```

giac [B] time = 0.21, size = 116, normalized size = 1.87

$$\frac{8\left(3\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^8+24\,ac^4\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+24\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+32\,ac^4+48\,bc^2\right)}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c*x + 1) -
sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 32*a*c^4 +
48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^3*c)
```

maple [A] time = 0.05, size = 37, normalized size = 0.60

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(2a\,c^2x^2+3b\,x^2+a\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

maxima [A] time = 1.23, size = 54, normalized size = 0.87

$$\frac{2\sqrt{c^2x^2-1}\,ac^2}{3\,x}+\frac{\sqrt{c^2x^2-1}\,b}{x}+\frac{\sqrt{c^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(c^2*x^2 -
1)*a/x^3
```

mupad [B] time = 2.44, size = 53, normalized size = 0.85

$$\frac{\sqrt{c\,x-1}\,\left(\left(\frac{2\,a\,c^3}{3}+b\,c\right)\,x^3+\left(\frac{2\,a\,c^2}{3}+b\right)\,x^2+\frac{a\,c\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*(a/3 + x^3*(b*c + (2*a*c^3)/3) + x^2*(b + (2*a*c^2)/3) + (
a*c*x)/3))/(x^3*(c*x + 1)^(1/2))
```

sympy [C] time = 61.44, size = 146, normalized size = 2.35

$$-\frac{ac^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bcG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**3*meijerg((((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg((((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2)) - b*c*meijerg((((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg((((1/2, 3/4, 1,
5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x*
*2))/(4*pi**(3/2))
```

3.357 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

```
[Out] 1/8*c^2*(3*a*c^2+4*b)*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/4*a*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/x^4+1/8*(3*a*c^2+4*b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
+ c*x]])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4}+\frac{1}{4}\left(4b+3ac^2\right)\int \frac{1}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2}+\frac{1}{8}\left(4b+3ac^2\right)\int \frac{}{x\sqrt{-1+}}\\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2}+\frac{1}{8}\left(c^2\left(4b+3ac^2\right)\right)\int \frac{}{x\sqrt{-}}\\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2}+\frac{1}{8}\left(c^3\left(4b+3ac^2\right)\right)\mathrm{Subst}\Big(\\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2}+\frac{1}{8}c^2\left(4b+3ac^2\right)\tan^{-1}\Big(\sqrt{-}\end{aligned}$$

Mathematica [A] time = 0.10, size = 102, normalized size = 1.03

$$\frac{\left(c^2x^2-1\right)\left(a\left(3c^2x^2+2\right)+4bx^2\right)-c^2x^4\sqrt{1-c^2x^2}\,\left(3ac^2+4b\right)\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{8x^4\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(4*b*x^2 + a*(2 + 3*c^2*x^2)) - c^2*(4*b + 3*a*c^2)*x^4*Sqr
t[1 - c^2*x^2]*ArcTanh[Sqrt[1 - c^2*x^2]])/(8*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

fricas [A] time = 1.31, size = 78, normalized size = 0.79

$$\frac{2\left(3\,ac^{4}+4\,bc^{2}\right)x^{4}\arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left(\left(3\,ac^{2}+4\,b\right)x^{2}+2\,a\right)\sqrt{cx+1}\,\sqrt{cx-1}}{8\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a*c^4 + 4*b*c^2)*x^4*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) +
 ((3*a*c^2 + 4*b)*x^2 + 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1))/x^4
```

giac [B] time = 0.22, size = 268, normalized size = 2.71

$$-\frac{\left(3\,ac^{5}+4\,bc^{3}\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}\right)+\frac{2\left(3\,ac^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+4\,bc^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+44\,ac^{5}\left(\sqrt{cx+1}-\sqrt{cx-\right.}{}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2
*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*(sqrt(c*x + 1) - sqr
t(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 + 16*b*c^3*(sq
rt(c*x + 1) - sqrt(c*x - 1))^10 - 176*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))
^6 - 64*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x + 1)
- sqrt(c*x - 1))^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((sqrt(c*
x + 1) - sqrt(c*x - 1))^4 + 4)^4)/c
```

maple [A] time = 0.07, size = 125, normalized size = 1.26

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3a\,c^{4}x^{4}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)+4b\,c^{2}x^{4}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)-3\sqrt{c^{2}x^{2}-1}\,a\,c^{2}x^{2}-4\sqrt{c^{2}x^{2}-1}\,b\,x^{2}\right.}{8\sqrt{c^{2}x^{2}-1}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*arctan(1/(c^2*x^2-1)^(1/2))*x^4*a*c^4+4
*arctan(1/(c^2*x^2-1)^(1/2))*x^4*b*c^2-3*(c^2*x^2-1)^(1/2)*x^2*a*c^2-4*(c^2
*x^2-1)^(1/2)*x^2*b-2*(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^4
```

maxima [A] time = 1.11, size = 85, normalized size = 0.86

$$-\frac{3}{8}\,ac^4\arcsin\left(\frac{1}{c|x|}\right)-\frac{1}{2}\,bc^2\arcsin\left(\frac{1}{c|x|}\right)+\frac{3\,\sqrt{c^2x^2-1}\,ac^2}{8\,x^2}+\frac{\sqrt{c^2x^2-1}\,b}{2\,x^2}+\frac{\sqrt{c^2x^2-1}\,a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a*c^4*arcsin(1/(c*abs(x))) - 1/2*b*c^2*arcsin(1/(c*abs(x))) + 3/8*sqrt
(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*x^2 - 1)*b/x^2 + 1/4*sqrt(c^2*x^2 -
1)*a/x^4
```

mupad [B] time = 21.45, size = 650, normalized size = 6.57

$$\frac{\frac{b\,c^2\,1i}{32}+\frac{b\,c^2\,{\left(\sqrt{c\,x-1}-i\right)}^2\,1i}{16\,{\left(\sqrt{c\,x+1}-1\right)}^2}-\frac{b\,c^2\,{\left(\sqrt{c\,x-1}-i\right)}^4\,15i}{32\,{\left(\sqrt{c\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{c\,x-1}-i\right)}^2}{{\left(\sqrt{c\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{c\,x-1}-i\right)}^4}{{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{c\,x-1}-i\right)}^6}{{\left(\sqrt{c\,x+1}-1\right)}^6}}-\frac{\frac{a\,c^4\,1i}{1024}-\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-i\right)}^2\,3i}{128\,{\left(\sqrt{c\,x+1}-1\right)}^2}-\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-i\right)}^4\,53i}{512\,{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{a\,c^4\,{\left(\sqrt{c\,x-1}-i\right)}^6\,87i}{256\,{\left(\sqrt{c\,x+1}-1\right)}^6}+\cdots}{\frac{{\left(\sqrt{c\,x-1}-i\right)}^4}{{\left(\sqrt{c\,x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{c\,x-1}-i\right)}^6}{{\left(\sqrt{c\,x+1}-1\right)}^6}+\frac{6\,{\left(\sqrt{c\,x-1}-i\right)}^8}{{\left(\sqrt{c\,x+1}-1\right)}^8}+\frac{4\,{\left(\sqrt{c\,x-1}-i\right)}^{10}}{{\left(\sqrt{c\,x+1}-1\right)}^{10}}+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((b*c^2*1i)/32 + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (b*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - ((a*c^4*1i)/1024 - (a*c^4*((c*x - 1)^(1/2) - 1i)^2*3i)/(128*((c*x
 + 1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*53i)/(512*((c*x + 1)^
(1/2) - 1)^4) + (a*c^4*((c*x - 1)^(1/2) - 1i)^6*87i)/(256*((c*x + 1)^(1/2)
- 1)^6) + (a*c^4*((c*x - 1)^(1/2) - 1i)^8*657i)/(1024*((c*x + 1)^(1/2) - 1)
^8) + (a*c^4*((c*x - 1)^(1/2) - 1i)^10*121i)/(256*((c*x + 1)^(1/2) - 1)^10)
)/((((c*x - 1)^(1/2) - 1i)^4/((c*x + 1)^(1/2) - 1)^4 + (4*((c*x - 1)^(1/2) -
 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (6*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(
1/2) - 1)^8 + (4*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + ((c*
x - 1)^(1/2) - 1i)^12/((c*x + 1)^(1/2) - 1)^12) - (a*c^4*log(((c*x - 1)^(1/
2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*3i)/8 - (b*c^2*log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^4*log(((c*x - 1)^(1/2) -
 1i)/((c*x + 1)^(1/2) - 1))*3i)/8 + (b*c^2*log(((c*x - 1)^(1/2) - 1i)/((c*x
 + 1)^(1/2) - 1))*1i)/2 + (a*c^4*((c*x - 1)^(1/2) - 1i)^2*7i)/(256*((c*x +
```

```
1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*1i)/(1024*((c*x + 1)^(1/
2) - 1)^4) + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(32*((c*x + 1)^(1/2) - 1)^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] Timed out
```

3.358 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=164

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{}}{}$$

```
[Out] 1/8*c^4*(6*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7+1/16*c^2
*(6*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/24*(6*a*d^2+5*b*c^2)
*x^3*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/6*b*x^5*(d*x-c)^(1/2)*(d*x+c)^(1/2)/
d^2
```

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 100, 12, 90, 63, 217, 206}

$$\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6) + ((5*b*c
^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4) + (b*x^5*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c + d
*x]/Sqrt[c + d*x]])/(8*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^5\sqrt{-c+dx}\,\sqrt{c+dx}}{6d^2} - \frac{1}{6}\left(-6a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\,\sqrt{c+dx}}{6d^2} + \frac{\left(5bc^2+6ad^2\right)\int \frac{}{\sqrt{-c}}}{24d^4}$$

$$= \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\,\sqrt{c+dx}}{6d^2} + \frac{\left(c^2\left(5bc^2+6ad^2\right)\right)\int}{8d^4}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-}}{}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-}}{}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-}}{}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-}}{}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\,\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-}}{}$$

Mathematica [A] time = 0.12, size = 148, normalized size = 0.90

$$\frac{3c^4\sqrt{d^2x^2-c^2}\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)+dx\left(d^2x^2-c^2\right)\left(6ad^2\left(3c^2+2d^2x^2\right)+b\left(15c^4+10c^2d^2x^2+8d^4\right.\right.}{48d^7\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*(-c^2 + d^2*x^2)*(6*a*d^2*(3*c^2 + 2*d^2*x^2) + b*(15*c^4 + 10*c^2*d^2
*x^2 + 8*d^4*x^4)) + 3*c^4*(5*b*c^2 + 6*a*d^2)*Sqrt[-c^2 + d^2*x^2]*ArcTanh
[(d*x)/Sqrt[-c^2 + d^2*x^2]])/(48*d^7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.24, size = 115, normalized size = 0.70

$$\frac{\left(8\,bd^5x^5+2\left(5\,bc^2d^3+6\,ad^5\right)x^3+3\left(5\,bc^4d+6\,ac^2d^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}-3\left(5\,bc^6+6\,ac^4d^2\right)\log\left(-dx+\sqrt{dx-}\right.}{48\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 + 2*(5*b*c^2*d^3 + 6*a*d^5)*x^3 + 3*(5*b*c^4*d + 6*a*c^2
*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^6 + 6*a*c^4*d^2)*log(-d*x +
 sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

giac [A] time = 0.28, size = 203, normalized size = 1.24

$$\frac{\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{5\,bc}{d^6}\right)+\frac{3\left(15\,bc^2d^{36}+2\,ad^{38}\right)}{d^{42}}\right)-\frac{55\,bc^3d^{36}+18\,acd^{38}}{d^{42}}\right)(dx+c)+\frac{85\,bc^4d^{36}+54\,ac^2d^{38}}{d^{42}}\right)(dx+c\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((d*x + c)*(4*(d*x + c)*((d*x + c)*b/d^6 - 5*b*c/d^6) + 3*(15*b*c
^2*d^36 + 2*a*d^38)/d^42) - (55*b*c^3*d^36 + 18*a*c*d^38)/d^42)*(d*x + c) +
 (85*b*c^4*d^36 + 54*a*c^2*d^38)/d^42)*(d*x + c) - 3*(11*b*c^5*d^36 + 10*a*
c^3*d^38)/d^42)*sqrt(d*x + c)*sqrt(d*x - c) - 6*(5*b*c^6 + 6*a*c^4*d^2)*log
(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)/d
```

maple [C] time = 0.10, size = 240, normalized size = 1.46

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\sqrt{d^2x^2-c^2}\,b\,d^5x^5\mathrm{csgn}(d)+12\sqrt{d^2x^2-c^2}\,a\,d^5x^3\mathrm{csgn}(d)+10\sqrt{d^2x^2-c^2}\,b\,c^2d^3x^3\mathrm{csgn}(d)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*(d^2*x^2-c^2)^(1/2)*b*d^5*x^5*csgn(d)+1
2*(d^2*x^2-c^2)^(1/2)*a*d^5*x^3*csgn(d)+10*(d^2*x^2-c^2)^(1/2)*b*c^2*d^3*x^
3*csgn(d)+18*(d^2*x^2-c^2)^(1/2)*a*c^2*d^3*x*csgn(d)+15*(d^2*x^2-c^2)^(1/2)
*b*c^4*d*x*csgn(d)+18*a*c^4*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d
))+15*b*c^6*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/d^7/(d^2
*x^2-c^2)^(1/2)
```

maxima [A] time = 0.57, size = 196, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}\,bx^5}{6\,d^2}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^2x^3}{24\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax^3}{4\,d^2}+\frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^7}+\frac{3\,ac^4\log\left(2\,d^2x+2\right.}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^2*x^3/d^4
+ 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^7 + 3/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5 + 5
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6 + 3/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^4
```

mupad [B] time = 42.66, size = 1682, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((5*b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2))) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2
))^3) + (311*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c
)^(1/2))^5) + (8361*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (
d*x - c)^(1/2))^7) + (42259*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(
1/2) - (d*x - c)^(1/2))^9) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(
2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1
/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (42259*b*c^6*((c + d*x)^(1
/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (8361*b*c^6*((c
+ d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (311*b*
c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/
2))^21) + (5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x -
c)^(1/2))^23))/(d^7 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) -
(d*x - c)^(1/2))^2 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (
d*x - c)^(1/2))^4 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (
d*x - c)^(1/2))^6 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (
d*x - c)^(1/2))^8 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) -
(d*x - c)^(1/2))^10 + (924*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2)
- (d*x - c)^(1/2))^12 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2
) - (d*x - c)^(1/2))^14 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1
/2) - (d*x - c)^(1/2))^16 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^
(1/2) - (d*x - c)^(1/2))^18 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)
^(1/2) - (d*x - c)^(1/2))^20 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c
)^(1/2) - (d*x - c)^(1/2))^22 + (d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^
(1/2) - (d*x - c)^(1/2))^24) - ((23*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2
*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (3*a*c^4*((c + d*x)^(1/2) - c^(1/2)))/
(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (333*a*c^4*((c + d*x)^(1/2) - c^(1/2))
^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (671*a*c^4*((c + d*x)^(1/2) - c^
(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (671*a*c^4*((c + d*x)^(1/2
) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (333*a*c^4*((c + d*x
```

```
)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (23*a*c^4*((
c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*a*
c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/
(d^5 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^
4 - (56*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6
 + (70*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
- (56*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10
 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^1
2 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^1
4 + (d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16)
 - (3*a*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)
)))/(2*d^5) - (5*b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
 - c)^(1/2))))/(4*d^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.359 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{2c^2\sqrt{dx-c}\,\sqrt{c+dx}\,\left(5ad^2+4bc^2\right)}{15d^6}+\frac{x^2\sqrt{dx-c}\,\sqrt{c+dx}\,\left(5ad^2+4bc^2\right)}{15d^4}+\frac{bx^4\sqrt{dx-c}\,\sqrt{c+dx}}{5d^2}$$

```
[Out] 2/15*c^2*(5*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6+1/15*(5*a*d^2+4*
b*c^2)*x^2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/5*b*x^4*(d*x-c)^(1/2)*(d*x+c)^
(1/2)/d^2
```

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{dx-c}\,\sqrt{c+dx}\,\left(5ad^2+4bc^2\right)}{15d^4}+\frac{2c^2\sqrt{dx-c}\,\sqrt{c+dx}\,\left(5ad^2+4bc^2\right)}{15d^6}+\frac{bx^4\sqrt{dx-c}\,\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c
^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(5*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}-\frac{1}{5}\left(-5a-\frac{4bc^2}{d^2}\right)\int\frac{x^3}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\
&= \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4}+\frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}+\frac{\left(4bc^2+5ad^2\right)\int\frac{}{\sqrt{-}}}{15d^4}\\
&= \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4}+\frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2}+\frac{\left(2c^2\left(4bc^2+5ad^2\right)\right.}{15}\\
&= \frac{2c^2\left(4bc^2+5ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{15d^6}+\frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4}+\frac{bx^4}{}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.74

$$\frac{\left(d^2x^2-c^2\right)\left(5ad^2\left(2c^2+d^2x^2\right)+b\left(8c^4+4c^2d^2x^2+3d^4x^4\right)\right)}{15d^6\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
[Out] ((-c^2 + d^2*x^2)*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c^4 + 4*c^2*d^2*x^2 + 3
*d^4*x^4)))/(15*d^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.10, size = 66, normalized size = 0.56

$$\frac{\left(3\,bd^4x^4+8\,bc^4+10\,ac^2d^2+\left(4\,bc^2d^2+5\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*b*d^4*x^4 + 8*b*c^4 + 10*a*c^2*d^2 + (4*b*c^2*d^2 + 5*a*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6

giac [A] time = 0.25, size = 124, normalized size = 1.05

$$\frac{\left(\left((dx+c)\left(3\,(dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{4\,bc}{d^5}\right)+\frac{22\,bc^2d^{25}+5\,ad^{27}}{d^{30}}\right)-\frac{10\left(2\,bc^3d^{25}+acd^{27}\right)}{d^{30}}\right)(dx+c)+\frac{15\left(bc^4d^{25}+ac^2d^{27}\right)}{d^{30}}\right)\sqrt{dx+c}\,\sqrt{dx-}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")

[Out] 1/15*(((d*x + c)*(3*(d*x + c)*((d*x + c)*b/d^5 - 4*b*c/d^5) + (22*b*c^2*d^25 + 5*a*d^27)/d^30) - 10*(2*b*c^3*d^25 + a*c*d^27)/d^30)*(d*x + c) + 15*(b*c^4*d^25 + a*c^2*d^27)/d^30)*sqrt(d*x + c)*sqrt(d*x - c)/d

maple [A] time = 0.05, size = 68, normalized size = 0.58

$$\frac{\sqrt{dx+c}\,\left(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4\right)\sqrt{dx-c}}{15d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)

[Out] 1/15*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x^2+10*a*c^2*d^2+8*b*c^4)/d^6*(d*x-c)^(1/2)

maxima [A] time = 0.55, size = 124, normalized size = 1.05

$$\frac{\sqrt{d^2x^2-c^2}\,bx^4}{5\,d^2}+\frac{4\,\sqrt{d^2x^2-c^2}\,bc^2x^2}{15\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax^2}{3\,d^2}+\frac{8\,\sqrt{d^2x^2-c^2}\,bc^4}{15\,d^6}+\frac{2\,\sqrt{d^2x^2-c^2}\,ac^2}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^2*x^2/d^4 + 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2 - c^2)*b*c^4/d^6 + 2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4

mupad [B] time = 2.70, size = 130, normalized size = 1.10

$$\frac{\sqrt{d\,x-c}\,\left(\frac{8\,b\,c^5+10\,a\,c^3\,d^2}{15\,d^6}+\frac{x^3\,\left(4\,b\,c^2\,d^3+5\,a\,d^5\right)}{15\,d^6}+\frac{x\,\left(8\,b\,c^4\,d+10\,a\,c^2\,d^3\right)}{15\,d^6}+\frac{b\,x^5}{5\,d}+\frac{x^2\,\left(4\,b\,c^3\,d^2+5\,a\,c\,d^4\right)}{15\,d^6}+\frac{b\,c\,x^4}{5\,d^2}\right)}{\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((8*b*c^5 + 10*a*c^3*d^2)/(15*d^6) + (x^3*(5*a*d^5 + 4*b*c
^2*d^3))/(15*d^6) + (x*(10*a*c^2*d^3 + 8*b*c^4*d))/(15*d^6) + (b*x^5)/(5*d)
 + (x^2*(4*b*c^3*d^2 + 5*a*c*d^4))/(15*d^6) + (b*c*x^4)/(5*d^2)))/(c + d*x)
^(1/2)
```

sympy [C] time = 70.84, size = 240, normalized size = 2.03

$$\frac{ac^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{bc^5G}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -
1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg(((-2,
 -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*e
xp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg(((-9/4, -
7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2
*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4, -2,
1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2
*x**2))/(4*pi**(3/2)*d**6)
```

3.360 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{c^2(4ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{x\sqrt{dx-c}\sqrt{c+dx}\,(4ad^2+3bc^2)}{8d^4}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

```
[Out] 1/4*c^2*(4*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5+1/8*(4*a
*d^2+3*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/4*b*x^3*(d*x-c)^(1/2)*(d*
x+c)^(1/2)/d^2
```

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {460, 90, 12, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\,(4ad^2+3bc^2)}{8d^4}+\frac{c^2(4ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b*x^3*Sqrt[
-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c
 + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} - \frac{1}{4}\left(-4a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(3bc^2+4ad^2\right)\int \frac{}{\sqrt{-c+}}}{8d^4}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\int}{8d^4}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{c^2\left(3bc^2+4ad^2\right)\tanh}{4d^5}$$

Mathematica [A] time = 0.10, size = 121, normalized size = 1.03

$$\frac{dx\left(d^2x^2-c^2\right)\left(4ad^2+3bc^2+2bd^2x^2\right)+c^2\sqrt{d^2x^2-c^2}\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{8d^5\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*(-c^2 + d^2*x^2)*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + c^2*(3*b*c^2 + 4*
a*d^2)*Sqrt[-c^2 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-c^2 + d^2*x^2]])/(8*d^5*Sqr
t[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 0.91, size = 90, normalized size = 0.76

$$\frac{\left(2\,bd^3x^3+\left(3\,bc^2d+4\,ad^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}-\left(3\,bc^4+4\,ac^2d^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 + (3*b*c^2*d + 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) -
(3*b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^5
```

giac [A] time = 0.23, size = 140, normalized size = 1.19

$$\frac{\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^4}-\frac{3\,bc}{d^4}\right)+\frac{9\,bc^2d^{16}+4\,ad^{18}}{d^{20}}\right)-\frac{5\,bc^3d^{16}+4\,acd^{18}}{d^{20}}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{2\left(3\,bc^4+4\,ac^2d^2\right)\log\left(\left|-\sqrt{dx+c}\right.\right.}{d^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^4 - 3*b*c/d^4) + (9*b*c^2*d^16
+ 4*a*d^18)/d^20) - (5*b*c^3*d^16 + 4*a*c*d^18)/d^20)*sqrt(d*x + c)*sqrt(d*
x - c) - 2*(3*b*c^4 + 4*a*c^2*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))
/d^4)/d
```

maple [C] time = 0.08, size = 182, normalized size = 1.54

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\sqrt{d^2x^2-c^2}\,b\,d^3x^3\mathrm{csgn}(d)+4a\,c^2d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-c}\right.}{8\sqrt{d^2x^2-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*(d^2*x^2-c^2)^(1/2)*b*d^3*x^3*csgn(d)+4*
(d^2*x^2-c^2)^(1/2)*a*d^3*x*csgn(d)+3*(d^2*x^2-c^2)^(1/2)*b*c^2*d*x*csgn(d)
+4*a*c^2*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))+3*b*c^4*ln((d*x+
(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/d^5/(d^2*x^2-c^2)^(1/2)
```

maxima [A] time = 0.64, size = 142, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}\,bx^3}{4\,d^2}+\frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^5}+\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^3}+\frac{3\,\sqrt{d^2x^2-c^2}\,bc^2x}{8\,d^4}+\frac{\sqrt{d^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2
- c^2)*d)/d^5 + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 3/8*
sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

mupad [B] time = 25.51, size = 1048, normalized size = 8.88

$$\frac{\frac{2\,a\,c^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}+\frac{14\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^5}+\frac{2\,a\,c^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^7}}{d^3-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^6}+\frac{d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^8}}-\frac{\frac{23\,b\,c^4\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{2\,{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^3}-\frac{3\,b\,c^4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}\cdots}{d^5-\frac{8\,d^5\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}^2}+\frac{28\,d^5\,\left(\sqrt{c+d\,x}\cdots\right.}{\left(\sqrt{-c}-\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*a*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*a*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*a*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*a*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) - ((23
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3)
- (3*b*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2)))
+ (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2
))^5) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c
)^(1/2))^7) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d
*x - c)^(1/2))^9) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/
2) - (d*x - c)^(1/2))^11) + (23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((
-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)
/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^5 - (8*d^5*((c + d*x)^(1/2) - c^
(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^5*((c + d*x)^(1/2) - c^(
1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^5*((c + d*x)^(1/2) - c^(1
/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^5*((c + d*x)^(1/2) - c^(1/
2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^5*((c + d*x)^(1/2) - c^(1/2
))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^5*((c + d*x)^(1/2) - c^(1/
2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^5*((c + d*x)^(1/2) - c^(1/
2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^5*((c + d*x)^(1/2) - c^(1/2)
)^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) - (2*a*c^2*atanh(((c + d*x)^(1/2)
- c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/d^3 - (3*b*c^4*atanh(((c + d*x)
^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
[Out] Timed out
```

3.361 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,(3ad^2+2bc^2)}{3d^4}+\frac{bx^2\sqrt{dx-c}\,\sqrt{c+dx}}{3d^2}$$

```
[Out] 1/3*(3*a*d^2+2*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^4+1/3*b*x^2*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/d^2
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {460, 74}

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,(3ad^2+2bc^2)}{3d^4}+\frac{bx^2\sqrt{dx-c}\,\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x^2*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*d^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2} - \frac{1}{3}\left(-3a-\frac{2bc^2}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$
$$= \frac{\left(2bc^2+3ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^4} + \frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.85

$$\frac{\left(d^2x^2-c^2\right)\left(3ad^2+2bc^2+bd^2x^2\right)}{3d^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((-c^2 + d^2*x^2)*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4*Sqrt[-c + d*x]*Sq
rt[c + d*x])
```

fricas [A] time = 1.14, size = 42, normalized size = 0.58

$$\frac{\left(bd^2x^2+2\,bc^2+3\,ad^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^2*x^2 + 2*b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^4
```

giac [A] time = 0.20, size = 65, normalized size = 0.90

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^3}-\frac{2\,bc}{d^3}\right)+\frac{3\left(bc^2d^9+ad^{11}\right)}{d^{12}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^3 - 2*b*c/d^3) +
3*(b*c^2*d^9 + a*d^11)/d^12)/d
```

maple [A] time = 0.04, size = 43, normalized size = 0.60

$$\frac{\sqrt{dx+c}\,\left(b\,d^2x^2+3a\,d^2+2b\,c^2\right)\sqrt{dx-c}}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)/d^4*(d*x-c)^(1/2)
```

maxima [A] time = 0.64, size = 69, normalized size = 0.96

$$\frac{\sqrt{d^2x^2-c^2}\,bx^2}{3\,d^2}+\frac{2\,\sqrt{d^2x^2-c^2}\,bc^2}{3\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2/d^4 + sqr
t(d^2*x^2 - c^2)*a/d^2
```

mupad [B] time = 2.66, size = 76, normalized size = 1.06

$$\frac{\sqrt{d\,x-c}\,\left(\frac{2\,b\,c^3+3\,a\,c\,d^2}{3\,d^4}+\frac{b\,x^3}{3\,d}+\frac{x\,\left(2\,b\,c^2\,d+3\,a\,d^3\right)}{3\,d^4}+\frac{b\,c\,x^2}{3\,d^2}\right)}{\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((2*b*c^3 + 3*a*c*d^2)/(3*d^4) + (b*x^3)/(3*d) + (x*(3*a*d
^3 + 2*b*c^2*d))/(3*d^4) + (b*c*x^2)/(3*d^2)))/(c + d*x)^(1/2)
```

sympy [C] time = 44.70, size = 223, normalized size = 3.10

$$\frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|{\frac{c^2}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|{\frac{c^2e^{2i\pi}}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bc^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, \ldots \\ -\frac{3}{2},-\frac{5}{4},-1 \ldots \end{matrix}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), (
)), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg(((-5/4, -3/4), (-1, -1,
 -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi*
*(3/2)*d**4) + I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4
, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(
3/2)*d**4)
```

3.362 $\int \frac{a+bx^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=68

$$\frac{\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\,\sqrt{c+dx}}{2d^2}$$

```
[Out] (2*a*d^2+b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3+1/2*b*x*(d*x-c)^(1
/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {389, 63, 217, 206}

$$\frac{\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\,\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} - \frac{\left(-bc^2-2ad^2\right)\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{2d^2} \\
&= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{\left(bc^2+2ad^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d^3} \\
&= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{\left(bc^2+2ad^2\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3} \\
&= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{\left(bc^2+2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 119, normalized size = 1.75

$$\frac{4\left(ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)-\frac{2bc^{5/2}\sqrt{\frac{dx}{c}+1}\,\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c+dx}}+bdx\sqrt{dx-c}\sqrt{c+dx}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] - (2*b*c^(5/2)*Sqrt[1 + (d*x)/c]*ArcSin
h[Sqrt[-c + d*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c + d*x] + 4*(b*c^2 + a*d^2)*ArcT
anh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(2*d^3)
```

fricas [A] time = 1.17, size = 63, normalized size = 0.93

$$\frac{\sqrt{dx+c}\sqrt{dx-c}\,bdx-\left(bc^2+2\,ad^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*b*d*x - (b*c^2 + 2*a*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^3
```

giac [A] time = 0.26, size = 79, normalized size = 1.16

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\left(\frac{(dx+c)b}{d^2}-\frac{bc}{d^2}\right)-\frac{2\,(bc^2+2\,ad^2)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*b/d^2 - b*c/d^2) - 2*(b*c^2 + 2
*a*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)/d
```

maple [C] time = 0.07, size = 124, normalized size = 1.82

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2a\,d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+b\,c^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)\right.}{2\sqrt{d^2x^2-c^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^3*((d^2*x^2-c^2)^(1/2)*csgn(d)*d*x*b+ln((
d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*b*c^2+2*ln((d*x+(d^2*x^2-c^2)^(1/
2)*csgn(d))*csgn(d))*a*d^2)/(d^2*x^2-c^2)^(1/2)*csgn(d)
```

maxima [A] time = 0.46, size = 89, normalized size = 1.31

$$\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^3}+\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d}+\frac{\sqrt{d^2x^2-c^2}\,bx}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + a*log(2*d^2*x + 2*sq
rt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*b*x/d^2
```

mupad [B] time = 10.80, size = 417, normalized size = 6.13

$$\frac{\frac{2bc^2(\sqrt{c+dx}-\sqrt{c})}{\sqrt{-c}-\sqrt{dx-c}}+\frac{14bc^2(\sqrt{c+dx}-\sqrt{c})^3}{(\sqrt{-c}-\sqrt{dx-c})^3}+\frac{14bc^2(\sqrt{c+dx}-\sqrt{c})^5}{(\sqrt{-c}-\sqrt{dx-c})^5}+\frac{2bc^2(\sqrt{c+dx}-\sqrt{c})^7}{(\sqrt{-c}-\sqrt{dx-c})^7}}{d^3-\frac{4d^3(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2}+\frac{6d^3(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{-c}-\sqrt{dx-c})^4}-\frac{4d^3(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{-c}-\sqrt{dx-c})^6}+\frac{d^3(\sqrt{c+dx}-\sqrt{c})^8}{(\sqrt{-c}-\sqrt{dx-c})^8}}+\frac{4a\,\mathrm{atan}\left(\frac{d(\sqrt{-c}-\sqrt{dx-c})}{\sqrt{-d^2}(\sqrt{c+dx}-\sqrt{c})}\right)}{\sqrt{-d^2}}-\frac{2bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*b*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*b*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*b*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*b*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) + (4*a
*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))))/(-d^2)^(1/2) - (2*b*c^2*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^
(1/2) - (d*x - c)^(1/2))))/d^3
```

sympy [C] time = 41.78, size = 199, normalized size = 2.93

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bc^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c
**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))
/(4*pi**(3/2)*d) + b*c**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1,
 -3/4, -1/2, -1/4, 0, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*c
**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1
, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.363 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{d^2}$$

```
[Out] a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c+b*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {460, 92, 205}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d
*x])/c])/c
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
```

```
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}+a\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}+(ad)\,\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\sqrt{c+dx}\right)\\ &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c}\end{aligned}$$

Mathematica [A] time = 0.04, size = 87, normalized size = 1.55

$$\frac{ad^2\sqrt{d^2x^2-c^2}\,\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)-bc^3+bcd^2x^2}{cd^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-(b*c^3) + b*c*d^2*x^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2
*x^2]/c])/(c*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 0.97, size = 61, normalized size = 1.09

$$\frac{2\,ad^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}\,bc}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + sqrt(d*x + c)*sqr
t(d*x - c)*b*c)/(c*d^2)
```

giac [A] time = 0.21, size = 55, normalized size = 0.98

$$-\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c}+\frac{\sqrt{dx+c}\sqrt{dx-c}\,b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x + c)*sqrt
(d*x - c)*b/d^2
```

maple [B] time = 0.08, size = 108, normalized size = 1.93

$$\frac{\left(-a\,d^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\right)\sqrt{dx-c}\,\sqrt{dx+c}}{\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^(1/2)*(d^2
*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(-c^2)
^(1/2)
```

maxima [A] time = 1.34, size = 37, normalized size = 0.66

$$-\frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c}+\frac{\sqrt{d^2x^2-c^2}\,b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2 - c^2)*b/d^2
```

mupad [B] time = 3.97, size = 108, normalized size = 1.93

$$\frac{b\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{d^2}-\frac{a\,\sqrt{-c}\,\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (b*(c + d*x)^(1/2)*(d*x - c)^(1/2))/d^2 - (a*(-c)^(1/2)*(log(((c + d*x)^(1/
2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2)
```

sympy [C] time = 39.16, size = 178, normalized size = 3.18

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{bcG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \cdots\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c
**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), (
)), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*c) + b*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.364 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=57

$$\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] 2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d+a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^
2/x
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 63, 217, 206}

$$\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x}+b\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\ &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d}\\ &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}\\ &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 1.58

$$\frac{\sqrt{d^2x^2-c^2}\left(\frac{a\sqrt{d^2x^2-c^2}}{c^2x}+\frac{b\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{d}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[-c^2 + d^2*x^2]*((a*Sqrt[-c^2 + d^2*x^2])/(c^2*x) + (b*ArcTanh[(d*x)/
Sqrt[-c^2 + d^2*x^2]])/d))/(Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.29, size = 68, normalized size = 1.19

$$-\frac{bc^2x\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)-ad^2x-\sqrt{dx+c}\,\sqrt{dx-c}\,ad}{c^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(b*c^2*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) - a*d^2*x - sqrt(d*x + c)
*sqrt(d*x - c)*a*d)/(c^2*d*x)
```

giac [A] time = 0.22, size = 66, normalized size = 1.16

$$\frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2} - b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*log((sqrt(d*x
 + c) - sqrt(d*x - c))^4))/d
```

maple [C] time = 0.07, size = 97, normalized size = 1.70

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(b\,c^2x\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^2x^2-c^2}\,ad\operatorname{csgn}(d)\right)\operatorname{csgn}(d)}{\sqrt{d^2x^2-c^2}\,c^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(b*c^2*x*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d
))*csgn(d))+(d^2*x^2-c^2)^(1/2)*a*d*csgn(d))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d/
x
```

maxima [A] time = 1.26, size = 55, normalized size = 0.96

$$\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d}+\frac{\sqrt{d^2x^2-c^2}\,a}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + sqrt(d^2*x^2 - c^2)*a/(c^2*x)
```

mupad [B] time = 2.94, size = 77, normalized size = 1.35

$$\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}}+\frac{a\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{c^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (4*b*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2)
 - c^(1/2)))))/(-d^2)^(1/2) + (a*(c + d*x)^(1/2)*(d*x - c)^(1/2))/(c^2*x)
```

sympy [C] time = 36.82, size = 165, normalized size = 2.89

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3
/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4
, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_p
olar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

3.365 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2c^2x^2}$$

```
[Out] 1/2*(a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+1/2*a*(d*x-c)
^(1/2)*(d*x+c)^(1/2)/c^2/x^2
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {454, 92, 205}

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)*ArcTan[(S
qrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
```

```
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{1}{2}\left(2b+\frac{ad^2}{c^2}\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{1}{2}\left(d\left(2b+\frac{ad^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\sqrt{c\right.$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{\left(2bc^2+ad^2\right)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}$$

Mathematica [A] time = 0.10, size = 102, normalized size = 1.34

$$\frac{x^2\sqrt{d^2x^2-c^2}\left(ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)+a\left(cd^2x^2-c^3\right)}{2c^3x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*(-c^3 + c*d^2*x^2) + (2*b*c^2 + a*d^2)*x^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[S
qrt[-c^2 + d^2*x^2]/c])/(2*c^3*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.27, size = 73, normalized size = 0.96

$$\frac{2\left(2\,bc^2+ad^2\right)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}\,ac}{2\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*b*c^2 + a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c)
 + sqrt(d*x + c)*sqrt(d*x - c)*a*c)/(c^3*x^2)
```

giac [B] time = 0.49, size = 141, normalized size = 1.86

$$-\frac{\frac{\left(2\,bc^2d+ad^3\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 +
 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c) -
sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^2))/d
```

maple [B] time = 0.07, size = 158, normalized size = 2.08

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(a\,d^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+2b\,c^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\right)}{2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(a*d^2*x^2*ln(-2*(c^2-(-c^2)^(1/2)*(d^
2*x^2-c^2)^(1/2))/x)+2*b*c^2*x^2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2
))/x)-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1
/2)
```

maxima [A] time = 1.29, size = 60, normalized size = 0.79

$$-\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c}-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}+\frac{\sqrt{d^2x^2-c^2}\,a}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arcsin(c/(d*abs(x)))/c - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c^3 + 1/2*sqrt(d
^2*x^2 - c^2)*a/(c^2*x^2)
```

mupad [B] time = 7.50, size = 457, normalized size = 6.01

$$\frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}}-\frac{b\,\sqrt{-c}\,\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}-\frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right.}{2\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (a*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))/(2*c^(9/2)) - (b*(-c)^(1/2)*(log(((c + d*x)^(1/2) - c^(1/2)
)^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2) - c^(1/2))
/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2) - (a*(-c)^(3/2)*d^2*log(((c + d*
x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(9/2)) - ((a*(-c)
^(3/2)*d^2)/(32*c^(9/2)) + (a*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)
/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*a*(-c)^(3/2)*d^2*((c +
 d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*
x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2)
- c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*d^2*((c + d*x)^(1/2) -
c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

sympy [C] time = 69.67, size = 162, normalized size = 2.13

$$-\frac{ad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}+\frac{iad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**3) + I*a*d**2*meijerg(((1, 5/4, 3/2,
7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), c**2*exp_polar(2*I*pi)/(d*
*2*x**2))/(4*pi**(3/2)*c**3) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/
2, 3/4, 1, 5/4, 3/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*b*meijer
g(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_
polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c)
```

3.366 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{3c^2x^3}$$

```
[Out] 1/3*a*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^3+1/3*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/
2)*(d*x+c)^(1/2)/c^4/x
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {454, 95}

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3} + \frac{1}{3}\left(3b+\frac{2ad^2}{c^2}\right)\int \frac{1}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3} + \frac{(3bc^2+2ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.88

$$-\frac{(c^2-d^2x^2)\left(a\left(c^2+2d^2x^2\right)+3bc^2x^2\right)}{3c^4x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -1/3*((c^2 - d^2*x^2)*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(c^4*x^3*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 0.67, size = 67, normalized size = 0.89

$$\frac{(3\,bc^2d+2\,ad^3)x^3+\left(ac^2+(3\,bc^2+2\,ad^2)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{3\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^4*x^3)
```

giac [B] time = 0.24, size = 137, normalized size = 1.83

$$\frac{8\left(3\,bd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+24\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^4d^2+3\right.}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x + c
) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^
4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)
```

maple [A] time = 0.04, size = 49, normalized size = 0.65

$$\frac{\sqrt{dx+c}\,\left(2a\,d^2x^2+3b\,c^2x^2+a\,c^2\right)\sqrt{dx-c}}{3c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^4*(d*x-c)^(1/2)
```

maxima [A] time = 1.32, size = 75, normalized size = 1.00

$$\frac{\sqrt{d^2x^2-c^2}\,b}{c^2x}+\frac{2\sqrt{d^2x^2-c^2}\,ad^2}{3\,c^4x}+\frac{\sqrt{d^2x^2-c^2}\,a}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(d^2*x^2 - c^2)*b/(c^2*x) + 2/3*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x) + 1/3
*sqrt(d^2*x^2 - c^2)*a/(c^2*x^3)
```

mupad [B] time = 2.77, size = 79, normalized size = 1.05

$$\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c}+\frac{x^2\,\left(3\,b\,c^3+2\,a\,c\,d^2\right)}{3\,c^4}+\frac{x^3\,\left(3\,b\,c^2\,d+2\,a\,d^3\right)}{3\,c^4}+\frac{a\,d\,x}{3\,c^2}\right)}{x^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c) + (x^2*(3*b*c^3 + 2*a*c*d^2))/(3*c^4) + (x^3*(2*a
*d^3 + 3*b*c^2*d))/(3*c^4) + (a*d*x)/(3*c^2)))/(x^3*(c + d*x)^(1/2))
```

sympy [C] time = 70.80, size = 170, normalized size = 2.27

$$-\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}-\frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4, 2
, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi)/(
d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2))
, ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b
*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

3.367 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=123

$$\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

```
[Out] 1/8*d^2*(3*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/4*a*(
d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2/x^4+1/8*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x
+c)^(1/2)/c^4/x^2
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[
-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{\left(4bc^2+3ad^2\right)\int \frac{}{x\sqrt{-c}}}{8c^4}$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{\left(d^2\left(4bc^2+3ad^2\right)\right)\int}{8c^4}$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{\left(d^3\left(4bc^2+3ad^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^4x^2} + \frac{d^2\left(4bc^2+3ad^2\right)\tan^{-}}{8c^5}$$

Mathematica [A] time = 0.11, size = 144, normalized size = 1.17

$$-\frac{\left(c^2-d^2x^2\right)\left(c^2\sqrt{1-\frac{d^2x^2}{c^2}}\left(2ac^2+3ad^2x^2+4bc^2x^2\right)+d^2x^4\left(3ad^2+4bc^2\right)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right)\right)}{8c^6x^4\sqrt{dx-c}\,\sqrt{c+dx}\,\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -1/8*((c^2 - d^2*x^2)*(c^2*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2)*Sqrt[1 - (
d^2*x^2)/c^2] + d^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTanh[Sqrt[1 - (d^2*x^2)/c^2]
]))/(c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*Sqrt[1 - (d^2*x^2)/c^2])
```

fricas [A] time = 1.08, size = 100, normalized size = 0.81

$$\frac{2\left(4\,bc^2d^2 + 3\,ad^4\right)x^4 \arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right) + \left(2\,ac^3 + \left(4\,bc^3 + 3\,acd^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{8\,c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x -
c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))
/(c^5*x^4)
```

giac [B] time = 0.27, size = 325, normalized size = 2.64

$$-\frac{\frac{\left(4\,bc^2d^3+3\,ad^5\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} + \frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3\,ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+44\,ac^2d^5\left(\sqrt{d\cdots}\right.\right.}{\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/
c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 3*a*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^10 + 44*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(
d*x + c) - sqrt(d*x - c))^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))
^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4
*c^4))/d
```

maple [B] time = 0.07, size = 227, normalized size = 1.85

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3a\,d^4x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+4b\,c^2d^2x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-3\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,\cdots\right.}{8\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^
2)^(1/2))/x)*x^4*a*d^4+4*b*c^2*d^2*x^4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2
)^(1/2))/x)-3*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^2-4*(-c^2)^(1/2)*(d^
2*x^2-c^2)^(1/2)*x^2*b*c^2-2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*c^2)/(d^2*x
^2-c^2)^(1/2)/x^4/(-c^2)^(1/2)
```

maxima [A] time = 1.21, size = 114, normalized size = 0.93

$$-\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}-\frac{3\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^5}+\frac{\sqrt{d^2x^2-c^2}\,b}{2\,c^2x^2}+\frac{3\,\sqrt{d^2x^2-c^2}\,ad^2}{8\,c^4x^2}+\frac{\sqrt{d^2x^2-c^2}\,a}{4\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c^3 - 3/8*a*d^4*arcsin(c/(d*abs(x)))/c^5 +
1/2*sqrt(d^2*x^2 - c^2)*b/(c^2*x^2) + 3/8*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x^
2) + 1/4*sqrt(d^2*x^2 - c^2)*a/(c^2*x^4)
```

mupad [B] time = 19.13, size = 1005, normalized size = 8.17

$$\frac{3\,a\,\sqrt{-c}\,d^4\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{8\,c^{11/2}}-\frac{\frac{b\,(-c)^{3/2}\,d^2}{32\,c^{9/2}}+\frac{b\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,b\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}-\frac{\frac{a\,\sqrt{-c}\,d^4}{1024\,c^{11/2}}-\frac{3\,a\,\sqrt{-c}\,d^4\,\left(\sqrt{c\ldots}\right.}{128\,c^{11/2}\,\left(\sqrt{-c\ldots}\right.}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)
^(1/2))))/(8*c^(11/2)) - ((b*(-c)^(3/2)*d^2)/(32*c^(9/2)) + (b*(-c)^(3/2)*d
^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2)
)^2) - (15*b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c
)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) -
 (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x
- c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2)
)^6) - ((a*(-c)^(1/2)*d^4)/(1024*c^(11/2)) - (3*a*(-c)^(1/2)*d^4*((c + d*x)
^(1/2) - c^(1/2))^2)/(128*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (53*
a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(11/2)*((-c)^(1/2) -
 (d*x - c)^(1/2))^4) + (87*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/
```

```
(256*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^6) + (657*a*(-c)^(1/2)*d^4*((c
 + d*x)^(1/2) - c^(1/2))^8)/(1024*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8
) + (121*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(11/2)*((-
c)^(1/2) - (d*x - c)^(1/2))^10))/((((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*
x - c)^(1/2))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)
^(1/2))^8 + (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2
))^10 + ((c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) -
 (b*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^
(1/2))))/(2*c^(9/2)) - (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^
2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(8*c^(11/2)) + (b*(-c)^(3/2)*d^2*l
og(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*
c^(9/2)) - (7*a*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*(-c)^(1/2)*c^(9/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^2) + (a*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(
1024*(-c)^(1/2)*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) + (b*d^2*((c + d*
x)^(1/2) - c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2)
)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.368 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{3c^2\left(4ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{3x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(4ad^2+5bc^2\right)}{8d^6}-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{dx-c}\,\sqrt{c+}$$

```
[Out] 3/4*c^2*(4*a*d^2+5*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^7-1/4*(4*a
*d^2+5*b*c^2)*x^3/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*b*x^5/d^2/(d*x-c)^(1/
2)/(d*x+c)^(1/2)+3/8*(4*a*d^2+5*b*c^2)*x*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^6
```

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {460, 98, 21, 90, 12, 63, 217, 206}

$$-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{3x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(4ad^2+5bc^2\right)}{8d^6}+\frac{3c^2\left(4ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{bx^5}{4d^2\sqrt{dx-c}\,\sqrt{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((5*b*c^2 + 4*a*d^2)*x^3)/(4*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^5)/(
4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*
x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(4*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{4}\left(-4a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(4a+\frac{5bc^2}{d^2}\right)\int \frac{x^2\left(-3c^2-3cdx\right)}{\sqrt{-c+dx}\,(c+dx)^{3/2}}}{4cd^2}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3\left(5bc^2+4ad^2\right)\right)\int \frac{x^2}{\sqrt{-c+dx}}}{4d^4}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}}{8d^6}$$

Mathematica [A] time = 0.16, size = 119, normalized size = 0.74

$$\frac{3c^3\sqrt{1-\frac{d^2x^2}{c^2}}\left(4ad^2+5bc^2\right)\sin^{-1}\left(\frac{dx}{c}\right)+4ad^3x\left(d^2x^2-3c^2\right)+bdx\left(-15c^4+5c^2d^2x^2+2d^4x^4\right)}{8d^7\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (4*a*d^3*x*(-3*c^2 + d^2*x^2) + b*d*x*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4)
 + 3*c^3*(5*b*c^2 + 4*a*d^2)*Sqrt[1 - (d^2*x^2)/c^2]*ArcSin[(d*x)/c])/(8*d^
7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.08, size = 190, normalized size = 1.18

$$\frac{8\,bc^{6} + 8\,ac^{4}d^{2} - 8\,\left(bc^{4}d^{2} + ac^{2}d^{4}\right)x^{2} + \left(2\,bd^{5}x^{5} + \left(5\,bc^{2}d^{3} + 4\,ad^{5}\right)x^{3} - 3\,\left(5\,bc^{4}d + 4\,ac^{2}d^{3}\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}}{8\,\left(d^{9}x^{2} - c^{2}d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*c^6 + 8*a*c^4*d^2 - 8*(b*c^4*d^2 + a*c^2*d^4)*x^2 + (2*b*d^5*x^5 +
 (5*b*c^2*d^3 + 4*a*d^5)*x^3 - 3*(5*b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x + c)
*sqrt(d*x - c) + 3*(5*b*c^6 + 4*a*c^4*d^2 - (5*b*c^4*d^2 + 4*a*c^2*d^4)*x^2
)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^9*x^2 - c^2*d^7)
```

giac [A] time = 0.40, size = 214, normalized size = 1.33

$$\frac{\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^{7}} - \frac{5\,bc}{d^{7}}\right) + \frac{25\,bc^{2}d^{35}+4\,ad^{37}}{d^{42}}\right) - \frac{35\,bc^{3}d^{35}+12\,acd^{37}}{d^{42}}\right)(dx+c) + \frac{2\left(7\,bc^{4}d^{35}+2\,ac^{2}d^{37}\right)}{d^{42}}\right)\sqrt{dx+c}}{8\,\sqrt{dx-c}} - \frac{3\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^7 - 5*b*c/d^7) + (25*b*c^2*d^35
 + 4*a*d^37)/d^42) - (35*b*c^3*d^35 + 12*a*c*d^37)/d^42)*(d*x + c) + 2*(7*b
*c^4*d^35 + 2*a*c^2*d^37)/d^42)*sqrt(d*x + c)/sqrt(d*x - c) - 3/8*(5*b*c^4
+ 4*a*c^2*d^2)*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^7 - 2*(b*c^5 + a*c^
3*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

maple [C] time = 0.09, size = 316, normalized size = 1.96

$$\frac{\left(2\sqrt{d^{2}x^{2}-c^{2}}\;b\,d^{5}x^{5}\mathrm{csgn}(d) + 12a\,c^{2}d^{4}x^{2}\ln\left(\left(dx + \sqrt{d^{2}x^{2}-c^{2}}\;\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right) + 4\sqrt{d^{2}x^{2}-c^{2}}\;a\,d^{5}x^{3}\mathrm{csgn}(d)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8*(2*(d^2*x^2-c^2)^(1/2)*b*d^5*x^5*csgn(d)+4*(d^2*x^2-c^2)^(1/2)*a*d^5*x^
3*csgn(d)+5*(d^2*x^2-c^2)^(1/2)*b*c^2*d^3*x^3*csgn(d)+12*ln((d*x+(d^2*x^2-c
^2)^(1/2)*csgn(d))*csgn(d))*x^2*a*c^2*d^4+15*ln((d*x+(d^2*x^2-c^2)^(1/2)*cs
gn(d))*csgn(d))*x^2*b*c^4*d^2-12*(d^2*x^2-c^2)^(1/2)*a*c^2*d^3*x*csgn(d)-15
*(d^2*x^2-c^2)^(1/2)*b*c^4*d*x*csgn(d)-12*a*c^4*d^2*ln((d*x+(d^2*x^2-c^2)^(
```

```
1/2)*csgn(d))*csgn(d))-15*b*c^6*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d
)))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 0.49, size = 196, normalized size = 1.22

$$\frac{bx^5}{4\sqrt{d^2x^2-c^2}\,d^2}+\frac{5\,bc^2x^3}{8\sqrt{d^2x^2-c^2}\,d^4}+\frac{ax^3}{2\sqrt{d^2x^2-c^2}\,d^2}-\frac{15\,bc^4x}{8\sqrt{d^2x^2-c^2}\,d^6}-\frac{3\,ac^2x}{2\sqrt{d^2x^2-c^2}\,d^4}+\frac{15\,bc^4\log\left(2\,d^2x+2\sqrt{d\right.}}{8\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d^
4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c^2
)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x + 2
*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)
*d)/d^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(b\,x^2+a\right)}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.369 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$

Optimal. Leaf size=115

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{3d^6}-\frac{x^2\left(3ad^2+4bc^2\right)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -1/3*(3*a*d^2+4*b*c^2)*x^2/d^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*b*x^4/d^2/(d
*x-c)^(1/2)/(d*x+c)^(1/2)+2/3*(3*a*d^2+4*b*c^2)*(d*x-c)^(1/2)*(d*x+c)^(1/2)
/d^6
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 98, 21, 74}

$$-\frac{x^2\left(3ad^2+4bc^2\right)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{2\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{3d^6}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^4)/(
3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]
*Sqrt[c + d*x])/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{3}\left(-3a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3a+\frac{4bc^2}{d^2}\right)\int \frac{x\left(-2c^2-2cdx\right)}{\sqrt{-c+dx}\,(c+dx)^{3/2}}}{3cd^2}$$

$$= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(2\left(3a+\frac{4bc^2}{d^2}\right)\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c}}}{3d^2}$$

$$= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{2\left(4bc^2+3ad^2\right)\sqrt{-c+dx}}{3d^6}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.63

$$\frac{-6ac^2d^2+3ad^4x^2-8bc^4+4bc^2d^2x^2+bd^4x^4}{3d^6\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x^4)/(3*d^6
*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.18, size = 80, normalized size = 0.70

$$\frac{\left(bd^4x^4 - 8\,bc^4 - 6\,ac^2d^2 + \left(4\,bc^2d^2 + 3\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\left(d^8x^2 - c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^4*x^4 - 8*b*c^4 - 6*a*c^2*d^2 + (4*b*c^2*d^2 + 3*a*d^4)*x^2)*sqrt(
d*x + c)*sqrt(d*x - c)/(d^8*x^2 - c^2*d^6)
```

giac [B] time = 0.43, size = 200, normalized size = 1.74

$$\frac{\left(2\,(dx+c)\left((dx+c)\left(\frac{(dx+c)b}{d^6} - \frac{4\,bc}{d^6}\right) + \frac{10\,bc^2d^{24}+3\,ad^{26}}{d^{30}}\right) - \frac{3\left(9\,bc^3d^{24}+5\,acd^{26}\right)}{d^{30}}\right)\sqrt{dx+c}}{6\,\sqrt{dx-c}} + \frac{2\left(b^2c\right.}{\left(bc^4\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x + c)*((d*x + c)*((d*x + c)*b/d^6 - 4*b*c/d^6) + (10*b*c^2*d^24
+ 3*a*d^26)/d^30) - 3*(9*b*c^3*d^24 + 5*a*c*d^26)/d^30)*sqrt(d*x + c)/sqrt(
d*x - c) + 2*(b^2*c^8 + 2*a*b*c^6*d^2 + a^2*c^4*d^4)/((b*c^4*(sqrt(d*x + c)
 - sqrt(d*x - c))^2 + a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^5
 + 2*a*c^3*d^2)*d^6)
```

maple [A] time = 0.05, size = 68, normalized size = 0.59

$$-\frac{-b\,d^4x^4 - 3a\,d^4x^2 - 4b\,c^2d^2x^2 + 6a\,c^2d^2 + 8b\,c^4}{3\sqrt{dx+c}\,\sqrt{dx-c}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -1/3*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^2+6*a*c^2*d^2+8*b*c^4)/(d*x+c)^(
1/2)/d^6/(d*x-c)^(1/2)
```

maxima [A] time = 0.46, size = 123, normalized size = 1.07

$$\frac{bx^4}{3\,\sqrt{d^2x^2-c^2}\,d^2} + \frac{4\,bc^2x^2}{3\,\sqrt{d^2x^2-c^2}\,d^4} + \frac{ax^2}{\sqrt{d^2x^2-c^2}\,d^2} - \frac{8\,bc^4}{3\,\sqrt{d^2x^2-c^2}\,d^6} - \frac{2\,ac^2}{\sqrt{d^2x^2-c^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2 - c^2)*d^
4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(d^2*x^2 - c^2)*d^6)
- 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

mupad [B] time = 2.80, size = 90, normalized size = 0.78

$$\frac{\sqrt{d\,x-c}\,\left(\frac{x^2\,\left(4\,b\,c^2\,d^2+3\,a\,d^4\right)}{3\,d^7}-\frac{8\,b\,c^4+6\,a\,c^2\,d^2}{3\,d^7}+\frac{b\,x^4}{3\,d^3}\right)}{x\,\sqrt{c+d\,x}-\frac{c\,\sqrt{c+d\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((x^2*(3*a*d^4 + 4*b*c^2*d^2))/(3*d^7) - (8*b*c^4 + 6*a*c^
2*d^2)/(3*d^7) + (b*x^4)/(3*d^3)))/(x*(c + d*x)^(1/2) - (c*(c + d*x)^(1/2))
/d)
```

sympy [C] time = 177.26, size = 226, normalized size = 1.97

$$a\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -1,0,\frac{1}{2},1\\ -\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},1 & \\ -\frac{5}{4},-\frac{3}{4} & -2,-\frac{3}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}\right)+b\left(\frac{c^3G_{6,6}^{6,2}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1)), ((-3/4, -1/2, -1/4, 0, 1/2, 0
), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg(((-2, -3/2, -5/4
, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/2, -1/2, 0)), c**2*exp_polar(2*
I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4)) + b*(c**3*meijerg(((-7/4, -5/4), (-2
, -1, -1/2, 1)), ((-7/4, -3/2, -5/4, -1, -1/2, 0), ()), c**2/(d**2*x**2))/(
2*pi**(3/2)*d**6) - I*c**3*meijerg(((-3, -5/2, -9/4, -2, -7/4, 1), ()), ((-
9/4, -7/4), (-3, -5/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi
**(3/2)*d**6))
```

3.370 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{dx-c}\,(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}}-\frac{c\,(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

```
[Out] (2*a*d^2+3*b*c^2)*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^5-1/2*c*(2*a*d^2+3
*b*c^2)/d^5/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*b*x^3/d^2/(d*x-c)^(1/2)/(d*x+c)
^(1/2)-1/2*(2*a*d^2+3*b*c^2)*(d*x-c)^(1/2)/d^5/(d*x+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 89, 12, 78, 63, 217, 206}

$$-\frac{\sqrt{dx-c}\,(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}}-\frac{c\,(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^3)/(2*
d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x])/(2
*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c +
d*x]])/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{2}\left(-2a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(-2a-\frac{3bc^2}{d^2}\right)\int \frac{cd^2x}{\sqrt{-c+dx}\,(c+dx)^{3/2}}}{2cd^3}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3bc^2+2ad^2\right)\int \frac{x}{\sqrt{-c+dx}\,(c+dx)}}{2d^3}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + $$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + $$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + $$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}} + $$

Mathematica [A] time = 0.12, size = 90, normalized size = 0.59

$$\frac{c\sqrt{1-\frac{d^2x^2}{c^2}}\left(2ad^2+3bc^2\right)\sin^{-1}\left(\frac{dx}{c}\right)-2ad^3x-3bc^2dx+bd^3x^3}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*b*c^2*d*x - 2*a*d^3*x + b*d^3*x^3 + c*(3*b*c^2 + 2*a*d^2)*Sqrt[1 - (d^2
*x^2)/c^2]*ArcSin[(d*x)/c])/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 0.98, size = 159, normalized size = 1.05

$$\frac{2\,bc^4+2\,ac^2d^2-2\left(bc^2d^2+ad^4\right)x^2+\left(bd^3x^3-\left(3\,bc^2d+2\,ad^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}+\left(3\,bc^4+2\,ac^2d^2-\left(3\,bc^2d^2\right.\right.}{2\left(d^7x^2-c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^4 + 2*a*c^2*d^2 - 2*(b*c^2*d^2 + a*d^4)*x^2 + (b*d^3*x^3 - (3*b*
c^2*d + 2*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^4 + 2*a*c^2*d^2 -
(3*b*c^2*d^2 + 2*a*d^4)*x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^7*
x^2 - c^2*d^5)
```

giac [A] time = 0.32, size = 147, normalized size = 0.97

$$\frac{\sqrt{dx+c}\left((dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{3\,bc}{d^5}\right)+\frac{bc^2d^{15}-ad^{17}}{d^{20}}\right)}{2\,\sqrt{dx-c}}-\frac{\left(3\,bc^2+2\,ad^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{2\,d^5}-\frac{2\left(bc^3+\right.}{\left(\left(\sqrt{dx+c}-\sqrt{dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + c)*((d*x + c)*((d*x + c)*b/d^5 - 3*b*c/d^5) + (b*c^2*d^15 -
a*d^17)/d^20)/sqrt(d*x - c) - 1/2*(3*b*c^2 + 2*a*d^2)*log((sqrt(d*x + c) -
sqrt(d*x - c))^2)/d^5 - 2*(b*c^3 + a*c*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c
))^2 + 2*c)*d^5)
```

maple [C] time = 0.08, size = 254, normalized size = 1.67

$$\frac{\left(2a\,d^4x^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+3b\,c^2d^2x^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2*((d^2*x^2-c^2)^(1/2)*b*d^3*x^3*csgn(d)+2*ln((d*x+(d^2*x^2-c^2)^(1/2)*cs
gn(d))*csgn(d))*x^2*a*d^4+3*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*x
^2*b*c^2*d^2-2*(d^2*x^2-c^2)^(1/2)*a*d^3*x*csgn(d)-3*(d^2*x^2-c^2)^(1/2)*b*
c^2*d*x*csgn(d)-2*a*c^2*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))-3
*b*c^4*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/(d^2*x^2-c^2)
^(1/2)/d^5/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 0.50, size = 138, normalized size = 0.91

$$\frac{bx^3}{2\,\sqrt{d^2x^2-c^2}\,d^2}-\frac{3\,bc^2x}{2\,\sqrt{d^2x^2-c^2}\,d^4}-\frac{ax}{\sqrt{d^2x^2-c^2}\,d^2}+\frac{3\,bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^5}+\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4)
 - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 -
 c^2)*d)/d^5 + a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \left(b x^2 + a\right)}{(c + d x)^{3/2} (d x - c)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.371 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+2bc^2\right)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(a/c^2+b/d^2)*x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2)+(a*d^2+2*b*c^2)*(d*x-c)^(1/2
)*(d*x+c)^(1/2)/c^2/d^4
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {458, 74}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+2bc^2\right)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2
)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 458

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*
c - a1*a2*d)*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p +
 1))/(a1*a2*b1*b2*e*n*(p + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n
*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m,
n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !Inte
gerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p +
 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}} - \left(-\frac{a}{c^2}-\frac{2b}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 0.59

$$\frac{-ad^2-2bc^2+bd^2x^2}{d^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.14, size = 56, normalized size = 0.74

$$\frac{\left(bd^2x^2-2\,bc^2-ad^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{d^6x^2-c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^2*x^2 - 2*b*c^2 - a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/(d^6*x^2 - c^2*d^
4)
```

giac [B] time = 0.28, size = 152, normalized size = 2.00

$$\frac{\sqrt{dx+c}\left(\frac{2\,(dx+c)b}{d^4}-\frac{5\,bc^2d^8+ad^{10}}{cd^{12}}\right)}{2\,\sqrt{dx-c}}+\frac{2\left(b^2c^4+2\,abc^2d^2+a^2d^4\right)}{\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,bc^3+2\,acd^2\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + c)*(2*(d*x + c)*b/d^4 - (5*b*c^2*d^8 + a*d^10)/(c*d^12))/sqr
t(d*x - c) + 2*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/((b*c^2*(sqrt(d*x + c) -
```

```
 sqrt(d*x - c))^2 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^3 + 2*a
*c*d^2)*d^4)
```

maple [A] time = 0.04, size = 43, normalized size = 0.57

$$-\frac{-b\,d^{2}x^{2}+a\,d^{2}+2b\,c^{2}}{\sqrt{dx+c}\,\sqrt{dx-c}\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -(-b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)
```

maxima [A] time = 0.58, size = 69, normalized size = 0.91

$$\frac{bx^{2}}{\sqrt{d^{2}x^{2}-c^{2}}\,d^{2}}-\frac{2\,bc^{2}}{\sqrt{d^{2}x^{2}-c^{2}}\,d^{4}}-\frac{a}{\sqrt{d^{2}x^{2}-c^{2}}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^4) - a/(sq
rt(d^2*x^2 - c^2)*d^2)
```

mupad [B] time = 2.75, size = 67, normalized size = 0.88

$$\frac{a\,d^{2}\,\sqrt{d\,x-c}\,+2\,b\,c^{2}\,\sqrt{d\,x-c}\,-b\,d^{2}\,x^{2}\,\sqrt{d\,x-c}}{d^{4}\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (a*d^2*(d*x - c)^(1/2) + 2*b*c^2*(d*x - c)^(1/2) - b*d^2*x^2*(d*x - c)^(1/2
))/(d^4*(c + d*x)^(1/2)*(c - d*x))
```

sympy [C] time = 136.30, size = 201, normalized size = 2.64

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}cd^{2}}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},1 & \\ -\frac{1}{4},\frac{1}{4} & -1,-\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}cd^{2}}\right)+b\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \ldots\\ -\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},\ldots\end{matrix}\right.}{2\pi^{\frac{3}{2}}d^{4}\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-1, -1/2, -1/4, 0, 1/4
, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*
x**2))/(2*pi**(3/2)*c*d**2)) + b*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1))
, ((-3/4, -1/2, -1/4, 0, 1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4)
 - I*c*meijerg(((-2, -3/2, -5/4, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/
2, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4))
```

3.372 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

```
[Out] 2*b*arctanh((d*x-c)^(1/2)/(d*x+c)^(1/2))/d^3-(a/c^2+b/d^2)*x/(d*x-c)^(1/2)/
(d*x+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {386, 63, 217, 206}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 386

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*c - a1*a2*d)*x*(a1
+ b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*b1*b2*n*(p + 1)), x
] - Dist[(a1*a2*d - b1*b2*c*(n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(
a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1
, a2, b2, c, d, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (LtQ[p
, -1] || ILtQ[1/n + p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{b\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx,x,\sqrt{-c+dx}\right)}{d^3}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}\\
&= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 86, normalized size = 1.37

$$\frac{2bc^{5/2}\sqrt{\frac{dx}{c}+1}\,\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\,\sqrt{c}}\right)-\frac{dx\left(ad^2+bc^2\right)}{\sqrt{dx-c}}}{c^2d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-((d*(b*c^2 + a*d^2)*x)/Sqrt[-c + d*x]) + 2*b*c^(5/2)*Sqrt[1 + (d*x)/c]*Ar
cSinh[Sqrt[-c + d*x]/(Sqrt[2]*Sqrt[c])])/(c^2*d^3*Sqrt[c + d*x])
```

fricas [B] time = 1.41, size = 129, normalized size = 2.05

$$\frac{bc^4+ac^2d^2-\left(bc^2d+ad^3\right)\sqrt{dx+c}\,\sqrt{dx-c}\,x-\left(bc^2d^2+ad^4\right)x^2-\left(bc^2d^2x^2-bc^4\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{c^2d^5x^2-c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*c^4 + a*c^2*d^2 - (b*c^2*d + a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - (b*c
^2*d^2 + a*d^4)*x^2 - (b*c^2*d^2*x^2 - b*c^4)*log(-d*x + sqrt(d*x + c)*sqrt
(d*x - c)))/(c^2*d^5*x^2 - c^4*d^3)
```

giac [B] time = 0.28, size = 113, normalized size = 1.79

$$-\frac{b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3}-\frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)cd^3}-\frac{\left(bc^2d^3+ad^5\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^3 + a*d^5)*sqrt(d*x
 + c)/(sqrt(d*x - c)*c^2*d^6)
```

maple [C] time = 0.07, size = 160, normalized size = 2.54

$$\frac{\left(b\,c^2d^2x^2\ln\left(\left(dx+\sqrt{(dx-c)\,(dx+c)}\ \mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-\sqrt{d^2x^2-c^2}\ a\,d^3x\,\mathrm{csgn}(d)-b\,c^4\ln\left(\left(dx+\sqrt{(dx-c)}\right.\right.}{\sqrt{d^2x^2-c^2}\ \sqrt{dx+c}\ \sqrt{dx-c}\ c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (ln((csgn(d)*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*x^2*b*c^2*d^2-(d^2*x^2-c
^2)^(1/2)*a*d^3*x*csgn(d)-(d^2*x^2-c^2)^(1/2)*b*c^2*d*x*csgn(d)-ln((csgn(d)
*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/c
^2/d^3/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 0.55, size = 76, normalized size = 1.21

$$-\frac{ax}{\sqrt{d^2x^2-c^2}\,c^2}-\frac{bx}{\sqrt{d^2x^2-c^2}\,d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2+a}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [C] time = 112.36, size = 182, normalized size = 2.89

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}\right)+b\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},\frac{1}{2},1\\ -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1,0 & \end{matrix}\right.}{2\pi^{\frac{3}{2}}d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg(((-1/2, 0, 1/4, 1/2, 3/4
, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2
))/(2*pi**(3/2)*c**2*d)) + b*(meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-
1/4, 0, 1/4, 1/2, 1, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meij
erg(((-3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0))
, c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

3.373 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c^3}$$

```
[Out] -a*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^3+(-a/c^2-b/d^2)/(d*x-c)^(1/2)/(
d*x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {458, 92, 205}

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d*
x]*Sqrt[c + d*x])/c])/c^3
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 458

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*
c - a1*a2*d)*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p +
 1))/(a1*a2*b1*b2*e*n*(p + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n
*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p +
```

```
1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m,
n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !Inte
gerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p +
 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{c^2}\\
&= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{(ad)\,\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\,\sqrt{c+dx}\right)}{c^2}\\
&= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 1.29

$$-\frac{ad^2\sqrt{d^2x^2-c^2}\,\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)+acd^2+bc^3}{c^3d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((b*c^3 + a*c*d^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2*x^2]
/c])/(c^3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]))
```

fricas [A] time = 1.10, size = 101, normalized size = 1.55

$$-\frac{\left(bc^3+acd^2\right)\sqrt{dx+c}\,\sqrt{dx-c}\,+2\left(ad^4x^2-ac^2d^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(a*d^4*x^2 - a*c^2*d^2)
*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*x^2 - c^5*d^2)
```

giac [B] time = 0.36, size = 115, normalized size = 1.77

$$\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^3d^2}+\frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*c^2 + a*d^
2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x +
c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

maple [B] time = 0.08, size = 188, normalized size = 2.89

$$\frac{a\,d^4x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-a\,c^2d^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b}{\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,\sqrt{dx+c}\,\sqrt{dx-c}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*d^4-ln(-2*(c^2
-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^
(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(-c^2)^(1/2)/(d^2*x^2-c
^2)^(1/2)/d^2/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 1.46, size = 58, normalized size = 0.89

$$\frac{a\arcsin\left(\frac{c}{d|x|}\right)}{c^3}-\frac{a}{\sqrt{d^2x^2-c^2}\,c^2}-\frac{b}{\sqrt{d^2x^2-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(c/(d*abs(x)))/c^3 - a/(sqrt(d^2*x^2 - c^2)*c^2) - b/(sqrt(d^2*x^2
- c^2)*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2+a}{x\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [C] time = 136.44, size = 172, normalized size = 2.65

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & 1,2,\frac{5}{2}\\ \frac{5}{4},\frac{3}{2},\frac{7}{4},2,\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{2},\frac{3}{4},1,\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & 0,\frac{1}{2},\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((5/4, 7/4, 1), (1, 2, 5/2)), ((5/4, 3/2, 7/4, 2, 5/2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c**3) - I*meijerg(((0, 1/2, 3/4, 1, 5/4, 1),
 ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(
2*pi**(3/2)*c**3)) + b*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2,
3/4, 1, 3/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-
1, -1/2, -1/4, 0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp
_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2))
```

3.374 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{a}{c^2x\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

```
[Out] a/c^2/x/(d*x-c)^(1/2)/(d*x+c)^(1/2)-(2*a*d^2+b*c^2)*x/c^4/(d*x-c)^(1/2)/(d*
x+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {454, 39}

$$\frac{a}{c^2x\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c^4*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{c^2x\sqrt{-c+dx}\,\sqrt{c+dx}} + \left(b+\frac{2ad^2}{c^2}\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$
$$= \frac{a}{c^2x\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(bc^2+2ad^2\right)x}{c^4\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{a\left(c^2-2d^2x^2\right)-bc^2x^2}{c^4x\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.00, size = 103, normalized size = 1.54

$$-\frac{\left(bc^2d^2+2\,ad^4\right)x^3-\left(ac^2d-\left(bc^2d+2\,ad^3\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}-\left(bc^4+2\,ac^2d^2\right)x}{c^4d^3x^3-c^6dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] -((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*sqrt(d*x
+ c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)
```

giac [B] time = 0.46, size = 219, normalized size = 3.27

$$-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^4d}-\frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right.}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6+2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] -1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c^2*(sqrt(d
*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*a*
c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((sqrt(d
```

```
*x + c) - sqrt(d*x - c))^6 + 2*c*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2*
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 8*c^3)*c^3*d)
```

maple [A] time = 0.05, size = 48, normalized size = 0.72

$$\frac{-2a\,d^{2}x^{2}-b\,c^{2}x^{2}+a\,c^{2}}{\sqrt{dx+c}\,\sqrt{dx-c}\,c^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (-2*a*d^2*x^2-b*c^2*x^2+a*c^2)/(d*x+c)^(1/2)/x/c^4/(d*x-c)^(1/2)
```

maxima [A] time = 1.36, size = 71, normalized size = 1.06

$$-\frac{bx}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}}-\frac{2\,ad^{2}x}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{4}}+\frac{a}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*x/(sqrt(d^2*x^2 - c^2)*c^2) - 2*a*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) + a/(s
qrt(d^2*x^2 - c^2)*c^2*x)
```

mupad [B] time = 2.87, size = 73, normalized size = 1.09

$$\frac{2\,a\,d^{2}\,x^{2}\,\sqrt{d\,x-c}\,-a\,c^{2}\,\sqrt{d\,x-c}\,+b\,c^{2}\,x^{2}\,\sqrt{d\,x-c}}{c^{4}\,x\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (2*a*d^2*x^2*(d*x - c)^(1/2) - a*c^2*(d*x - c)^(1/2) + b*c^2*x^2*(d*x - c)^
(1/2))/(c^4*x*(c + d*x)^(1/2)*(c - d*x))
```

sympy [C] time = 136.13, size = 165, normalized size = 2.46

$$a\left(-\frac{dG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{3}{2},\frac{5}{2},3\\ \frac{7}{4},2,\frac{9}{4},\frac{5}{2},3 & 0\end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}c^{4}}+\frac{idG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},1,\frac{5}{4},\frac{3}{2},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & \frac{1}{2},1,2,0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}c^{4}}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c}{d^{2}}\right.}{2\pi^{\frac{3}{2}}c^{2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-d*meijerg((((7/4, 9/4, 1), (3/2, 5/2, 3)), ((7/4, 2, 9/4, 5/2, 3), (0,))
, c**2/(d**2*x**2))/(2*pi**(3/2)*c**4) + I*d*meijerg((((1/2, 1, 5/4, 3/2, 7/
4, 1), ()), ((5/4, 7/4), (1/2, 1, 2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2
))/(2*pi**(3/2)*c**4)) + b*(-meijerg((((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4,
 1, 5/4, 3/2, 2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg
(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp
_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d))
```

3.375 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{(3ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^5}-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{2c^2x^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

```
[Out] -1/2*(3*a*d^2+2*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^5+1/2*(-3*a*
d^2-2*b*c^2)/c^4/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/2*a/c^2/x^2/(d*x-c)^(1/2)/(d
*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 104, 21, 92, 205}

$$-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{(3ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^5}+\frac{a}{2c^2x^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(2*b*c^2 + 3*a*d^2)/(2*c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*c^2*x^2*Sq
rt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[-c + d*x]*S
qrt[c + d*x])/c])/(2*c^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{2}\left(2b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(-2b-\frac{3ad^2}{c^2}\right)\int \frac{cd+d^2}{x\sqrt{-c+dx}\,(c}}{2c^2d}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(2bc^2+3ad^2\right)\int \frac{1}{x\sqrt{-c+dx}}}{2c^4}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(d\left(2bc^2+3ad^2\right)\right)\text{Subst}}{}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(2bc^2+3ad^2\right)\tan^{-1}\left(\frac{\sqrt{-c}}{}\right.}{2c^5}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.64

$$\frac{ac^2 - x^2\left(3ad^2 + 2bc^2\right) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1 - \frac{d^2x^2}{c^2}\right)}{2c^4x^2\sqrt{dx - c}\sqrt{c + dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (a*c^2 - (2*b*c^2 + 3*a*d^2)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (d^2*x
^2)/c^2])/(2*c^4*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 1.07, size = 138, normalized size = 1.18

$$\frac{\left(ac^3 - \left(2\,bc^3 + 3\,acd^2\right)x^2\right)\sqrt{dx + c}\,\sqrt{dx - c} - 2\left(\left(2\,bc^2d^2 + 3\,ad^4\right)x^4 - \left(2\,bc^4 + 3\,ac^2d^2\right)x^2\right)\arctan\left(-\frac{dx - \sqrt{dx + c}\,\sqrt{d}}{c}\right.}{2\left(c^5d^2x^4 - c^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*c^3 - (2*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*((
2*b*c^2*d^2 + 3*a*d^4)*x^4 - (2*b*c^4 + 3*a*c^2*d^2)*x^2)*arctan(-(d*x - sq
rt(d*x + c)*sqrt(d*x - c))/c))/(c^5*d^2*x^4 - c^7*x^2)
```

giac [B] time = 0.54, size = 211, normalized size = 1.80

$$\frac{\left(2\,bc^2 + 3\,ad^2\right)\arctan\left(\frac{\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5} - \frac{\left(bc^2 + ad^2\right)\sqrt{dx + c}}{2\sqrt{dx - c}\,c^5} + \frac{2\left(bc^2 + ad^2\right)}{\left(\left(\sqrt{dx + c} - \sqrt{dx - c}\right)^2 + 2\,c\right)c^4} + \frac{2\left(ad^2\left(\sqrt{dx +\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 - 1
/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) + 2*(b*c^2 + a*d^2)/((
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4) + 2*(a*d^2*(sqrt(d*x + c) - s
qrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*
x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^4)
```

maple [B] time = 0.09, size = 315, normalized size = 2.69

$$\frac{3a\,d^4x^4\ln\left(-\frac{2\left(c^2 - \sqrt{-c^2}\,\sqrt{d^2x^2 - c^2}\right)}{x}\right) + 2b\,c^2d^2x^4\ln\left(-\frac{2\left(c^2 - \sqrt{-c^2}\,\sqrt{d^2x^2 - c^2}\right)}{x}\right) - 3a\,c^2d^2x^2\ln\left(-\frac{2\left(c^2 - \sqrt{-c^2}\,\sqrt{d^2x^2 - c^2}\right)}{x}\right) - 2b}{2\sqrt{-c^2}\,\sqrt{d^2x^2 - c^2}\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2/c^4*(3*a*d^4*x^4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)+2*b*c^
2*d^2*x^4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)-3*ln(-2*(c^2-(-c^
2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*c^2*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d
^2*x^2-c^2)^(1/2))/x)*x^2*b*c^4-3*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*d^2*x^
2-2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*b*c^2*x^2+(-c^2)^(1/2)*(d^2*x^2-c^2)^(
1/2)*a*c^2)/(-c^2)^(1/2)/x^2/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2
)
```

maxima [A] time = 1.26, size = 104, normalized size = 0.89

$$\frac{b \arcsin\left(\frac{c}{d|x|}\right)}{c^3} + \frac{3\,ad^2 \arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5} - \frac{b}{\sqrt{d^2x^2-c^2}\,c^2} - \frac{3\,ad^2}{2\,\sqrt{d^2x^2-c^2}\,c^4} + \frac{a}{2\,\sqrt{d^2x^2-c^2}\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*arcsin(c/(d*abs(x)))/c^3 + 3/2*a*d^2*arcsin(c/(d*abs(x)))/c^5 - b/(sqrt(d
^2*x^2 - c^2)*c^2) - 3/2*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4) + 1/2*a/(sqrt(d^2*
x^2 - c^2)*c^2*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{x^3\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.376 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

```
[Out] 1/3*a/c^2/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/3*(4*a*d^2+3*b*c^2)/c^4/x/(d*x-
c)^(1/2)/(d*x+c)^(1/2)-2/3*d^2*(4*a*d^2+3*b*c^2)*x/c^6/(d*x-c)^(1/2)/(d*x+c
)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 103, 12, 39}

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*S
qrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{3}\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{1}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{2d^2}{(-c+dx)^{3/2}(c+}}{3c^2}$$

$$= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(2d^2\left(3b+\frac{4ad^2}{c^2}\right)\right)\int \frac{}{(-c+d}}{3c^2}$$

$$= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{2d^2\left(3bc^2+4ad^2\right)x}{3c^6\sqrt{-c+dx}\,\sqrt{c+dx}}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.65

$$\frac{a\left(c^4+4c^2d^2x^2-8d^4x^4\right)+3bc^2x^2\left(c^2-2d^2x^2\right)}{3c^6x^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c^
6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 0.88, size = 132, normalized size = 1.11

$$-\frac{2\left(3\,bc^2d^3+4\,ad^5\right)x^5-2\left(3\,bc^4d+4\,ac^2d^3\right)x^3-\left(ac^4-2\left(3\,bc^2d^2+4\,ad^4\right)x^4+\left(3\,bc^4+4\,ac^2d^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{}}{3\left(c^6d^2x^5-c^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*b*c^2*d^3 + 4*a*d^5)*x^5 - 2*(3*b*c^4*d + 4*a*c^2*d^3)*x^3 - (a*
c^4 - 2*(3*b*c^2*d^2 + 4*a*d^4)*x^4 + (3*b*c^4 + 4*a*c^2*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^6*d^2*x^5 - c^8*x^3)
```

giac [B] time = 0.73, size = 242, normalized size = 2.03

$$-\frac{\left(bc^2d+ad^3\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^6}-\frac{2\left(bc^2d+ad^3\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^5}-\frac{8\left(3\,bc^2d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+3\,ad^3\left(\sqrt{dx+c}-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c^2*d + a*d
^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(sqrt(
d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 2
4*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*a*c^2*d^3*(sqrt(d*x + c) -
 sqrt(d*x - c))^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt(d*x
- c))^4 + 4*c^2)^3*c^4)
```

maple [A] time = 0.05, size = 73, normalized size = 0.61

$$\frac{-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4}{3\sqrt{dx+c}\,\sqrt{dx-c}\,c^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/3*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c^2*d^2*x^2+3*b*c^4*x^2+a*c^4)/(d*x+c
)^(1/2)/x^3/c^6/(d*x-c)^(1/2)
```

maxima [A] time = 1.34, size = 125, normalized size = 1.05

$$-\frac{2\,bd^2x}{\sqrt{d^2x^2-c^2}\,c^4}-\frac{8\,ad^4x}{3\,\sqrt{d^2x^2-c^2}\,c^6}+\frac{b}{\sqrt{d^2x^2-c^2}\,c^2x}+\frac{4\,ad^2}{3\,\sqrt{d^2x^2-c^2}\,c^4x}+\frac{a}{3\,\sqrt{d^2x^2-c^2}\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) - 8/3*a*d^4*x/(sqrt(d^2*x^2 - c^2)*c^6
) + b/(sqrt(d^2*x^2 - c^2)*c^2*x) + 4/3*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*x) +
 1/3*a/(sqrt(d^2*x^2 - c^2)*c^2*x^3)
```

mupad [B] time = 2.90, size = 104, normalized size = 0.87

$$\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c^2\,d}+\frac{x^2\,\left(3\,b\,c^4+4\,a\,c^2\,d^2\right)}{3\,c^6\,d}-\frac{x^4\,\left(6\,b\,c^2\,d^2+8\,a\,d^4\right)}{3\,c^6\,d}\right)}{x^4\,\sqrt{c+d\,x}-\frac{c\,x^3\,\sqrt{c+d\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c^2*d) + (x^2*(3*b*c^4 + 4*a*c^2*d^2))/(3*c^6*d) - (
x^4*(8*a*d^4 + 6*b*c^2*d^2))/(3*c^6*d)))/(x^4*(c + d*x)^(1/2) - (c*x^3*(c +
 d*x)^(1/2))/d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.377 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^7}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

```
[Out] -3/8*d^2*(5*a*d^2+4*b*c^2)*arctan((d*x-c)^(1/2)*(d*x+c)^(1/2)/c)/c^7-3/8*d^
2*(5*a*d^2+4*b*c^2)/c^6/(d*x-c)^(1/2)/(d*x+c)^(1/2)+1/4*a/c^2/x^4/(d*x-c)^(
1/2)/(d*x+c)^(1/2)+1/8*(5*a*d^2+4*b*c^2)/c^4/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2
)
```

Rubi [A] time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {454, 103, 12, 104, 21, 92, 205}

$$\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^7}+\frac{a}{4c^2x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(4*c^
2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*Sqrt[-
c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(8*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{4}\left(4b+\frac{5ad^2}{c^2}\right)\int \frac{1}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(4bc^2+5ad^2\right)\int \frac{}{x(-c+d}}{8c^4}$$

$$= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3d^2\left(4bc^2+5ad^2\right)\right)\int}{8c^4}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+d}}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+d}}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+d}}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+d}}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.47

$$\frac{ac^4-d^2x^4\left(5ad^2+4bc^2\right)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{d^2x^2}{c^2}\right)}{4c^6x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (a*c^4 - d^2*(4*b*c^2 + 5*a*d^2)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (d
^2*x^2)/c^2])/(4*c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

fricas [A] time = 0.69, size = 165, normalized size = 0.99

$$\frac{\left(2\,ac^5-3\left(4\,bc^3d^2+5\,acd^4\right)x^4+\left(4\,bc^5+5\,ac^3d^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}\,-6\left(\left(4\,bc^2d^4+5\,ad^6\right)x^6-\left(4\,bc^4d^2+5\,a\right.\right.}{8\left(c^7d^2x^6-c^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4)*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c^4*d^2 + 5*a*c^2*d^4)*x^4)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^7*d^2*x^6 - c^9*x^4)

giac [B] time = 0.79, size = 402, normalized size = 2.42

$$\frac{3\left(4\,bc^2d^2+5\,ad^4\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{4\,c^7}-\frac{\left(bc^2d^2+ad^4\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^7}+\frac{2\left(bc^2d^2+ad^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^6}+\frac{4\,bc^2d^2(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")

[Out] 3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x - c)*c^7) + 2*(b*c^2*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 7*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^2*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^6)

maple [B] time = 0.08, size = 387, normalized size = 2.33

$$\frac{15a\,d^6x^6\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+12b\,c^2d^4x^6\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-15a\,c^2d^4x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)

[Out] 1/8/c^6*(15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*a*d^6+12*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*b*c^2*d^4-15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*c^2*d^4-12*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^4*d^2-15*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^4*a*d^4-12*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^4*b*c^2*d^2+5*x^2*a*c^2*d^2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)+4*x^2*b*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)+2*a*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/(-c^2)^(1/2)/x^4/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2)

maxima [A] time = 1.48, size = 162, normalized size = 0.98

$$\frac{3\,bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5}+\frac{15\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^7}-\frac{3\,bd^2}{2\,\sqrt{d^2x^2-c^2}\,c^4}-\frac{15\,ad^4}{8\,\sqrt{d^2x^2-c^2}\,c^6}+\frac{b}{2\,\sqrt{d^2x^2-c^2}\,c^2x^2}+\frac{5\,ad^2}{8\,\sqrt{d^2x^2-c^2}\,c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*d^2*arcsin(c/(d*abs(x)))/c^5 + 15/8*a*d^4*arcsin(c/(d*abs(x)))/c^7 -
3/2*b*d^2/(sqrt(d^2*x^2 - c^2)*c^4) - 15/8*a*d^4/(sqrt(d^2*x^2 - c^2)*c^6)
+ 1/2*b/(sqrt(d^2*x^2 - c^2)*c^2*x^2) + 5/8*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*
x^2) + 1/4*a/(sqrt(d^2*x^2 - c^2)*c^2*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{x^5\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.378 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=40

$$\sqrt{cx-1}\,\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)$$

```
[Out] arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {460, 92, 205}

$$\sqrt{cx-1}\,\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \sqrt{-1+cx}\,\sqrt{1+cx} + \int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$
$$= \sqrt{-1+cx}\,\sqrt{1+cx} + c\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\,\sqrt{1+cx}\right)$$
$$= \sqrt{-1+cx}\,\sqrt{1+cx} + \tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.40

$$\frac{c^2x^2 + \sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right) - 1}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-1 + c^2*x^2 + Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

fricas [A] time = 1.24, size = 39, normalized size = 0.98

$$\sqrt{cx+1}\,\sqrt{cx-1} + 2\,\arctan\left(-cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) + 2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
```

giac [A] time = 0.17, size = 40, normalized size = 1.00

$$\sqrt{cx+1}\,\sqrt{cx-1} - 2\,\arctan\left(\frac{1}{2}\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^
2)
```

maple [A] time = 0.07, size = 53, normalized size = 1.32

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)+\sqrt{c^2x^2-1}\right)}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*((c^2*x^2-1)^(1/2)-arctan(1/(
c^2*x^2-1)^(1/2)))
```

maxima [A] time = 1.35, size = 23, normalized size = 0.58

$$\sqrt{c^2x^2-1}-\arcsin\left(\frac{1}{c|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c^2*x^2 - 1) - arcsin(1/(c*abs(x)))
```

mupad [B] time = 3.65, size = 72, normalized size = 1.80

$$\sqrt{c\,x-1}\,\sqrt{c\,x+1}-\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}+\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i - log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i + (c*x - 1)^(1/2)*(c*x + 1)^(1/2)
```

sympy [C] time = 30.11, size = 148, normalized size = 3.70

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| {\frac{1}{c^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} - \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| {\frac{1}{c^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \right.}{4 \pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()),
1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2,
 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((-1, -
3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar
(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2))
```

3.379 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\,\sqrt{a+bx}}\,dx$

Optimal. Leaf size=53

$$\sqrt{bx-a}\,\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

```
[Out] (c/a^2+d/b^2)*(b*x-a)^(1/2)*(b*x+a)^(1/2)/(x^(b^2*c/(a^2*d+b^2*c)))
```

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {450}

$$\sqrt{bx-a}\,\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[
a + b*x]),x]
```

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

Rule 450

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] &
& EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1
), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\,\sqrt{a+bx}}\,dx=\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\,\sqrt{a+bx}$$

Mathematica [C] time = 0.31, size = 244, normalized size = 4.60

$$\frac{\sqrt{1-\frac{b^2x^2}{a^2}}\left(a^2d+b^2c\right)x^{-\frac{b^2c}{a^2d+b^2c}}\left(b^2dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2da^2+b^2c}{2da^2+2b^2c};\frac{4da^2+3b^2c}{2da^2+2b^2c};\frac{b^2x^2}{a^2}\right)-\left(2a^2d+b^2c\right)\,{}_2F_1\left(\frac{1}{2},-\frac{b^2c}{2(da^2+b^2c)};\frac{2da^2+b}{2da^2+2}\right.\right.}{b^2\sqrt{bx-a}\sqrt{a+bx}\left(2a^2d+b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]
*Sqrt[a + b*x]),x]
```

```
[Out] ((b^2*c + a^2*d)*Sqrt[1 - (b^2*x^2)/a^2]*(-((b^2*c + 2*a^2*d)*Hypergeometri
c2F1[1/2, -1/2*(b^2*c)/(b^2*c + a^2*d), (b^2*c + 2*a^2*d)/(2*b^2*c + 2*a^2*
d), (b^2*x^2)/a^2]) + b^2*d*x^2*Hypergeometric2F1[1/2, (b^2*c + 2*a^2*d)/(2
*b^2*c + 2*a^2*d), (3*b^2*c + 4*a^2*d)/(2*b^2*c + 2*a^2*d), (b^2*x^2)/a^2])
)/(b^2*(b^2*c + 2*a^2*d)*x^((b^2*c)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[a
+ b*x])
```

fricas [A] time = 1.01, size = 65, normalized size = 1.23

$$\frac{\left(b^2c+a^2d\right)\sqrt{bx+a}\,\sqrt{bx-a}\,x}{a^2b^2x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*c + a^2*d)
/(b^2*c + a^2*d)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\sqrt{bx+a}\,\sqrt{bx-a}\,x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^
2*c + a^2*d))), x)
```

maple [A] time = 0.05, size = 66, normalized size = 1.25

$$\frac{\left(a^{2} d+b^{2} c\right) \sqrt{b x+a}\, \sqrt{b x-a}\, x\, x^{-\frac{a^{2} d+2 b^{2} c}{a^{2} d+b^{2} c}}}{a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/
2),x)
```

```
[Out] x*(a^2*d+b^2*c)*(b*x+a)^(1/2)/b^2/a^2/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))*(
b*x-a)^(1/2)
```

maxima [A] time = 1.24, size = 79, normalized size = 1.49

$$\frac{\left(b^{2} c+a^{2} d\right) \sqrt{b x+a}\, \sqrt{b x-a}\, x e^{\left(-\frac{2 b^{2} c \log(x)}{b^{2} c+a^{2} d}-\frac{a^{2} d \log(x)}{b^{2} c+a^{2} d}\right)}}{a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/(b^2*c + a
^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

mupad [B] time = 3.27, size = 96, normalized size = 1.81

$$-\frac{\frac{x\left(d\, a^{4}+c\, a^{2}\, b^{2}\right)}{a^{2}\, b^{2}}-\frac{x^{3}\left(d\, a^{2}\, b^{2}+c\, b^{4}\right)}{a^{2}\, b^{2}}}{x^{\frac{d\, a^{2}+2\, c\, b^{2}}{d\, a^{2}+c\, b^{2}}}\, \sqrt{a+b\, x}\, \sqrt{b\, x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x
 - a)^(1/2)),x)
```

```
[Out] -((x*(a^4*d + a^2*b^2*c))/(a^2*b^2) - (x^3*(b^4*c + a^2*b^2*d))/(a^2*b^2))/
(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/2
)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.380 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}\ \sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{1-x}\ \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

```
[Out] arcsin(x)*(1-x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {519, 41, 216}

$$\frac{\sqrt{1-x}\ \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}\,\sqrt{1+x}}\,dx = \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$
$$= \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x^2}}\,dx}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$
$$= \frac{\sqrt{1-x}\,\sin^{-1}(x)}{\sqrt{-1-\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{\sqrt{1-x}\,\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\,\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

fricas [C] time = 0.92, size = 69, normalized size = 1.92

$$-i\log\left(\frac{\sqrt{x+1}\,\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}+i\,x-1}{x}\right)+i\log\left(\frac{\sqrt{x+1}\,\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}-i\,x-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] -I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I*
log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}\,\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}\,\sqrt{-\sqrt{x}-1}\,\sqrt{\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

```
[Out] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}\,\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\,\sqrt{\sqrt{x}-1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

3.381 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

```
[Out] -2*arctan((-b^2*x+a^2)^(1/2)/(b^2*x+a^2)^(1/2))*(-b^2*x+a^2)^(1/2)/b^2/(a-b
*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {519, 63, 217, 203}

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx &= \frac{\sqrt{a^2-b^2x}\int \frac{1}{\sqrt{a^2-b^2x}\,\sqrt{a^2+b^2x}}\,dx}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\\
&= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a^2-x^2}}\,dx, x, \sqrt{a^2-b^2x}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\\
&= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}\\
&= -\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 1.00

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

fricas [A] time = 1.32, size = 50, normalized size = 0.67

$$-\frac{2 \arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a
))/(b^2*x))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{-b\sqrt{x}+a}\sqrt{b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(b*x^(1/2)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}\,\sqrt{a-b\sqrt{x}}\,\sqrt{a^2+x\,b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2
),x)
```

```
[Out] Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)), x
)
```

3.382 $\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx$

Optimal. Leaf size=113

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\left(c+dx^{2n}\right)^q\left(\frac{dx^{2n}}{c}+1\right)^{-q}F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q*AppellF1(1/2/n,-p,-q,1+1/2/n,b^2*
x^(2*n)/a^2,-d*x^(2*n)/c)/((1-b^2*x^(2*n)/a^2)^p)/((1+d*x^(2*n)/c)^q)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {519, 430, 429}

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\left(c+dx^{2n}\right)^q\left(\frac{dx^{2n}}{c}+1\right)^{-q}F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n), -p, -q,
(2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/((1 - (b^2*x^(2*n))/a
^2)^p*(1 + (d*x^(2*n))/c)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
```

```
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(c+dx^{2n}\right)^q \, dx$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(c+dx^{2n}\right)^q \, dx$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q}\right) \int \left(1-\frac{b}{}\right.$$
$$= x (a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q} F_1\left(\frac{1}{2n};-\right.$$

Mathematica [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
[Out] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q, x]
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="fricas")
[Out] integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="giac")
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

maple [F] time = 1.60, size = 0, normalized size = 0.00

$$\int (-b\,x^n+a)^p\,(b\,x^n+a)^p\left(d\,x^{2n}+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^n+a)^p*(b*x^n+a)^p*(c+d*x^(2*n))^q,x)
[Out] int((-b*x^n+a)^p*(b*x^n+a)^p*(c+d*x^(2*n))^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx^{2\,n}+c\right)^q(bx^n+a)^p(-bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="maxima")
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c+d\,x^{2\,n}\right)^q\,(a+b\,x^n)^p\,(a-b\,x^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p,x)
[Out] int((c + d*x^(2*n))^q*(a + b*x^n)^p*(a - b*x^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
[Out] Timed out
```

3.383 $\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=87

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

```
[Out] x*(a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p*hypergeom([-p, 1/4/n],[1+1/4/
n],b^4*x^(4*n)/a^4)/((1-b^4*x^(4*n)/a^4)^p)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {519, 253, 246, 245}

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(a^2+b^2x^{2n}\right)^p$$

$$= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(a^4-b^4x^{4n}\right)^{-p}\right) \int \left(a^4-b^4x^{4n}\right)^p$$

$$= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}\right) \int \left(1-\frac{b^4x^{4n}}{a^4}\right)^p$$

$$= x (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(\right.\right.$$

Mathematica [A] time = 0.05, size = 87, normalized size = 1.00

$$x (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;1+\frac{1}{4n};\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, 1 + 1/(4*n), (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^{2n}+a^2\right)^p (bx^n+a)^p (-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{2n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int (-b\,x^n + a)^p\,(b\,x^n + a)^p\left(b^2x^{2n} + a^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^n+a)^p*(b*x^n+a)^p*(a^2+b^2*x^(2*n))^p,x)
[Out] int((-b*x^n+a)^p*(b*x^n+a)^p*(a^2+b^2*x^(2*n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^{2n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="maxima"
)
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x^n)^p\,(a - b\,x^n)^p\left(a^2 + b^2\,x^{2\,n}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x)
[Out] int((a + b*x^n)^p*(a - b*x^n)^p*(a^2 + b^2*x^(2*n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
[Out] Timed out
```

3.384 $\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx$

Optimal. Leaf size=76

$$\frac{x\left(c+dx^{2n}\right)^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

```
[Out] x*(c+d*x^(2*n))^p*AppellF1(1/2/n,1,-p,1+1/2/n,b^2*x^(2*n)/a^2,-d*x^(2*n)/c)
/a^2/((1+d*x^(2*n)/c)^p)
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {517, 430, 429}

$$\frac{x\left(c+dx^{2n}\right)^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))
/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
```

```
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^{2n}\right)^p}{\left(a-bx^n\right)\left(a+bx^n\right)}\,dx &= \int \frac{\left(c+dx^{2n}\right)^p}{a^2-b^2x^{2n}}\,dx \\
&= \left(\left(c+dx^{2n}\right)^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}\right)\int \frac{\left(1+\frac{dx^{2n}}{c}\right)^p}{a^2-b^2x^{2n}}\,dx \\
&= \frac{x\left(c+dx^{2n}\right)^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}
\end{aligned}
$$

Mathematica [B] time = 0.43, size = 258, normalized size = 3.39

$$
\frac{a^2c(2n+1)x\left(c+dx^{2n}\right)^pF_1\left(\frac{1}{2n};-p,1;1+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)}{\left(a^2-b^2x^{2n}\right)\left(2a^2dnpx^{2n}F_1\left(1+\frac{1}{2n};1-p,1;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+2b^2cnx^{2n}F_1\left(1+\frac{1}{2n};-p,2;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)\right.}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n),
-((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*n))*(2*a^2*d*n*p*x^(
2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^
(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*AppellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n), -
((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n), -p,
 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(-\frac{\left(dx^{2n}+c\right)^p}{b^2x^{2n}-a^2},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^(2*n) + c)^p/(b^2*x^(2*n) - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^{2n}+c\right)^p}{(bx^n+a)(bx^n-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="giac")
[Out] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

maple [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{2n}+c\right)^p}{(-b\,x^n+a)\,(b\,x^n+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^(2*n)+c)^p/(-b*x^n+a)/(b*x^n+a),x)
[Out] int((d*x^(2*n)+c)^p/(-b*x^n+a)/(b*x^n+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^{2n}+c\right)^p}{(bx^n+a)(bx^n-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="maxima")
[Out] -integrate((d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int -\frac{\left(c+d\,x^{2\,n}\right)^p}{a^2-b^2\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^(2*n))^p/((a + b*x^n)*(a - b*x^n)),x)
[Out] -int(-(c + d*x^(2*n))^p/(a^2 - b^2*x^(2*n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.385 $\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}$

Optimal. Leaf size=96

$$-\frac{b^2x(np+n+1)\left(a-bx^{n/2}\right)^{p+1}\left(a+bx^{n/2}\right)^{p+1}\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

```
[Out] -b^2*(n*p+n+1)*x*(a-b*x^(1/2*n))^(1+p)*(a+b*x^(1/2*n))^(1+p)/a^4/d/n/(1+p)/
((-a^2*d*n*(1+p)/b^2/(n*p+n+1)+d*x^n)^((n*p+n+1)/n))
```

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {519, 381}

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2*
n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^((1 +
n + n*p)/n)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
```

```
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \left(\left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(a^2-b^2x^n\right)^{-p}\right)\int \left(a\right.$$

$$= -\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(a^2\right.}{a^4dn(1+p}$$

Mathematica [A] time = 0.39, size = 103, normalized size = 1.07

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(d\left(x^n-\frac{a^2n(p+1)}{b^2(np+n+1)}\right)\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(d*(-((a^2*n*(1 + p))/(b^2*(1 + n + n*p))) + x^n))^((1 +
n + n*p)/n)))
```

fricas [A] time = 1.24, size = 180, normalized size = 1.88

$$\frac{\left(\left(b^4np+b^4n+b^4\right)xx^{2\,n}-\left(2\,a^2b^2np+2\,a^2b^2n+a^2b^2\right)xx^n+\left(a^4np+a^4n\right)x\right)\left(bx^{\frac{1}{2}\,n}+a\right)^p\left(-bx^{\frac{1}{2}\,n}+a\right)^p}{\left(a^4np+a^4n\right)\left(-\frac{a^2dnp+a^2dn-\left(b^2dnp+b^2dn+b^2d\right)x^n}{b^2np+b^2n+b^2}\right)^{\frac{np+2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="fricas")
```

```
[Out] ((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n + a^2*b^2
)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/((a
```

```
^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2*d*n*p + b^2*d*n + b^2*d)*x^n)
/(b^2*n*p + b^2*n + b^2))^((n*p + 2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

maple [F] time = 2.17, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{\frac{n}{2}}+a\right)^{p}\left(b\,x^{\frac{n}{2}}+a\right)^{p}\left(d\,x^{n}+\frac{\left(p+1\right)a^{2}d}{\left(\frac{-np-2n-1}{n}+1\right)b^{2}}\right)^{\frac{-np-2n-1}{n}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(p+1)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

```
[Out] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(p+1)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x^{n/2}\right)}^p\,{\left(a-b\,x^{n/2}\right)}^p}{{\left(d\,x^n-\frac{a^2\,d\,\left(p+1\right)}{b^2\,\left(\frac{2\,n+n\,p+1}{n}-1\right)}\right)}^{\frac{2\,n+n\,p+1}{n}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n),x)
```

```
[Out] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-n
*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```